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Likelihood Ratio Determination for Stochastic Processes Recognition Problem with Respect to the Set of Continuous and Discrete Memory Observations

Nikolas DYOMIN
Department of Applied Mathematics and Cybernetics, Tomsk State University
36 Lenin ave., Tomsk, Russia
e-mail: svr@hm.tpu.ru

Svetlana ROZHKOVA, Olga ROZHKOVA
Department of Natural Sciences and Mathematics, Tomsk Politechnical University
30 Lenin ave., Tomsk, Russia
e-mail: svr@hm.tpu.ru

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Abstract. This paper considers the problem of likelihood ratio determination for recognition of the stochastic processes with continuous time on the set continuous and discrete time memory observations. The research of memory influence on the detection quality of anomalous noises in the discrete channel observation with applying the general obtained results is realized for the one particular problem.

Key words: likelihood ratio, recognition, a posteriori density function, a posteriori probalitity, filtering, estimation.

1. Introduction

For the recognition problem the likelihood ratio on the class of nonrandomized Bayes strategies are sufficient statistics for construction of the decision rules (Middleton, 1960; Van Trees, 1971; Sage and Melse, 1972; Fukunaga, 1972). Therefore determination of likelihood ratio is a basic problem for the given problem solution. The modern stage in the synthesis theory of the algorithms for stochastic process treatment began from papers (Kalman, 1960; 1961). In the Kalman systems the pair of processes $\{x_t; y_t\}$ with continuous and discrete time, where x_t is an unobservable process, and y_t is an observable process, is the basic mathematical object. For the case where x_t and y_t are the processes with the discrete time and x_t is a Gaussian process, the problem of likelihood ratio determination in the detection problem, which is a particular case of recognition problem, was considered in (Schweppe, 1965; Mc Lenndon and Sage, 1970; Sage and Melse, 1972).

For the case where x_t and y_t are the discrete time processes and x_t is the Gaussian process, the likelihood ratio determination problem in the detection problem was considered in (Kailath, 1969; Sage and Melse, 1972). The likelihood ratio determination problem in the general problem of recognition was considered in (Willsky and Jones, 1976) for the case where x_t and y_t are the discrete time processes, in (Grene, 1978) for the case where x_t and y_t are the continuous time processes. The likelihood ratio determination problem where $y_t = y(t, t_m) = \{z_t, \eta(t_m)\}$, i.e., the observable process is a set of the processes with continuous z_t and discrete $\eta(t_m)$ time, was considered in (Dyomin, 1979).

A new class of problems is about the situation when the observable processes z_t and $\eta(t_m)$ possess the memory relatively unobservable process, i.e., z_t and $\eta(t_m)$ depend not only on the current values but also on the arbitrary number N of the former values processes x_t. For the similar class of processes $\{x_t; z_t; \eta(t_m)\}$ and for the single multiplicity memory ($N = 1$) the filtering problem was considered in (Kulman and Hametv, 1978; Dyomin, 1987), the extrapolation problem was considered in (Dyomin, 1992), the problem of transmission schotastic signals through continuous-discrete channels was considered in (Dyomin and Korotkevich, 1987), the recognition problem was considered in (Dyomin, 1985). For the arbitrary multiplicity memory, where $N \geqslant 1$ the filtering problem was considered in (Abakumova *et al.*, 1995a; 1995b) and the extrapolation problem was considered in (Dyomin *et al.*, 1997; Dyomin *et al.*, 1999). The present paper considers the likelihood ratio determination problem in the general recognition problem of arbitrary quantity of hypotheses, when the unobservable process x_t is the process with continuous time and the observable process is a set of the processes with continuous z_t and discrete $\eta(t_m)$ time with arbitrary multiplicity memory. This paper together with (Abakumova *et al.*, 1995a; 1995b; Dyomin *et al.*, 1997; Dyomin *et al.*, 1999; Dyomin and Rozhkova, 1999) solves problems of optimal algorithms synthesis for treatment of the continuous time stochastic processes with respect to continuous-discrete observations with the arbitrary multiplicity memory.

Used notations: $\mathcal{P}\{\cdot\}$ is event probability; $M\{\cdot\}$ is expectation; $\mathcal{N}\{y; a, B\}$ detotes Gaussian probability density function with given parameters a and B; I and O are single and zero matrix of appropriate sizes; $\ll T \gg$ denote transpose of a matrix, as the upper right index; $|\cdot|$ and $\mathrm{tr}[\cdot]$ are determinant and trace of matrix; $B > 0$ $(B < 0; B \geqslant 0)$ are positively (negatively, nonnegatively) defined matrix.

2. Problem Formulation

Let the unobservable n-dimensional process x_t and the observable l-dimensional process z_t with continuous time denoted by the stochastic differential equations

$$dx_t = f(t, x_t, z_t, \theta)dt + \Phi_1(t, x_t, z_t, \theta)dw_t, \quad t \geqslant 0, \tag{2.1}$$

$$dz_t = h(t, x_t, x_{\tau_1}, \cdots, x_{\tau_N}, z_t, \theta)dt + \Phi_2(t, z_t)dv_t, \tag{2.2}$$

and the observable q-dimensional discrete time process $\eta(t_m)$ is expressed by

$$\eta(t_m) = g(t_m, x_{t_m}, x_{\tau_1}, \cdots, x_{\tau_N}, \theta) + \xi(t_m), \quad m = 0, 1, \ldots, \tag{2.3}$$

where $0 \leqslant t_0 < \tau_N < \cdots < \tau_1 < t_m \leqslant t$. The parameter θ is the hypothesis identifier and can accept values from the set $\Omega_\theta = \{\theta_0, \theta_1, \cdots, \theta_r\}$ with the a priori probabilities $p_0(\theta_j) = \mathcal{P}\{\theta = \theta_j\}$, $j = \overline{0;r}$. It is assumed: 1) w_t and v_t are standard Wiener processes of sizes r_1 and r_2; 2) for all $\theta \in \Omega_\theta$ coefficients of the equations (2.1), (2.2) satisfy to conditions (Liptser and Shiryayev, 1977; 1978; Kallianpur, 1980) and $g(\cdot)$ is continuous for all arguments; 3) $\xi(t_m)$ is q-dimensional Gaussian sequence with $M\{\xi(t_m)|\theta = \theta_j\} = b_j(t_m)$, $M\{\xi(t_m)\xi^T(t_m)|\theta = \theta_j\} = V(t_m, \theta)$ and x_0, w_t, v_t, $\xi(t_m)$, θ are assumed to be statistically independent; 4) $Q(\cdot) = \Phi_1(\cdot)\Phi_1^T(\cdot) > 0$, $R(\cdot) = \Phi_2(\cdot)\Phi_2^T(\cdot) > 0$, $V(\cdot) > 0$ for all $\theta \in \Omega_\theta$; 5) the initial density functions $p_0(x_0|\theta_j) = \partial\mathcal{P}\{x_0 \leqslant x|\theta = \theta_j\}/\partial x$, $j = \overline{0;r}$ are given. The problem is raised as follows: with respect to the realizations set $z_0^t = \{z(\sigma);\ 0 \leqslant \sigma \leqslant t\}$ and $\eta_0^m = \{\eta(t_0), \eta(t_1), \ldots, \eta(t_m)\}$ find the likelihood ratio $\Lambda_t(\theta_j : \theta_\alpha)$ for the hypotheses recognition problem $\mathcal{H}_j\{\theta = \theta_j\}$ and $\mathcal{H}_\alpha\{\theta = \theta_\alpha\}$, $j = \overline{0;r}$, $\alpha = \overline{0;r}$.

The models of the processes z_t and $\eta(t_m)$ of (2.2), (2.3) are adequate to the observations with fixed memory if $\tau_k = \mathrm{const}$ and observations with sliding memory if $\tau_k = t - t_k^*$ in (2.2) and $\tau_k = t_m - t_k^*$ in (2.3), where $t_k^* = \mathrm{const}$, $k = \overline{1;N}$. The present problem is considered for the case of fixed memory.

3. The General Relations

With a view of compact representation of mathematical expressions we shall enter extended processes $\widetilde{x}_\tau^N$, $\widetilde{x}_{t,\tau}^{N+1}$ and variables $\widetilde{x}_N$, $\widetilde{x}_{N+1}$ as

$$\widetilde{x}_\tau^N = \begin{bmatrix} x_{\tau_1} \\ x_{\tau_2} \\ \vdots \\ x_{\tau_N} \end{bmatrix}, \quad \widetilde{x}_{t,\tau}^{N+1} = \begin{bmatrix} x_t \\ \widetilde{x}_\tau^N \end{bmatrix}, \quad \widetilde{x}_N = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}, \quad \widetilde{x}_{N+1} = \begin{bmatrix} x \\ \widetilde{x}_N \end{bmatrix}. \tag{3.1}$$

The method of evalution $\Lambda_t(\theta_j : \theta\alpha)$ is based on the formula [1]

$$\Lambda_t(\theta_j : \theta_\alpha) = [p_0(\theta_\alpha)/p_0(\theta_j)]P_t(\theta_j : \theta_\alpha), \tag{3.2}$$

which unite the likelihood ratio and the a posteriori probabilities ratio

$$P_t(\theta_j : \theta_\alpha) = p_t(\theta_j)/p_t(\theta_\alpha), \tag{3.3}$$

where

$$p_t(\theta_j) = \mathcal{P}\{\theta = \theta_j | z_0^t, \eta_0^m\}, \quad j = \overline{0;r}. \tag{3.4}$$

PROPOSITION 1. The a posteriori probabilities of hypotheses $\mathcal{H}_j\{\theta = \theta_j\}$ for $t_m \leqslant t < t_{m+1}$ are denoted by the equations

$$d_t p_t(\theta_j) = p_t(\theta_j)\big[\overline{h(t, z_t|\theta j)} - \overline{h(t, z_t)}\big]^T R^{-1}(t, z_t)[dz_t - \overline{h(t, z_t)}dt], \tag{3.5}$$

with the initial conditions

$$p_{t_m}(\theta_j) = \Big[C\big(\eta(t_m), z_{t_m}|\theta_j\big)/C\big(\eta(t_m), z_{t_m}\big)\Big]p_{t_m-0}(\theta_j), \tag{3.6}$$

where

$$\overline{h(t, z_t|\theta_j)} = M\big\{h(t, x_t, \widetilde{x}_\tau^N, z_t, \theta)|\theta = \theta_j, z_0^t, \eta_0^m\big\}, \tag{3.7}$$

$$\overline{h(t, z_t)} = M\big\{h(t, x_t, \widetilde{x}_\tau^N, z_t, \theta)|z_0^t, \eta_0^m\big\}, \tag{3.8}$$

$$C\big(\eta(t_m), z_{t_m}|\theta_j\big) = M\Big\{C\big(\eta(t_m), z_{t_m}, x_{t_m}, \widetilde{x}_\tau^N, \theta\big)|\theta = \theta_j, z_0^{t_m}, \eta_0^{m-1}\Big\}, \tag{3.9}$$

$$C\big(\eta(t_m), z_{t_m}\big) = M\Big\{C\big(\eta(t_m), z_{t_m}, x_{t_m}, \widetilde{x}_\tau^N, \theta\big)|z_0^{t_m}, \eta_0^{m-1}\Big\}, \tag{3.10}$$

$$\begin{aligned} &C\big(\eta(t_m), z_{t_m}, x, \widetilde{x}_N, \theta_j\big)\\ &\quad = \big|V(t_m, \theta_j)\big|^{-1/2} \exp\bigg\{-\frac{1}{2}\big[\eta(t_m) - b_j(t_m) - g(t_m, x, \widetilde{x}_N, z_{t_m}, \theta_j)\big]^T\\ &\qquad \times V^{-1}(t_m, \theta_j)\big[\eta(t_m) - b_j(t_m) - g(t_m, x, \widetilde{x}_N, z_{t_m}, \theta_j)\big]\bigg\}, \end{aligned} \tag{3.11}$$

and $p_{t_m-0}(\theta_j) = \lim p_t(\theta_j)$ subject to $t \uparrow t_m$.

Proof. For

$$p_t(x, \widetilde{x}_N, \theta_j) = \partial^{N+1}\mathcal{P}\{x_t \leqslant x, \widetilde{x}_\tau^N \leqslant \widetilde{x}_N, \theta = \theta_j|z_0^t, \eta_0^m\}/\partial x\partial\widetilde{x}_N \tag{3.12}$$

on the time intervals $t_m \leqslant t < t_{m+1}$ we have the equation

$$\begin{aligned} d_t p_t(x, \widetilde{x}_N, \theta_j) \;=\; & L_{t,x}\big[p_t(x, \widetilde{x}_N, \theta_j)\big]dt + p_t(x, \widetilde{x}_N, \theta_j)\big[h(t, x, \widetilde{x}_N, z_t, \theta_j)\\ & - \overline{h(t, z_t)}\big]^T R^{-1}(t, z_t)\big[dz_t - \overline{h(t, z_t)}dt\big], \end{aligned} \tag{3.13}$$

where $L_{t,x}[\cdot]$ denotes Kolmogorov operator and is adequate to to the process x_t subject to $\theta = \theta_j$. The equation (3.13) follows from the equation (II 26) in (Abakumova *et al.*, 1995a). Since

$$p_t(x, \widetilde{x}_N, \theta_j) = p_t(x, \widetilde{x}_N|\theta_j)p_t(\theta_j), \tag{3.14}$$

where

$$p_t(x, \widetilde{x}_N|\theta_j) = \partial^{N+1}\mathcal{P}\big\{x_t \leqslant x, \widetilde{x}_\tau^N \leqslant \widetilde{x}_N|\theta = \theta_j, z_0^t, \eta_0^m\big\}/\partial x\partial\widetilde{x}_N, \tag{3.15}$$

then integrating (3.13) with respect to $\{x, \widetilde{x}_N\}$ taking into account (3.14) yields (3.5). By the use of the Baies formula reduce

$$p_{t_m}(x, \widetilde{x}_N, \theta_j) =$$

$$= \frac{p(\eta(t_m)|x_{t_m}=x, \widetilde{x}_\tau^N=\widetilde{x}_N, \theta=\theta_j, z_0^{t_m}, \eta_0^{m-1})p_{t_m-0}(x,\widetilde{x}_N,\theta_j)}{\sum\limits_{k=0}^{N}\int p\big(\eta(t_m)|x_{t_m}=x, \widetilde{x}_\tau^N=\widetilde{x}_N, \theta=\theta_k, z_0^{t_m}, \eta_0^{m-1}\big)p_{t_m-0}(x,\widetilde{x}_N,\theta_k)\,\mathrm{d}x\mathrm{d}\widetilde{x}_N}, \tag{3.16}$$

where $p_{t_m-0}(x,\widetilde{x}_N,\theta_j) = \lim p_t(x,\widetilde{x}_N,\theta_j)$ subject to $t \uparrow t_m$. From (2.3) taking into account suppositions 2), 3) reduce

$$\begin{aligned} &p\big(\eta(t_m)|x,\widetilde{x}_N,\theta_j,z_0^{t_m},\eta_0^{m-1}\big) \\ &\quad = \mathcal{N}\big\{\eta(t_m); b_j(t_m) + g(t_m,x,\widetilde{x}_N,z_{t_m},\theta_j), V(t_m,\theta_j)\big\}. \end{aligned} \tag{3.17}$$

Making use of (3.17) in (3.16) taking into account (3.10), (3.11) yields

$$\begin{aligned} &p_{t_m}(x,\widetilde{x}_N,\theta_j) \\ &\quad = \Big[C\big(\eta(t_m),z_{t_m},x,\widetilde{x}_N,\theta_j\big)/C\big(\eta(t_m),z_{t_m}\big)\Big]p_{t_m-0}(x,\widetilde{x}_N,\theta_j). \end{aligned} \tag{3.18}$$

Integrating (3.18) with respect to $\{x,\widetilde{x}_N\}$ taking into account (3.9), (3.14) yields (3.6).

COROLLARY 1. For $p_t(\theta_j)$ we have

$$\begin{aligned} p_t(\theta_j) = p_{\tau_1-0}(\theta_j)\exp\bigg\{ &\sum_{\tau_1\leqslant t_i\leqslant t} \ln\left[\frac{C(\eta(t_i),z_{t_i}|\theta_j)}{C(\eta(t_i),z_{t_i})}\right] \\ +\int_{\tau_1}^{t}&\Big[\overline{h(s,z_s|\theta_j)}-\overline{h(s,z_s)}\Big]^T R^{-1}(s,z_s)\Big[\mathrm{d}z_s-\frac{1}{2}\overline{h(s,z_s|\theta_j)}\mathrm{d}s-\frac{1}{2}\overline{h(s,z_s)}\mathrm{d}s\Big]\bigg\}. \end{aligned} \tag{3.19}$$

Proof. Let us denote $\widetilde{p}_t(\theta_j) = \ln\{p_t(\theta_j)\}$. As process $\widetilde{z}_t$, differential of which has representation

$$d\widetilde{z}_t = dz_t - \overline{h(t,z_t)}dt, \tag{3.20}$$

is the Wiener process (Liptser and Shiryayev, 1977; 1978; Kallianpur, 1980) that differentiating according to Ito formula, taking into account (3.5) yields for $t_m \leqslant t < t_{m+1}$

$$\begin{aligned} d_t\widetilde{p}_t(\theta_j) = &\Big[\overline{h(t,z_t|\theta j)} - \overline{h(t,z_t)}\Big]^T R^{-1}(t,z_t) \\ &\times \left[dz_t - \frac{1}{2}\overline{h(t,z_t|\theta_j)}dt - \frac{1}{2}\overline{h(t,z_t)}dt\right]. \end{aligned} \tag{3.21}$$

From (3.6) it follows that

$$\widetilde{p}_{t_m}(\theta_j) = \widetilde{p}_{t_m-0}(\theta_j) + \ln\frac{C\big(\eta(t_m),z_{t_m}|\theta_j\big)}{C\big(\eta(t_m),z_{t_m}\big)}. \tag{3.22}$$

As $p_t(\theta_j) = \exp\{\widetilde{p}_t(\theta_j)\}$ then (3.19) for $t \geqslant \tau_1$ follows from (3.21), (3.22).

Theorem 1. *The likelihood ratio* $\Lambda_t(\theta_j : \theta_\alpha)$ *in the problem of hypotheses recognition* $\mathcal{H}_j\{\theta = \theta_j\}$ *and* $\mathcal{H}_\alpha\{\theta = \theta_\alpha\}$, $j = \overline{0;r}$, $\alpha = \overline{0;r}$, *subject to* $t_m \leqslant t < t_{m+1}$ *is denoted by the equation*

$$
\begin{aligned}
d_t\Lambda_t(\theta_j : \theta_\alpha) \;=\; & \Lambda_t(\theta_j : \theta_\alpha)\big[\overline{h(t, z_t|\theta j)} - \overline{h(t, z_t|\theta_\alpha)}\big]^T \\
& \times R^{-1}(t, z_t)\big[dz_t - \overline{h(t, z_t|\theta_\alpha)}dt\big],
\end{aligned} \tag{3.23}
$$

with the initial condition

$$
\Lambda_{t_m}(\theta_j : \theta_\alpha) = \Big[C\big(\eta(t_m), z_{t_m}|\theta_j\big)/C\big(\eta(t_m), z_{t_m}|\theta_\alpha\big)\Big]\Lambda_{t_m-0}(\theta_j : \theta_\alpha), \tag{3.24}
$$

where $\Lambda_{t_m-0}(\theta_j : \theta_\alpha) = \lim \Lambda_t(\theta_j : \theta_\alpha)$ *subject to* $t \uparrow t_m$.

Proof. Differentiating (3.3) arcording to Ito formula taking into account (3.5), (3.20) yields

$$
\begin{aligned}
d_tP_t(\theta_j : \theta_\alpha) \;=\; & P_t(\theta_j : \theta_\alpha)\big[\overline{h(t, z_t|\theta_j)} - \overline{h(t, z_t|\theta_\alpha)}\big]^T \\
& \times R^{-1}(t, z_t)\big[dz_t - \overline{h(t, z_t|\theta_\alpha)}dt\big].
\end{aligned} \tag{3.25}
$$

Substituting (3.2) into (3.25) yields (3.23). Substituting (3.6) into (3.3) taking into account (3.2) we have (3.24).

COROLLARY 2. For $\Lambda_t(\theta_j : \theta_\alpha)$ we have

$$
\begin{aligned}
\Lambda_t(\theta_j : \theta_\alpha) \;=\; & \Lambda_{\tau_1-0}(\theta_j : \theta_\alpha)\exp\Bigg\{\sum_{\tau_1 \leqslant t_i \leqslant t} \ln\left[\frac{C(\eta(t_i), z_{t_i}|\theta_j)}{C(\eta(t_i), z_{t_i}|\theta_\alpha)}\right] \\
& + \int_{\tau_1}^{t} \big[\overline{h(s, z_s|\theta_j)} - \overline{h(s, z_s|\theta_\alpha)}\big]^T R^{-1}(s, z_s) \\
& \times \left[\mathrm{d}z_s - \frac{1}{2}\overline{h(s, z_s|\theta_j)}\mathrm{d}s - \frac{1}{2}\overline{h(s, z_s|\theta_\alpha)}\mathrm{d}s\right]\Bigg\}.
\end{aligned} \tag{3.26}
$$

Proof. Equation (3.23) is rewritten as

$$
\begin{aligned}
d_t\Lambda_t(\theta_j : \theta_\alpha) \;=\; & \Lambda_t(\theta_j : \theta_\alpha)\big[\overline{h(t, z_t|\theta_j)} - \overline{h(t, z_t|\theta_\alpha)}\big]^T \\
& \times R^{-1}(t, z_t)\big[\overline{h(t, z_t)} - \overline{h(t, z_t|\theta_\alpha)}\big]\,dt \\
& + \Lambda_t(\theta_j : \theta_\alpha)\big[\overline{h(t, z_t|\theta_j)} - \overline{h(t, z_t|\theta_\alpha)}\big]^T \\
& \times R^{-1}(t, z_t)\big[dz_t - \overline{h(t, z_t)}dt\big].
\end{aligned} \tag{3.27}
$$

Let us denote $\widetilde{\Lambda}_t(\theta_j : \theta_\alpha) = \ln\{\Lambda_t(\theta_j : \theta_\alpha)\}$. Differentiating according to Ito formula taking into account (3.20), (3.27) yields for $t_m \leqslant t < t_{m+1}$

$$
d_t\widetilde{\Lambda}_t(\theta_j : \theta_\alpha) \;=\; \big[\overline{h(t,z_t|\theta_j)} - \overline{h(t,z_t|\theta_\alpha)}\big]^T R^{-1}(t,z_t)
$$

$$\times \left[dz_t - \frac{1}{2}\overline{h(t,z_t|\theta_j)}dt - \frac{1}{2}\overline{h(t,z_t|\theta_\alpha)}dt \right]. \tag{3.28}$$

From (3.24) it follows that

$$\widetilde{\Lambda}_{t_m}(\theta_j : \theta_\alpha) = \widetilde{\Lambda}_{t_m - 0}(\theta_j : \theta_\alpha) + \ln \frac{C(\eta(t_m), z_{t_m}|\theta_j)}{C(\eta(t_m), z_{t_m}|\theta_\alpha)}. \tag{3.29}$$

As $\Lambda_t(\theta_j : \theta_\alpha) = \exp\{\widetilde{\Lambda}_t(\theta_j : \theta_\alpha)\}$, then (3.26) for $t \geqslant \tau_1$ follows from (3.28), (3.29).

From the Theorem 1 it follows that the effective calculation $\Lambda_t(\theta_j : \theta_\alpha)$ is realized provided that there is a possibility of effective calculation $\overline{h(t, z_t|\theta_j)}$ and $C(\eta(t_m), z_{t_m}|\theta_j)$. In the folowing item the particular case of the processes x_t, z_t, $\eta(t_m)$ supposing such possibility is considered.

4. The case of Effective Calculation $\Lambda_t(\theta_j : \theta_\alpha)$

Let us denote $\widetilde{\tau}_N = (\tau_1, \tau_2, \cdots, \tau_N)$,

$$\begin{aligned}
&\mu(t|\theta_j) = M\{x_t|\theta = \theta_j, z_0^t, \eta_0^m\}, \quad \mu(\tau_k, t|\theta_j) = M\{x_{\tau_k}|\theta = \theta_j, z_0^t, \eta_0^m\},\\
&\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t|\theta_j) = \begin{bmatrix} \mu(t|\theta_j) \\ \widetilde{\mu}_N(\widetilde{\tau}_N, t|\theta_j) \end{bmatrix} = \begin{bmatrix} \mu(t|\theta_j) \\ \mu(\tau_k, t|\theta_j) \end{bmatrix}, \quad k = \overline{1; N},
\end{aligned} \tag{4.1}$$

$$\begin{aligned}
&\Gamma(t|\theta_j) = M\Big\{ \big[x_t - \mu(t|\theta_j)\big] \big[x_t - \mu(t|\theta_j)\big]^T |z_0^t, \eta_0^m \Big\},\\
&\Gamma_{kk}(\tau_k, t|\theta_j) = M\Big\{ \big[x_{\tau_k} - \mu(\tau_k, t|\theta_j)\big] \big[x_{\tau_k} - \mu(\tau_k, t|\theta_j)\big]^T |z_0^t, \eta_0^m \Big\},\\
&\Gamma_{0k}(\tau_k, t|\theta_j) = M\Big\{ \big[x_t - \mu(t|\theta_j)\big] \big[x_{\tau_k} - \mu(\tau_k, t|\theta_j)\big]^T |z_0^t, \eta_0^m \Big\},\\
&\Gamma_{lk}(\tau_l, \tau_k, t|\theta_j) = M\Big\{ \big[x_{\tau_l} - \mu(\tau_l, t|\theta_j)\big] \big[x_{\tau_k} - \mu(\tau_k, t|\theta_j)\big]^T |z_0^t, \eta_0^m \Big\},
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t|\theta_j) &= \begin{bmatrix} \Gamma(t|\theta_j) & \widetilde{\Gamma}_{0N}(\widetilde{\tau}_N, t|\theta_j) \\ \widetilde{\Gamma}_{0N}^T(\cdot) & \widetilde{\Gamma}_N(\widetilde{\tau}_N, t|\theta_j) \end{bmatrix}\\
&= \begin{bmatrix} \Gamma(t|\theta_j) & \Gamma_{01}(\tau_1, t|\theta_j) & \Gamma_{0k}(\tau_k, t|\theta_j) \\ \Gamma_{01}^T(\cdot) & \Gamma_{11}(\tau_1, t|\theta_j) & \Gamma_{lk}(\tau_l, \tau_k, t|\theta_j) \\ \Gamma_{0k}^T(\cdot) & \Gamma_{lk}^T(\cdot) & \Gamma_{kk}(\tau_k, t|\theta_j) \end{bmatrix}, \; l = \overline{1; N-1}, \; k = \overline{2; N}, \; k > l.
\end{aligned}$$

PROPOSITION 2. Assume (see (2.1)–(2.3)):

$$\begin{aligned}
&f(\cdot) = F(t, z_t, \theta)x_t, \;\; \Phi_1(\cdot) = \Phi_1(t, z_t, \theta),\\
&h(\cdot) = H_0(t, z_t, \theta)x_t + \sum_{k=1}^{N} H_k(t, z_t, \theta)x_{\tau_k},\\
&g(\cdot) = G_0(t, z_t, \theta)x_{t_m} + \sum_{k=1}^{N} G_k(t_m, z_{t_m}, \theta)x_{\tau_k},
\end{aligned} \tag{4.3}$$

$$p_0(x|\theta_j) = \mathcal{N}\{x; \mu_0^j, \Gamma_0^j\}, \quad j = \overline{0;r}.$$

Then (see (3.1), (3.15))

$$p_t(\widetilde{x}_{N+1}|\theta_j) = \mathcal{N}\big\{\widetilde{x}_{N+1}; \widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t|\theta_j), \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t|\theta_j)\big\}, \quad j = \overline{0;r}. \tag{4.4}$$

Block components of the distribution (4.4) parameters $\widetilde{\mu}_{N+1}(\cdot)$ and $\widetilde{\Gamma}_{N+1}(\cdot)$ (see (4.1), (4.2)) for $t_m \leqslant t < t_{m+1}$ are defined by the equations

$$d_t\mu(t|\theta_j) = F(t, z_t, \theta_j)\mu(t|\theta_j)dt + \widetilde{H}_0^T(t, z_t|\theta_j)R^{-1}(t, z_t)d\widetilde{z}_t(\theta_j), \tag{4.5}$$

$$d_t\mu(\tau_k, t|\theta_j) = \widetilde{H}_k^T(t, z_t|\theta_j)R^{-1}(t, z_t)d\widetilde{z}_t(\theta_j), \quad k = \overline{1;N}, \tag{4.6}$$

$$\begin{aligned} d\Gamma(t|\theta_j)/dt = {} & F(t, z_t, \theta_j)\Gamma(t|\theta_j) + \Gamma(t|\theta_j)F^T(t, z_t, \theta_j) + Q(t, z_t, \theta_j) \\ & -\widetilde{H}_0^T(t, z_t|\theta_j)R^{-1}(t, z_t)\widetilde{H}_0(t, z_t|\theta_j), \end{aligned} \tag{4.7}$$

$$d\Gamma_{kk}(\tau_k, t|\theta_j)/dt = -\widetilde{H}_k^T(t, z_t|\theta_j)R^{-1}(t, z_t)\widetilde{H}_k(t, z_t|\theta_j), \quad k = \overline{1;N}, \tag{4.8}$$

$$\begin{aligned} d\Gamma_{0k}(\tau_k, t|\theta_j)/dt = {} & F(t, z_t, \theta_j)\Gamma_{0k}(\tau_k, t|\theta_j) \\ & -\widetilde{H}_0^T(t, z_t|\theta_j)R^{-1}(t, z_t)\widetilde{H}_k(t, z_t|\theta_j), \quad k = \overline{1;N}, \end{aligned} \tag{4.9}$$

$$d\Gamma_{lk}(\tau_l, \tau_k, t|\theta_j)/dt = -\widetilde{H}_l^T(t, z_t|\theta_j)R^{-1}(t, z_t)\widetilde{H}_k(t, z_t|\theta_j), \tag{4.10}$$

$l = \overline{1;N-1}$, $k = \overline{2;N}$, $k > l$, with the initial conditions

$$\begin{aligned} \mu(t_m|\theta_j) = {} & \mu(t_m - 0|\theta_j) + \widetilde{G}_0^T(t_m, z_{t_m}|\theta_j) \\ & \times W^{-1}(t_m, z_{t_m}|\theta_j)[\widetilde{\eta}(t_m|\theta_j) - b_j(t_m)], \end{aligned} \tag{4.11}$$

$$\begin{aligned} \mu(\tau_k, t_m|\theta_j) = {} & \mu(\tau_k, t_m - 0|\theta_j) + \widetilde{G}_k^T(t_m, z_{t_m}|\theta_j) \\ & \times W^{-1}(t_m, z_{t_m}|\theta_j)[\widetilde{\eta}(t_m|\theta_j) - b_j(t_m)], \end{aligned} \tag{4.12}$$

$$\begin{aligned} \Gamma(t_m|\theta_j) = {} & \Gamma(t_m - 0|\theta_j) - \widetilde{G}_0^T(t_m, z_{t_m}|\theta_j) \\ & \times W^{-1}(t_m, z_{t_m}|\theta_j)\widetilde{G}_0(t_m, z_{t_m}|\theta_j), \end{aligned} \tag{4.13}$$

$$\begin{aligned} \Gamma_{kk}(\tau_k, t_m|\theta_j) = {} & \Gamma_{kk}(\tau_k, t_m - 0|\theta_j) - \widetilde{G}_k^T(t_m, z_{t_m}|\theta_j) \\ & \times W^{-1}(t_m, z_{t_m}|\theta_j)\widetilde{G}_k(t_m, z_{t_m}|\theta_j), \end{aligned} \tag{4.14}$$

$$\begin{aligned} \Gamma_{0k}(\tau_k, t_m|\theta_j) = {} & \Gamma_{0k}(\tau_k, t_m - 0|\theta_j) - \widetilde{G}_0^T(t_m, z_{t_m}|\theta_j) \\ & \times W^{-1}(t_m, z_{t_m}|\theta_j)\widetilde{G}_k(t_m, z_{t_m}|\theta_j), \end{aligned} \tag{4.15}$$

$$\begin{aligned} \Gamma_{lk}(\tau_l, \tau_k, t_m|\theta_j) = {} & \Gamma_{lk}(\tau_l, \tau_k, t_m - 0|\theta_j) - \widetilde{G}_l^T(t_m, z_{t_m}|\theta_j) \\ & \times W^{-1}(t_m, z_{t_m}|\theta_j)\widetilde{G}_k(t_m, z_{t_m}|\theta_j), \end{aligned} \tag{4.16}$$

where

$$d\widetilde{z}_t(\theta_j) = dz_t - \left[H_0(t, z_t, \theta_j)\mu(t|\theta_j) + \sum_{i=1}^{N} H_i(t, z_t, \theta_j)\mu(\tau_i, t|\theta_j)\right]dt, \tag{4.17}$$

$$\widetilde{H}_0(t, z_t|\theta_j) = H_0(t, z_t, \theta_j)\Gamma(t|\theta_j) + \sum_{i=1}^{N} H_i(t, z_t, \theta_j)\Gamma_{0i}^T(\tau_i, t|\theta_j), \tag{4.18}$$

$$\widetilde{H}_k(t,z_t|\theta_j) = H_k(t,z_t,\theta_j)\Gamma_{kk}(\tau_k,t|\theta_j) + \sum_{i\neq k}^{N} H_i(t,z_t,\theta_j)\Gamma_{ki}^T(\tau_k,\tau_i,t|\theta_j), \tag{4.19}$$

$$\widetilde{\eta}(t_m|\theta_j) = \eta(t_m) - G_0(t_m,z_{t_m},\theta_j)\mu(t_m-0|\theta_j) - \sum_{i=1}^{N} G_i(t_m,z_{t_m},\theta_j)\mu(\tau_i,t_m-0|\theta_j), \tag{4.20}$$

$$\widetilde{G}_0(t_m,z_{t_m}|\theta_j) = G_0(t_m,z_{t_m},\theta_j)\Gamma(t_m-0|\theta_j) + \sum_{i=1}^{N} G_i(t_m,z_{t_m},\theta_j)\Gamma_{0i}^T(\tau_i,t_m-0|\theta_j), \tag{4.21}$$

$$\widetilde{G}_k(t_m,z_{t_m}|\theta_j) = G_k(t_m,z_{t_m},\theta_j)\Gamma_{kk}(\tau_k,t_m-0|\theta_j) + \sum_{i\neq k}^{N} G_i(t_m,z_{t_m},\theta_j)\Gamma_{ki}^T(\tau_k,\tau_i,t_m-0|\theta_j), \tag{4.22}$$

$$W(t_m,z_{t_m}|\theta_j) = V(t_m,\theta_j) + G_{0N}(t_m,z_{t_m},\theta_j)\widetilde{\Gamma}_{N+1} \times(\widetilde{\tau}_N,t_m-0|\theta_j)G_{0N}^T(t_m,z_{t_m},\theta_j), \tag{4.23}$$

$$G_{0N}(t_m,z_{t_m},\theta_j)=[G_0(t_m,z_{t_m},\theta_j)\vdots G_1(t_m,z_{t_m},\theta_j)\vdots\cdots\vdots G_N(t_m,z_{t_m},\theta_j)], \tag{4.24}$$

and $\varphi(\cdot,t_m-0) = \lim\varphi(\cdot,t)$ by $t\uparrow t_m$ is the solution of the appropriate differential equation on the previous interval time calculated at the point $t=t_m$.

As the fixed memory is a particular case of sliding memory then the formulated proposition is referred to as a particular case from the results (Abakumova *et al.*, 1995b).

Theorem 2. *The likelihood ratio* $\Lambda_t(\theta_j:\theta_\alpha)$ *is denoted by the Theorem* 1 *expressions, where*

$$\overline{h(t,z_t|\theta_j)} = H_0(t,z_t,\theta_j)\mu(t|\theta_j) + \sum_{k=1}^{N} H_k(t,z_t,\theta_j)\mu(\tau_k,t|\theta_j), \tag{4.25}$$

$$\begin{aligned} C(\eta(t_m),z_{t_m}|\theta_j) &= \exp\Big\{-\frac{1}{2}\big[\eta(t_m)-b_j(t_m)\big]^T V^{-1}(t_m,\theta_j) \\ &\quad\times\big[\eta(t_m)-b_j(t_m)\big]\Big\}\big|V(t_m,\theta_j)\big|^{-1/2} \\ &\quad\times\frac{\big|\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N,t_m|\theta_j)\big|^{1/2}}{\big|\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N,t_m-0|\theta_j)\big|^{1/2}} \\ &\quad\times\frac{\exp\Big\{\frac{1}{2}\widetilde{\mu}_{N+1}^T(\widetilde{\tau}_N,t_m|\theta_j)\widetilde{\Gamma}_{N+1}^{-1}(\widetilde{\tau}_N,t_m|\theta_j)\widetilde{\mu}_{N+1}(\widetilde{\tau}_N,t_m|\theta_j)\Big\}}{\exp\Big\{\frac{1}{2}\widetilde{\mu}_{N+1}^T(\widetilde{\tau}_N,t_m-0|\theta_j)\widetilde{\Gamma}_{N+1}^{-1}(\widetilde{\tau}_N,t_m-0|\theta_j)\widetilde{\mu}_{N+1}(\widetilde{\tau}_N,t_m-0|\theta_j)\Big\}}. \end{aligned} \tag{4.26}$$

Proof. The formula (4.25) follows immediately from (3.7) taking into account (4.1), (4.3). Using (3.9), (3.15) we obtain

$$C(\eta(t_m), z_{t_m}|\theta_j) = \int C\big(\eta(t_m), z_{t_m}, \widetilde{x}_{N+1}, \theta_j\big) p_{t_m-0}(\widetilde{x}_{N+1}|\theta_j) d\widetilde{x}_{N+1}. \quad (4.27)$$

From (3.1), (3.11), (4.3), (4.4), (4.24) it follows that

$$\begin{aligned} &C(\eta(t_m), z_{t_m}, \widetilde{x}_{N+1}, \theta_j\big) p_{t_m-0}(\widetilde{x}_{N+1}|\theta_j) \\ &\quad = (2\pi)^{-n(N+1)/2} \big|V(t_m, \theta_j)\big|^{-1/2} C_1 \exp\Big\{ -\frac{1}{2}E\Big\}, && (4.28) \\ &C_1 = \big|\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\big|^{-1/2}, && (4.29) \\ &E = \big[\eta(t_m) - b_j(t_m) - G_{0N}(t_m, z_{t_m}, \theta_j)\widetilde{x}_{N+1}\big]^T V^{-1} \\ &\quad \times (t_m, \theta_j)\big[\eta(t_m) - b_j(t_m) - G_{0N}(t_m, z_{t_m}, \theta_j)\widetilde{x}_{N+1}\big] \\ &\quad + \big[\widetilde{x}_{N+1} - \widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\big]^T \widetilde{\Gamma}_{N+1}^{-1} \\ &\quad \times (\widetilde{\tau}_N, t_m - 0|\theta_j)\big[\widetilde{x}_{N+1} - \widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\big]. && (4.30) \end{aligned}$$

Quadratic form $[\widetilde{x}_{N+1} - a]^T \Psi [\widetilde{x}_{N+1} - a]$ selection in (4.30) with consequent substitution into (4.28) is given by

$$\begin{aligned} &C(\eta(t_m), z_{t_m}, \widetilde{x}_{N+1}, \theta_j) p_{t_m-0}(\widetilde{x}_{N+1}|\theta_j) = \big|V(t_m, \theta_j)\big|^{-1/2} C_1 C_2 C_3 C_4 C_5 \\ &\quad \times \mathcal{N}\big\{\widetilde{x}_{N+1}; \widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j), \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)\big\}, && (4.31) \end{aligned}$$

where

$$\begin{aligned} &C_2 = \big|\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)\big|^{1/2}, \\ &C_3 = \exp\Big\{ -\frac{1}{2}\widetilde{\mu}_{N+1}^T(\widetilde{\tau}_N, t_m - 0|\theta_j)\widetilde{\Gamma}_{N+1}^{-1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\Big\}, \\ &C_4 = \exp\Big\{ \frac{1}{2}\widetilde{\mu}_{N+1}^T(\widetilde{\tau}_N, t_m|\theta_j)\widetilde{\Gamma}_{N+1}^{-1}(\widetilde{\tau}_N, t_m|\theta_j)\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)\Big\}, && (4.32) \\ &C_5 = \exp\Big\{ -\frac{1}{2}\big[\eta(t_m) - b_j(t_m)\big]^T V^{-1}(t_m, z_{t_m})\big[\eta(t_m) - b_j(t_m)\big]\Big\}, \\ &\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j) = \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)\Big[G_{0N}^T(t_m, z_{t_m}, \theta_j)V^{-1}(t_m, z_{t_m}) \\ &\quad \times \big[\eta(t_m) - b_j(t_m)\big] + \widetilde{\Gamma}_{N+1}^{-1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j)\Big], && (4.33) \\ &\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j) = \big[\widetilde{\Gamma}_{N+1}^{-1}(\widetilde{\tau}_N, t_m - 0|\theta_j) + G_{0N}^T(t_m, z_{t_m}, \theta_j) \\ &\quad \times V^{-1}(t_m, z_{t_m})G_{0N}(t_m, z_{t_m}, \theta_j)\big]^{-1}. && (4.34) \end{aligned}$$

Substituting (4.31) into (4.27) by the use of (4.29), (4.32) yields (4.26).

REMARK 1. Using the matrix identity $[A + BCB^T]^{-1} = A^{-1} - A^{-1}B[C^{-1} + B^T A^{-1} B]^{-1} B^T A^{-1}$ into (4.34) we have for $\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)$ equivalent (4.34) formula

$$\widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j) = \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j) - \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j) G_{0N}^T(t_m, z_{t_m}, \theta_j) \times W^{-1}(t_m, z_{t_m}|\theta_j) G_{0N}(t_m, z_{t_m}, \theta_j) \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j), \quad (4.35)$$

where $W(t_m, z_{t_m}|\theta_j)$ is denoted by (4.23). Using (4.35) into (4.33) yields for $\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)$ an equivalent (4.33) formula

$$\widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j) = \widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j) + \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m - 0|\theta_j) G_{0N}^T(t_m, z_{t_m}, \theta_j) \times W^{-1}(t_m, z_{t_m}|\theta_j)\big[\widetilde{\eta}(t_m|\theta_j) - b_j(t_m)\big], \quad (4.36)$$

where $\widetilde{\eta}(t_m|\theta_j)$ is denoted by (4.20). Block rewriting (4.35), (4.36) taking into account (4.1), (4.2), (4.21)–(4.24) result in (4.11)–(4.16). From (3.6), (3.14), (3.18) it follows that

$$p_{t_m}(\widetilde{x}_{N+1}|\theta_j) = \Big[C\big(\eta(t_m), z_{t_m}, \widetilde{x}_{N+1}, \theta_j\big) / C\big(\eta(t_m), z_{t_m}|\theta_j\big)\Big] p_{t_m - 0}(\widetilde{x}_{N+1}|\theta_j). \quad (4.37)$$

From (4.37) taking into account (4.26), (4.31) it folows that

$$p_{t_m}(\widetilde{x}_{N+1}|\theta_j) = \mathcal{N}\big\{\widetilde{x}_{N+1}; \widetilde{\mu}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j), \widetilde{\Gamma}_{N+1}(\widetilde{\tau}_N, t_m|\theta_j)\big\}. \quad (4.38)$$

Therefore (4.4) for $t = t_m$ and (4.11)–(4.16) is proved immediately in the process of Theorem 2 proof.

REMARK 2. Theorem 2 together with Proposition 2 gives a closed system of differential-reccurent expressions for determination $\Lambda_t(\theta_j : \theta_\alpha)$ as in the general case of memory continuous-discrete observations and for foliowing particular cases: á) discrete observations are absent ($G_0(\cdot) \equiv O$, $G_k(\cdot) \equiv O$, $k = \overline{1;N}$) and continuous observations are with memory, without memory ($H_k(\cdot) \equiv O$, $k = \overline{1;N}$), with pure lag time ($H_0(\cdot) \equiv O$); b) continuous observations are absent ($H_0(\cdot) \equiv O$, $H_k(\cdot) \equiv O$, $k = \overline{1;N}$) and discrete observations are with memory, without memory ($G_k(\cdot) \equiv O$, $k = \overline{1;N}$), with the pure lag time ($G_0(\cdot) \equiv O$); c) various combinations of situations, when are present both continuous and discrete observations (for example, continuous observations are without memory, discrete observations are with pure lag time). The results (Sage and Melse, 1972) for cases, when the process x_t is continuous time process and also results (Kailath, 1969; Greene, 1978; Dyomin, 1979, 1985) folow as particular cases from the formulated above Theorems and Corollaries.

REMARK 3. The obtained results are obviously extended for the case when the coefficients dependence into (2.1)–(2.3) of θ is not only functional, but also index, i.e.,

$$f(\cdot) = f^\theta(t, x_t, z_t) \in \big\{f^0(\cdot), f^1(\cdot), \cdots, f^r(\cdot)\big\},$$

$$\begin{aligned}
&\Phi_1(\cdot) = \Phi_1^\theta(t, x_t, z_t) \in \{\Phi_1^0(\cdot), \Phi_1^1(\cdot), \cdots, \Phi_1^r(\cdot)\}, \\
&h(\cdot) = h^\theta(t, x_t, x_{\tau_1}, \cdots, x_{\tau_N}, z_t) \in \{h^0(\cdot), h^1(\cdot), \cdots, h^r(\cdot)\}, \\
&g(\cdot) = g^\theta(t_m, x_{t_m}, x_{\tau_1}, \cdots, x_{\tau_N}, z_{t_m}) \in \{g^0(\cdot), g^1(\cdot), \cdots, g^r(\cdot)\}.
\end{aligned} \tag{4.39}$$

5. Detection of Anomalous Noises

The obtained results allow by virtue of $\Lambda_t(\theta_j : \theta_\alpha)$ make the optimal minimum average risk solution (Middleton, 1960; Van Trees, 1971; Sage and Melse, 1972; Fukunaga, 1972) in any time $t \geqslant 0$ while obtaining the observations $\{z_t; \eta(t_m)\}$ but an analysis of the recognition procedure quality in the general case is difficult. In some particular cases for some problems such analysis is possible. In this section we consider a similar problem.

Let the process x_t be a scalar stationary process and described by the equation

$$dx_t = -ax_t dt + \Phi_1 dw_t, \quad a > 0. \tag{5.1}$$

The given process, wich is known as process Ornstein-Uhlenbeck (Davis, 1977) is Gaussian with the correlation function in a stationary state $\mathcal{K}(s) = [Q/2a]\exp\{-a|s|\}$, $Q = \Phi_1^2$, and the correlation time $s_k = 1/a$. Let contituous observations be absent (situation b) from Remark 2), and the process $\eta(t_m)$ is scalar stationary process with the single multiplicity memory ($N = 1$, $\tau_1 = \tau$)

$$\eta(t_m) = G_0 x_{t_m} + G_1 x_\tau + \xi_0(t_m) + \theta\xi_1(t_m), \tag{5.2}$$

where $\theta \in \Omega_\theta = \{\theta_0; \theta_1\} = \{0; 1\}$, $\xi_0(t_m)$ and $\xi_1(t_m)$ are independent white Gaussian sequences with the expectations $b_0 = 0$, $b_1 \neq 0$ and the intensities V_0, V_1. The noise $\xi_0(t_m)$ is a regular noise and $\xi_1(t_m)$ is an anomalous noise. Therefore the problem of hypotheses recognition $\mathcal{H}_1\{\theta = \theta_1\}$ and $\mathcal{H}_0\{\theta = \theta_0\}$ is a problem of anomalous noise detection. The case of rare observations are considered when on the intervals $t \in (t_m, t_{m+1})$ solutions of the equations (4.7)–(4.9) for matrix $\widetilde{\Gamma}_2(\widetilde{\tau}_2, t)$ elements $\gamma(t)$, $\gamma_{01}(\tau, t)$, $\gamma_{11}(\tau, t)$ (see (4.2)) reach stationary values

$$\gamma = \gamma_{11} = \frac{Q}{2a}, \quad \gamma_{01} = \gamma \exp\{-at^*\}, \tag{5.3}$$

where $t^* = t - \tau$ is the memory depth. For lack of continuous observations $d_t\Lambda_t(\theta_1 : \theta_0) = 0$, then in the case of rare observations $\eta(t_m)$ it is meaningful to make a decision with respect to the values $\Lambda_{t_m}(\theta_1 : \theta_0)$, which is according to Theorem 1 as follows

$$\Lambda_{t_m}(\theta_1 : \theta_0) = \frac{C(\eta(t_m)|\theta_1)}{C(\eta(t_m)|\theta_0)}. \tag{5.4}$$

The calculations with respect to Theorem 2, Proposition 2 and (5.3) performed for the considered problem regarding (5.4) give the following expression for $\Lambda_{t_m}(\theta_1:\theta_0)$

$$\Lambda_{t_m}(\theta_1:\theta_0)=\left(\frac{W_0}{W_1}\right)^{1/2}\exp\left\{-\frac{[\widetilde{\eta}(t_m)-q\sqrt{V_0}]^2}{2W_1}\right\}\exp\left\{\frac{\widetilde{\eta}^2(t_m)}{2W_0}\right\}, \tag{5.5}$$

$$\widetilde{\eta}(t_m)=\eta(t_m)-G_0\mu(t_m-0)-G_1\mu(\tau,t_m-0), \tag{5.6}$$

$$W_0=V_0+g(t^*)\gamma,\quad W_1=(l+1)V_0+g(t^*)\gamma,$$
$$g(t^*)=G_0^2+G_1^2+2G_0G_1\exp\{-at^*\}. \tag{5.7}$$

The scale expectation b_1 and the intencity V_1 of the anomalous noise $\xi_1(t_m)$ as regards the intencity V_0 of regular noise as $V_1=lV_0$, $b_1=q\sqrt{V_0}$ are used in (5.5), (5.6). Let $\alpha=\mathcal{P}\{\theta=\theta_1|\theta=\theta_0\}$ be a probability of false detection and $\beta=\mathcal{P}\{\theta=\theta_0|\theta=\theta_1\}$ be a probability of anomalous noise omission. Let $\widetilde{\Lambda}_{t_m}(\theta_1:\theta_0)=\ln\{\Lambda_{t_m}(\theta_1:\theta_0)\}$ and

$$\begin{aligned}I_{t_m}(1:0)&=M\{\widetilde{\Lambda}_{t_m}(\theta_1:\theta_0)|\theta=\theta_1\},\\ I_{t_m}(0:1)&=-M\{\widetilde{\Lambda}_{t_m}(\theta_1:\theta_0)|\theta=\theta_0\}\end{aligned} \tag{5.8}$$

be Kullback divergences (Kullback, 1960). Then α, β and $I_{t_m}(1:0)$, $I_{t_m}(0:1)$ are linked by inequalities

$$\begin{aligned}I_{t_m}(1:0)&\geqslant\beta\ln[\beta/(1-\alpha)]+(1-\beta)\ln[(1-\beta)/\alpha],\\ I_{t_m}(0:1)&\geqslant\alpha\ln[\alpha/(1-\beta)]+(1-\alpha)\ln[(1-\alpha)/\beta],\end{aligned} \tag{5.9}$$

from which the lower boundaries $\alpha^*=\inf\alpha$ subject to fixed β and $\beta^*=\inf\beta$ subject to fixed α can be defined. From (5.2), (5.6) are obtained (see (4.1), (4.24))

$$\widetilde{\eta}(t_m)=G_{01}\widetilde{\varepsilon}_2(\widetilde{\tau}_2,t_m-0)+\xi_0(t_m)+\theta\xi_1(t_m), \tag{5.10}$$

where

$$\widetilde{\varepsilon}_2(\widetilde{\tau}_2,t_m-0)=\begin{bmatrix}\varepsilon(t_m-0)\\ \varepsilon(\tau,t_m-0)\end{bmatrix}=\begin{bmatrix}x_{t_m}-\mu(t_m-0)\\ x_\tau-\mu(\tau,t_m-0)\end{bmatrix},\quad G_{01}=[G_0\vdots G_1]. \tag{5.11}$$

Then from (4.4), (4.23), (5.7), (5.10) follows that

$$p_{t_m}(\widetilde{\eta}|\theta_0)=\mathcal{N}\{\widetilde{\eta};0,W_0\},\quad p_{t_m}(\widetilde{\eta}|\theta_1)=\mathcal{N}\{\widetilde{\eta};q\sqrt{V_0},W_1\}. \tag{5.12}$$

The calculations with respect to (5.8) with the help of (5.5), (5.12) yield

$$I_{t_m}(1:0)=\frac{1}{2}\left[\ln\left(\frac{V_0+g(t^*)\gamma}{(l+1)V_0+g(t^*)\gamma}\right)+\frac{(q^2+l)V_0}{V_0+g(t^*)\gamma}\right], \tag{5.13}$$

$$I_{t_m}(0:1)=\frac{1}{2}\left[\ln\left(\frac{(l+1)V_0+g(t^*)\gamma}{V_0+g(t^*)\gamma}\right)+\frac{(q^2-l)V_0}{(l+1)V_0+g(t^*)\gamma}\right]. \tag{5.14}$$

Let $\widetilde{I}_{t_m}(1:0)$, $\widetilde{I}_{t_m}(0:1)$, $\widetilde{\alpha}$, $\widetilde{\beta}$, $\widetilde{\alpha}^*$, $\widetilde{\beta}^*$ denote the corresponding values in the case of observance channel without memory, when $G_1 = 0$. It is obvious that $\widetilde{I}_{t_m}(1:0)$ and $\widetilde{I}_{t_m}(0:1)$ are expressed by (5.13), (5.14), where $g(t^*)$ are substituted by G_0^2. Then the values $\Delta I_{t_m}(1:0) = I_{t_m}(1:0) - \widetilde{I}_{t_m}(1:0)$, $\Delta I_{t_m}(0:1) = I_{t_m}(0:1) - \widetilde{I}_{t_m}(0:1)$ and the adequate values $\Delta\alpha^* = \alpha^* - \widetilde{\alpha}^*$, $\Delta\beta^* = \beta^* - \widetilde{\beta}^*$ will characterize the observation effectiveness with memory as regards the observations without memory in the problem of anomalous noise detection. If $\Delta I_{t_m}(1:0) > 0$, which yields $\Delta\alpha^* < 0$, then a channel with memory is more effective than a channel without memory with respect to the false detection. If $\Delta I_{t_m}(0:1) > 0$, which yields $\Delta\beta^* < 0$, then a channel with memory is more effective than a channel without memory with respect to the anomalous noise omission. The carried out research yields the following

PROPOSITION 3. Let

$$(G_0, G_1) \in M = M^+ \cup M^- = \{(G_0, G_1):\ G_1^2 + 2G_0G_1 < 0\}. \tag{5.15}$$

Then $\Delta I_{t_m}(1:0)$ and $\Delta I_{t_m}(0:1)$ subject to $t^* \uparrow_0^\infty$ are monotonically diminishing from the values $\Delta I_{t_m}^0(1:0) > 0$ and $\Delta I_{t_m}^0(0:1) > 0$ denoted by

$$\begin{aligned}\Delta I_{t_m}^0(1:0) \;&=\; \frac{1}{2}\ln[C_2^0/C_1^0] + \frac{1}{2}(q^2+l)V_0\big([V_0 + (G_0+G_1)^2\gamma]^{-1} \\ &\quad - [V_0 + G_0^2\gamma]^{-1}\big), \qquad (5.16)\\ \Delta I_{t_m}^0(0:1) \;&=\; \frac{1}{2}\ln[C_1^0/C_2^0] + \frac{1}{2}(q^2-l)V_0\Big(\big[(l+1)V_0 + (G_0+G_1)^2\gamma\big]^{-1} \\ &\quad - [(l+1)V_0 + G_0^2\gamma]^{-1}\Big), \qquad (5.17)\end{aligned}$$

where

$$\begin{aligned}C_1^0 &= \big[(l+1)V_0 + (G_0+G_1)^2\gamma\big][V_0 + G_0^2\gamma],\\ C_2^0 &= \big[V_0 + (G_0+G_1)^2\gamma\big]\big[(l+1)V_0 + G_0^2\gamma\big],\end{aligned} \tag{5.18}$$

up to the values $\Delta I_{t_m}^\infty(1:0) < 0$ and $\Delta I_{t_m}^\infty(0:1) < 0$ denoted by

$$\begin{aligned}\Delta I_{t_m}^\infty(1:0) \;&=\; \frac{1}{2}\ln[C_2^\infty/C_1^\infty] + \frac{1}{2}(q^2+l)V_0\Big(\big[V_0 + (G_0^2+G_1^2)\gamma\big]^{-1} \\ &\quad - [V_0 + G_0^2\gamma]^{-1}\Big), \qquad (5.19)\\ \Delta I_{t_m}^\infty(0:1) \;&=\; \frac{1}{2}\ln[C_1^\infty/C_2^\infty] + \frac{1}{2}(q^2-l)V_0\Big(\big[(l+1)V_0 + (G_0^2+G_1^2)\gamma\big]^{-1} \\ &\quad - [(l+1)V_0 + G_0^2\gamma]^{-1}\Big), \qquad (5.20)\end{aligned}$$

where

$$\begin{aligned}C_1^\infty &= \big[(l+1)V_0 + (G_0^2+G_1^2)\gamma\big][V_0 + G_0^2\gamma],\\ C_2^\infty &= \big[V_0 + (G_0^2+G_1^2)\gamma\big]\big[(l+1)V_0 + G_0^2\gamma\big].\end{aligned} \tag{5.21}$$

The value t^* subject to $\Delta I_{t_m}(1:0) = 0$, $\Delta I_{t_m}(0:1) = 0$ and which can be defined the effective memory depth t^*_{eff} expressed by

$$t^*_{eff} = \frac{1}{a} \ln \left(\frac{2|G_0|}{|G_1|} \right). \tag{5.22}$$

If $(G_0, G_1) \in \overline{M} = \{(G_0, G_1) :\ G_1^2 + 2G_0G_1 > 0\}$ then $\Delta I_{t_m}(1:0) < 0$, $\Delta I_{t_m}(0:1) < 0$ $(\Delta\alpha^* > 0,\ \Delta\beta^* > 0)$ for all $t^* \geqslant 0$, i.e., in this case the observations without memory is more effective than the observations with memory.

A physical interpretation of this result is the following. If $(G_0, G_1) \in M$, then $|G_0 + G_1| < |G_0|$. From (5.2) it follows that $\eta(t_m) = (G_0+G_1)x_{t_m} + \xi_0(t_m) + \theta\xi_1(t_m)$ subject to $t^* = 0$. This allows the anomal noise $\xi_1(t_m)$ to dominate a greater degree over the signal $Y(t_m) = (G_0+G_1)x_{t_m} + \xi_0(t_m)$ rather than the signal $\widetilde{Y}(t_m) = G_0x_{t_m} + \xi_0(t_m)$ provided that $(G_0, G_1) \in M$ since intencities of the signal $X(t_m) = (G_0 + G_1)x_{t_m}$ and $\widetilde{X}(t_m) = G_0x_{t_m}$ are proportional to $|G_0+G_1|^2$ and $|G_0|^2$, respectively. This explains the property $\Delta I^0_{t_m}(1:0) > 0$, $\Delta I^0_{t_m}(0:1) > 0$ and $\Delta\alpha^*_0 < 0$, $\Delta\beta^*_0 < 0$, respectively. The property $t^* \gg s_k$, where $s_k = 1/a$ is a correlation time of the process x_t, is adeguate to the case of $t^* \to \infty$. With the correlations lack between x_{t_m} and x_τ the signal intencity $X(t_m, \tau) = G_0x_{t_m} + G_1x_\tau$ is proportional to the value $G^2 = G_0^2 + G_1^2$. Thus, by a great depth of the memory, when $t^* \to \infty$, the anomal interference $\xi_1(t_m)$ dominates to a greater degree over the signal $\widetilde{Y}(t_m) = G_0x_{t_m} + \xi_0(t_m)$ rather than over the signal $Y(\tau, t_m) = G_0x_{t_m} + G_1x_\tau + \xi_0(t_m)$. By virtue of this the property $\Delta I^\infty_{t_m}(1:0) < 0$, $\Delta I^\infty_{t_m}(0:1) < 0$ is explained and $\Delta\alpha^*_\infty > 0$, $\Delta\beta^*_\infty > 0$, respectively.

COROLLARY 3. Let (5.15) be satisfied. Then $\Delta\alpha^*$ subject to fixed β and $\Delta\beta^*$ subject to fixed α are monotonically increaseing provided that $t^* \uparrow_0^\infty$ from the values $\Delta\alpha^*_0 < 0$ and $\Delta\beta^*_0 < 0$ to the values $\Delta\alpha^*_\infty > 0$ and $\Delta\beta^*_\infty > 0$ is equal zero in the point $t^* = t^*_{eff}$ denoted by (5.22).

This property arises from Proposition 3 taking into account (5.9). The results of Proposition 3 and Corollary 3 are illustrated by the performed calculations. In Fig. 1, 2 $\Delta I_{t_m}(1:0)$ and $\Delta\alpha^*$ are shown as functions of the memory depth t^* subject to various l and a. As it is subjected for a minor of the memory depth, when $t^* \to 0$, the behaviour of $\Delta I_{t_m}(1:0)$ and $\Delta I_{t_m}(0:1)$ depends on the ratio between G_0 and G_1, then the dependences for $\Delta I^0_{t_m}(1:0)$ from G_0 subject to fixed G_1 and from G_1 subject to fixed G_0 are shown in Fig. 3 and 4, where $\Delta I^0_{t_m}(1:0) = \lim \Delta I_{t_m}(1:0)$ provided that $t^* \uparrow_0^\infty$. The behaviour $\Delta\alpha^*_0 = \lim \Delta\alpha^*_0$ subject to $t^* \to 0$ adequate to these dependences is obvious. The dependences shown in the Fig. 3 and 4 correspond to the carried out researches. Analogous dependence are obtained for $\Delta I^0_{t_m}(0:1)$, $\Delta\beta^*_0$.

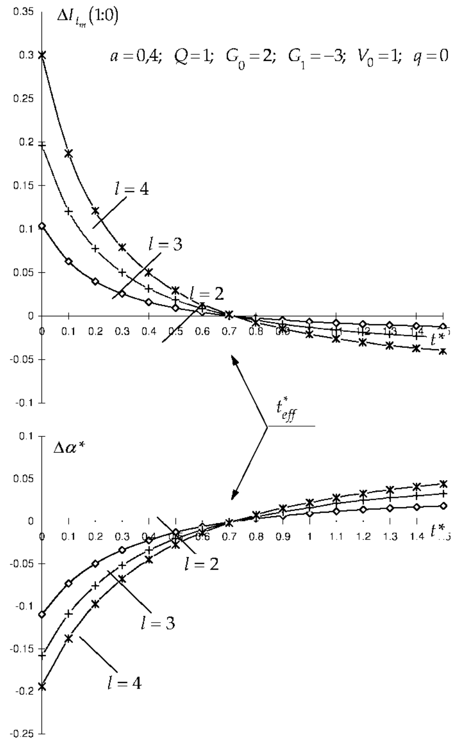

Fig. 1. Dependence of $\Delta I_{t_m}(1:0)$ and $\Delta\alpha^*$ from the memory depth subject to various l.

Proposition 3 and Copollary 3 characterize a potential effectiveness of observation with memory relative to those without memory. The following result deals with the properties of detector

$$\widetilde{\Lambda}_{t_m}(\theta_1 : \theta_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} d. \tag{5.23}$$

PROPOSITION 4. Let (5.15) be satisfied and $l = 0$. Then $\Delta\alpha$ and $\Delta\beta$ are denoted by

$$\Delta\alpha = \Phi\left(\frac{d-\widetilde{h}}{\widetilde{b}\sqrt{\widetilde{\widetilde{W}}}}\right) - \Phi\left(\frac{d-h}{b\sqrt{W_0}}\right), \tag{5.24}$$

$$\Delta\beta = \Phi\left(\frac{(d-h)-bq\sqrt{V_0}}{b\sqrt{W_0}}\right) - \Phi\left(\frac{(d-\widetilde{h})-\widetilde{b}q\sqrt{V_0}}{\widetilde{b}\sqrt{\widetilde{\widetilde{W}}}}\right), \tag{5.25}$$

where $\widetilde{W} = V_0 + \gamma G_0^2$, $b = q\sqrt{V_0}/W_0$, $\widetilde{b} = q\sqrt{V_0}/\widetilde{W}$, $h = -q^2V_0/2W_0$, $\widetilde{h} =$

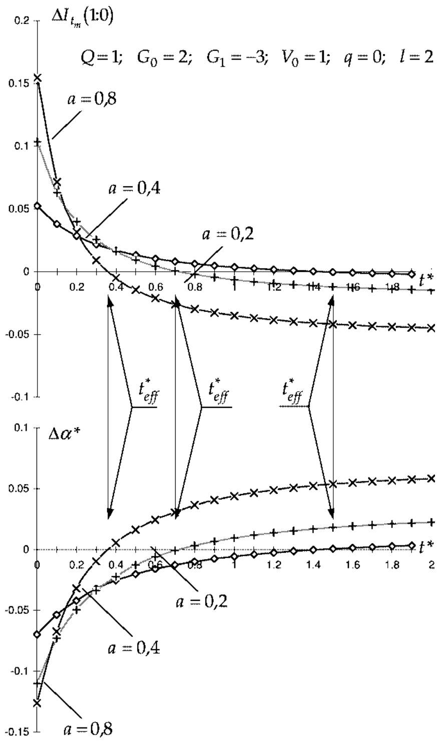

Fig. 2. Dependence of $\Delta I_{t_m}(1:0)$ and $\Delta\alpha^*$ from the memory depth subject to various α.

$-q^2V_0/2\widetilde{W}$, W_0 are described in (5.7) and

$$\Phi(y) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{y} \exp\left\{ -\frac{1}{2}s^2 \right\} \mathrm{d}s. \tag{5.26}$$

The functions $\Delta\alpha(t^*)$ and $\Delta\beta(t^*)$ are monotonically increaseing subject to $t^* \uparrow_0^\infty$ from the values $\Delta\alpha_0 < 0$ and $\Delta\beta_0 < 0$ to the values $\Delta\alpha_\infty > 0$ and $\Delta\beta_\infty > 0$, whilst $\Delta\alpha_0$, $\Delta\beta_0$ are expressed by (5.24), (5.25), where $W_0 = V_0 + \gamma(G_0 + G_1)^2$, and $\Delta\alpha_\infty$, $\Delta\beta_\infty$ are expressed by the same formulae, where $W_0 = V_0 + \gamma(G_0^2 + G_1^2)$. The property $\Delta\alpha(t^*) = 0$ and $\Delta\beta(t^*) = 0$ are justified at the point $t^* = t^*_{eff}$ denoted by (5.22).

Proof. From (5.5)–(5.7) by $l = 0$ it follows that $\widetilde{\Lambda}_{t_m}(\theta_1 : \theta_0) = b\widetilde{\eta}(t_m) + h$. Then according to (5.12)

$$p_{t_m}(\widetilde{\Lambda}|\theta_0) = \mathcal{N}\{\widetilde{\Lambda}; h, b^2W_0\}, \quad p_{t_m}(\widetilde{\Lambda}|\theta_1) = \mathcal{N}\{\widetilde{\Lambda}; bq\sqrt{V_0} + h, b^2W_0\}. \tag{5.27}$$

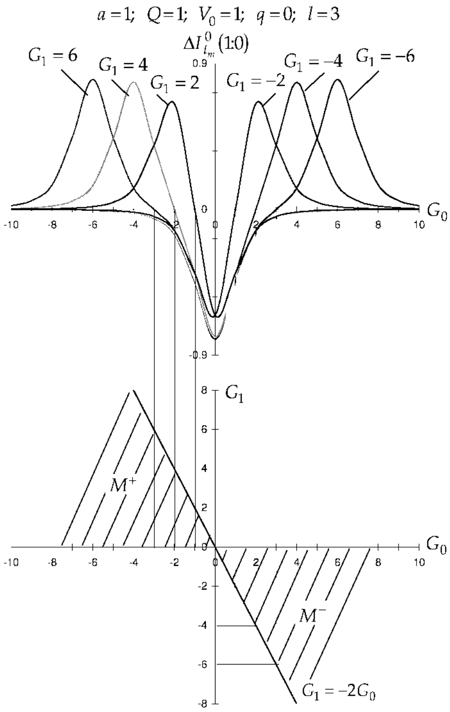

Fig. 3. Dependence of $\Delta I^0_{t_m}(1:0)$ from G_1 subject to fixed G_0.

Since

$$\alpha = \int\limits_{d}^{\infty} p_{t_m}(\widetilde{\Lambda}|\theta_0)\ d\widetilde{\Lambda}, \quad \beta = \int\limits_{-\infty}^{d} p_{t_m}(\widetilde{\Lambda}|\theta_1)\,\mathrm{d}\widetilde{\Lambda}, \tag{5.28}$$

then making use of (5.27) in (5.28) taking into account (5.26) yields

$$\alpha = 1 - \Phi\left(\frac{d-h}{b\sqrt{W_0}}\right), \quad \beta = \Phi\left(\frac{(d-h) - bq\sqrt{V_0}}{b\sqrt{W_0}}\right). \tag{5.29}$$

Then (5.24), (5.25) arise from (5.29). The properties $\Delta\alpha(t^*) = 0$ and $\Delta\beta(t^*) = 0$ are proved immediately taking into account the properties $\Phi(y)$.

COROLLARY 4. Let suppositions of Proposition 4 be satisfied and $d = 0$. Then $\Delta\alpha = \Delta\beta = \Delta$ and $\Delta(t^*)$ is monotonically increaseing subject to $t^* \uparrow_0^\infty$ from $\Delta_0 < 0$ to $\Delta_\infty > 0$, where

$$\Delta_0 = \Phi\left(\frac{1}{2}\frac{q\sqrt{V_0}}{\sqrt{V_0+\gamma G_0^2}}\right) - \Phi\left(\frac{1}{2}\frac{q\sqrt{V_0}}{\sqrt{V_0+\gamma(G_0+G_1)^2}}\right), \tag{5.30}$$

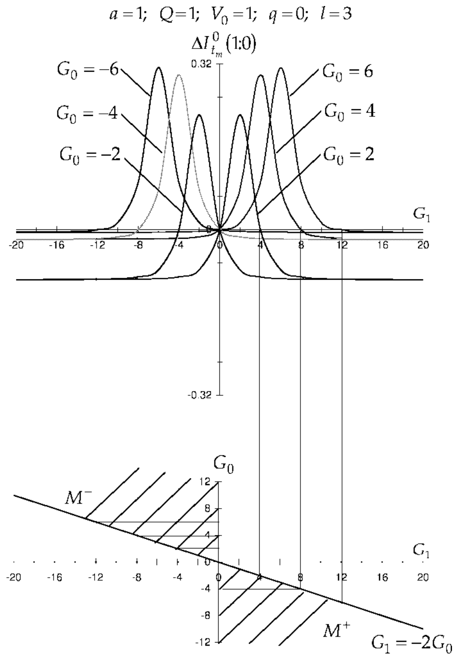

Fig. 4. Dependence of $\Delta I^0_{t_m}(1:0)$ from G_0 subject to fixed G_1.

$$\Delta_\infty = \Phi\left(\frac{1}{2}\frac{q\sqrt{V_0}}{\sqrt{V_0+\gamma G_0^2}}\right) - \Phi\left(\frac{1}{2}\frac{q\sqrt{V_0}}{\sqrt{V_0+\gamma(G_0^2+G_1^2)}}\right). \tag{5.31}$$

The property $\Delta(t^*) = 0$ is satisfied at the point $t^* = t^*_{eff}$ denoted by (5.22).

The formulated result arises immediately from Proposition 4 subject to $d = 0$ taking into account the property $\Phi(-y) = 1 - \Phi(y)$.

6. Estimation

In the framework of the considered recognition problem a solution for the problem of determining the mean-root-square optimal estimation $\widehat{\theta}(t)$ for the parameter θ, the filtering estimation $\mu(t)$ and the interpolation estimations $\mu(\tau_k, t)$, $k = \overline{1;N}$ for the process x_t has been obtained. Indeed, since a posteriori mean (Liptser and Shiryayev, 1977; 1978; Kallianpur, 1980) is in the mean-root-square optimal estimation, i.e.,

$$\widehat{\theta}(t)=M\{\theta|z_0^t,\eta_0^m\},\quad \mu(t)=M\{x_t|z_0^t,\eta_0^m\},\quad \mu(\tau_k,t)=M\{x_{\tau_k}|z_0^t,\eta_0^m\}, \tag{6.1}$$

then according to (3.4), (3.14), (3.15)

$$\begin{aligned}&\widehat{\theta}(t)=\sum_{j=0}^{r}\theta_j p_t(\theta_j),\quad \mu(t)=\sum_{j=0}^{r}\mu(t|\theta_j)p_t(\theta_j),\\&\mu(\tau_k,t)=\sum_{j=0}^{r}\mu(\tau_k,t|\theta_j)p_t(\theta_j),\end{aligned}\tag{6.2}$$

where $\mu(t|\theta_j)$ and $\mu(\tau_k,t|\theta_j)$ are defined in (4.1). Consequently subject to (4.3) are satisfied with respect to Propositions 1 and 2, where $\overline{h(t,z_t|\theta_j)}$ and $C(\eta(t_m),z_{t_m}|\theta_j)$ are denoted by (4.25), (4.26),

$$\overline{h(t,z_t)}=\sum_{j=0}^{r}\overline{h(t,z_t|\theta_j)}p_t(\theta_j),\tag{6.3}$$

$$C\big(\eta(t_m),z_{t_m}\big)=\sum_{j=0}^{r}C\big(\eta(t_m),z_{t_m}|\theta_j\big)p_{t_m-0}(\theta_j),\tag{6.4}$$

and arise from (3.7)–(3.10). Thus, in the Lainiotis sense adaptive estimations (Lainiotis, 1971) of the filtering $\mu(t)$ and the interpolation $\mu(\tau_k,t)$, $k=\overline{1;N}$, for the process x_t in the case of continuous-discrete observations with fixed memory have been obtained.

7. Conclusion

1. The derived problem solution of likelihood ratio determination in the general hypothesises recognition problem, when are observed the set of the processes with continuous and discrete time which depend both on current and former values of an unobservable process includes a solution of particular problems (see Remark 2).
2. An approach applied to the recognition problem consideration allowed to solve a combined problem of adaptive filtering and adaptive interpolation.
3. As it follows from the particular problem considered on the basis of general results in i.5 the presence of memory can either improve or worsen the quality of recognition procedure, which is determined by a memory depth, a ratio between the transmission coefficients of signal through the observation channels and its statistical properties.
4. The obtained theoretical results can be applied at solving practical recognition problems of complex systems, characterized by the continuous-discrete information availability and inertial observation channels (inertial information transmission channels).

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N.S. Dyomin graduated from Department of Radio and Physics, Tomsk State University, Russia, in 1967. He received the Ph.D. degree in Technical Cybernetics and Information Theory from Tomsk State University in 1987 and the Dr.Sci. degree in Application of Mathematical Methods in Scince Investigations from the same university in 1990. He is an Full Professor in Departmant of Applied Mathematics and Cybernetics of Tomsk State University. Present research interests: filtering, interpolation, extrapolation and recognition of stochastic processes; transmission of stochastic signals along continuous-discrete observation channels with memory; detection of anomalous noises; mathematical theory of financial analysis and security trading.

S.V. Rozhkova graduated from Departmant of Applied Mathematics and Cybernetics of Tomsk State University, Russia, in 1993. She received the Ph.D. degree in Application of Mathematical Methods in Science Investigations from Tomsk State University in 1996. She is an Associated Professor in Departmant of Natural Sciences and Mathematics of Tomsk Politechnical University. Present research interests: filtering, interpolation, extrapolation and recognition of stochastic processes; transmission of stochastic signals along continuous-discrete observation channels with memory; detection of anomalous noises.

O.V. Rozhkova graduated from Departmant of Applied Mathematics and Cybernetics of Tomsk State University, Russia, in 1988. She is an Assistant Professor in Departmant of Natural Sciences and Mathematics of Tomsk Politechnical University. Present research interests: filtering, interpolation, extrapolation and recognition of stochastic processes; detection of anomalous noises.

Tikėtinumo santykio nustatymas stochastinių procesų atpažinimo uždaviniuose, priimant domėn aibę įsimintų tolygių ir diskrečių stebėjimų

Nikolas DYOMIN, Svetlana ROZHKOVA, Olga ROZHKOVA

Straipsnyje nagrinėjamas tikėtinumo santykio nustatymas tolygaus laiko stochastinių procesų atpažinimo uždaviniuose, priimant domėn aibę įsimintų tolygių ir diskrečių stebėjimų. Ištirta tokios atminties įtaka anomalių triukšmų nustatymo kokybei stebint diskretinį kanalą.