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DETECTION OF EMERGENCE OF SLOW LINEAR CHANGES IN THE PROPERTIES OF RANDOM PROCESSES

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Abstract. The problem of change point detection when the properties of the random process observed suddenly begin changing slowly is considered. The most probable time moments of changes are investigated. Random processes are described by autoregression equations. The situation is studied when slow changes in the properties of a random process take place according to the linear law. An example of solving the problem is presented, realized by computer.

Key words: random process, autoregression, detection, most probable change.

1. Introduction. The functioning of many dynamic systems of animate and inanimate nature is accompanied by random phenomena, which represent random processes. These random processes reflect in a certain way the functional states of dynamic systems. Therefore, while analysing the properties of the random processes observed one can define the functional states of dynamic systems, generating them.

There is a great many of methods meant for detection of jumpwise changes in the states of dynamic systems. They are based on the solution of problems of jumpwise change detection in the properties of random processes (Willsky, 1978; Shaban, 1980; Kassam, 1980; Kligienė and Telksnys, 1983). However, in practice we come upon another important group of problems, how to detect at random arising slow changes in the properties of stochastic dynamic systems or random processes. It is necessary to make most probable decisions thereby on the appearing changes. Such a decision making is necessitated, for example, in the determination of the outset of drug effect on the organism, in the exposure of deterioration of mechanism units, determination of the onset of increasing concentration of environment pollution.

Unfortunately, there are no methods for detecting most probable instants of the emergence of slow changes in the properties of stochastic dynamic systems or random processes up till now. The investigations considering the problems of detection of slowly changing properties in random processes or dynamic systems do not provide answers to these questions (Safaryan, 1988; Darhovskii, 1989).

That is why this paper deals with the problem of detection of the time moment at which slow changes in the properties of the observed random process begin all of a sudden. The emergence of a most probable change point in the properties is being sought for. Random processes are described by autoregression equations. The changes in the properties take place according to the linear law. Algorithms for solving the problem are presented, convenient to realize by computers. The results of experiments are also given.

2. Statement of the problem. Let us consider a situation when stochastic dynamic systems can be in two states. From the beginning of observations $t = n_0 + 1$ up till a random time moment $t = U$ a stochastic dynamic system is in state 1. From the time moment $t = U + r + 1$ up till the end of observation $t = n$ it is in state 2. In the interval $t = U + 1, \ldots, U + r$

the properties of the stochastic dynamical system are changing slowly according to the linear law, passing from state 1 to state 2.

The behavior of dynamic systems under consideration is represented through its output coordinate and is expressed by the difference equation:

$$X_t(U) =$$

$$= \begin{cases} -\sum_{i=1}^{P_1} a_i^{(1)} x_{t-i} + b_1 V_t & (t = n_0 + p_1 + 1, \ldots, U) \\ -\sum_{i=1}^{P_2} \alpha_i x_{t-i} + \beta V_t & (t = U + 1, \ldots, U + r) \\ -\sum_{i=1}^{P_2} a_i^{(2)} x_{t-i} + b_2 V_t & (t = U + r + 1, \ldots, n), \end{cases} \quad (1)$$

where

$$a_i^{(s)}(i = 1, \ldots, p_s; \quad s = 1, 2),$$
$$\alpha_i = a_i^{(1)} + (a_i^{(2)} - a_i^{(1)})(i - u)r^{-1} \quad (i = 1, \ldots, p_2),$$
$$\beta = b_1 + (b_2 - b_1)(i - u)r^{-1},$$
$$b_s(s = 1, 2)$$

are the parameters of equation (1);
apart from that the parameters $a_i^{(s)}(i = 1, \ldots, p_s;\ s = 1, 2)$ and $\alpha_i(i = 1, \ldots, p_2)$ are such that the roots $z_{si}(i = 1, \ldots, p_s;$ $s = 1, 2)$ of characteristic equations

$$z_s^{p_s} + a_1^{(s)} z_s^{p_s - 1} + \cdots + a_{p_s}^{(s)} = 0 \quad (s = 1, 2)$$

as well as the roots z_i $(i = 1, \ldots, p_z)$ of the characteristic equation

$$z_s^{p_2} + \alpha_1 z_s^{p_2 - 1} + \cdots + \alpha_{p_2} = 0$$

are within the unit circle, i.e. $|z_{si}| < 1$ ($i = 1, \ldots, p_s$; $s = 1, 2$) and $|z_i| < 1$ ($i = 1, \ldots, p_2$).

V_t is a sequence of normal independent random variables with zero expectance and a unitary variance, i.e. $EV_t = 0$, $EV_t^2 = 1$.

X_t ($t = n_0 + 1, \ldots, n_0 + p_1$) are random initial conditions, described by covariance matrices:

$$K_1 = [k_{ij}^{(1)}] \quad (i, j = 1, \ldots, p_1), \tag{2}$$

where

$$k_{ij}^{(1)} = k_{ji}^{(1)} = k_{i,i+\rho}^{(1)} = k_\rho^{(1)} = k_0^{(1)} r(\rho) \; (\rho = 1, \ldots, p_1 - 1),$$
$$r(\rho) = r_\rho (\rho = 1, \ldots, p_1 - 1)$$

are defined by the solution of equation (Kligienė, 1973).

$$\begin{pmatrix} 1 + a_2^{(1)} & a_3^{(1)} & a_4^{(1)} & \ldots & a_{p_1-1}^{(1)} & a_{p_1}^{(1)} \\ a_1^{(1)} + a_3^{(1)} & 1 + a_4^{(1)} & a_5^{(1)} & \ldots & a_{p_1}^{(1)} & 0 \\ a_2^{(1)} + a_4^{(1)} & a_1^{(1)} + a_5^{(1)} & 1 + a_6^{(1)} & \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ a_{p_1-3}^{(1)} + a_{p_1-1}^{(1)} & a_{p_1-4}^{(1)} + a_{p_1}^{(1)} & a_{p_1-5}^{(1)} & \ldots & 1 & 0 \\ a_{p_1-2}^{(1)} + a_{p_1-3}^{(1)} & a_{p_1-3}^{(1)} & a_{p_1-4}^{(1)} & \ldots & a_1^{(1)} & 0 \end{pmatrix}$$

$$\times \begin{pmatrix} r(1) \\ r(2) \\ r(3) \\ \vdots \\ r(p_1 - 2) \\ r(p_1 - 1) \end{pmatrix} = \begin{pmatrix} a_1^{(1)} \\ a_2^{(1)} \\ a_3^{(1)} \\ \vdots \\ a_{p_1-2}^{(1)} \\ a_{p_1-1}^{(1)} \end{pmatrix}, \tag{3}$$

$$r(p_1) = -a_1^{(1)} r(p_1 - 1) - a_2^{(1)} r(p_1 - 2) - \cdots - a_{p_1-1}^{(1)} r(1) - a_{p_1}^{(1)},$$

$$k_0^{(1)} = b_1^2 [1 + a_1^{(1)} r(1) + \cdots + a_{p_1}^{(1)} r(p_1)]^{-1}.$$

A sample $x_t(u)\,(t = n_0 + 1, \ldots, n)$ of the random sequence $X_t(U = u)(t = n_0 + 1, \ldots, n)$, described by equation (1), is being observed. The parameters of equation (1) $a_i^{(s)}(i = 1, \ldots, p_s; s = 1, 2)$, $\alpha_i(i = 1, \ldots, p_2)$, β, $b_s(s = 1, 2)$ $p_s(s = 1, 2)$ and r are known.

It is necessary to define the most probable estimate u^* of the $t = u$ at which the properties of the stochastic dynamic system begin changing.

3. The method for the solution of the problem. Let us define the most probable time moment u^* at which the properties of the stochastic dynamic system (and the properties of the observed random process $x_t(t = n_0 + 1, \ldots, n)$ at the same time) begin changing from the maximum of the logarithm of the a posteriori probability density function $F(u|x_{n_0+1}, \ldots, x_n) = p(u) f(u|x_{n_0+1}, \ldots, x_n)$, i.e.

$$u^* = \underset{n_0+p_1+1 \leqslant u \leqslant n-r}{\arg\max} \{\ln f(u|x_{n_0+1}, \ldots, x_n) + \ln p(u)\}, \quad (4)$$

where $\ln f(u|x_{n_0+1}, \ldots, x_n) = \ln(y_{n_0+1} = x_{n_0+1}, \ldots, y_n = x_n|u)$ is the likelihood function.

The random sequence $X_t(U = u)$ is conditionally normal. In this case the conditional probability density function can be written like this:

$$p(y_{n_0+1}, \ldots, y_n|u) = (2\pi)^{-\frac{1}{2}(n-n_0)} (\det K)^{-\frac{1}{2}} \times \exp\Big\{ -\frac{1}{2} \sum_{i,j=n_0+1}^{n} \varkappa_{ij} x_i x_j \Big\}. \quad (5)$$

$\det K$ is a determinant of the matrix $K = [k_{ij}]$ $(i, j = n_0 + 1, \dots, n)$ whose elements represent the values of the correlation function $k_{ij} = E(X_i X_j)(i, j = n_0 + 1, \dots, n)$; $\varkappa_{ij}$ are the elements of the matrix $K^{-1} = [\varkappa_{ij}](i, j = n_0+1, \dots, n)$ inverse to the matrix $K = [k_{ij}](i, j = n_0 + 1, \dots, n)$;

$p(u)$ is an a priori probability density function of event $t = U$.

When solving practical problems the number of components of the observed random sequence $x_t = (t = n_0+1, \dots, n)$ amounts to thousands and even more. Therefore, it is actually impossible to make use of formula (4) directly. The computations are extremely labour-consuming and cumbersome. That is why we transform expressions (4) in such a way that it were possible in fact to estimate u^* by means of computers.

4. Transformation of the expressions. In order to avoid the difficulties of calculations while determining the most probable time moment u^*, at which a gradual change in the properties of the observed random sequence starts, we present the conditional probability density function $p(x_{n_0+1}, \dots, x_n | u)$ in such a manner:

$$p(y_{n_0+1}, \dots, y_n | u) = p(y_{n_0+1}, \dots, y_{n_0+p_1}) \times p(y_{n_0+p_1+1}, \dots, y_n | u,\ x_{n_0+1}, \dots, x_{n_0+p_1}), \tag{6}$$

where $p(y_{n_0+1}, \dots, y_{n_0+p_1})$is the probability density function of the first p_1 components of the random sequence $x_t(t = n_0 + 1, \dots, n_0 + p_1)$;

$$p(y_{n_0+p_1+1}, \dots, y_n | u,\ x_{n_0+1}, \dots, x_{n_0+p_1}) = \tag{7}$$

$$= \prod_{t=n_0+p_1+1}^{u} p(y_t | x_{t-1}, \dots, x_{t-p_1}) \times \tag{8}$$

$$\times \prod_{t=u+1}^{u+r} p(y_t | x_{t-1}, \dots, x_{t-p_2}) \times \tag{9}$$

$$\times \prod_{t=u+r+1}^{n-r} p(y_t|x_{t-1},\dots,x_{t-p_2}). \tag{10}$$

Further we consider expression (6) more in detail.

The first component $p(y_{n_0+1},\dots,y_{p_1})$ of expression (6) has the following form:

$$p(y_{n_0+1},\dots,y_n) =$$
$$= (2\pi)^{-\frac{p_1}{2}}[\det K_1]^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\sum_{i,j=n_0+1}^{n_0+p_1} \varkappa_{ij}^{(1)} y_i y_j\right], \tag{11}$$

where $\det K_1$ represents a determinant of the covariance matrix (2); $\varkappa_{ij}$ is the element of the matrix $K_1^{-1} = [\varkappa_{ij}^{(1)}](i,j = n_0+1,\dots,p_1)$ inverse to K_1.

Let us consider the components of (8), (9), (10) of the second factor of expression (6) separately and express them as follows:

$$p(y_t|x_{t-1},\dots,x_{t-p_1}) =$$
$$= (2\pi D_1)^{-\frac{1}{2}} \exp\left[-\frac{1}{2D_1}(y_t - E_1)^2\right]$$
$$(t = n_0 + p_1 + 1,\dots,u); \tag{12}$$

$$p(y_t|x_{t-1},\dots,x_{t-p_2}) =$$
$$= (2\pi D_{12})^{-\frac{1}{2}} \exp\left[-\frac{1}{2D_{12}}(y_t - E_{12})^2\right]$$
$$(t = u+1,\dots,u+r); \tag{13}$$

$$p(y_t|x_{t-1},\dots,x_{t-p_2}) =$$
$$= (2\pi D_2)^{-\frac{1}{2}} \exp\left[-\frac{1}{2D_2}(y_t - E_2)^2\right]$$
$$(t = u+r+1,\dots,n-r). \tag{14}$$

In (12), (13), (14) E_1, E_{12}, E_2 denote conditional expectances, and D_1, D_{12}, D_2 stand for conditional variances, defined by the formulas:

$$\begin{aligned}
E_1 &= E\{X_t|x_{t-1},\dots,x_{t-p_1}\} = \\
&= E\Big\{-\sum_{i=1}^{p_1} a_i^{(1)} x_{t-i} + b_1 V_t\Big\} = \\
&= \sum_{i=1}^{p_1} a_i^{(1)} x_{t-i} (t = n_0 + p + 1, \dots, u); \qquad (15)\\
E_{12} &= E\{X_t|x_{t-1},\dots,x_{t-p_2}\} = \\
&= E\Big\{-\sum_{i=1}^{p_2} \alpha_i^{(1)} x_{t-i} + \beta_1 V_t\Big\} = \\
&= -\sum_{i=1}^{p_2} \alpha_i^{(1)} x_{t-i} (t = u + 1, \dots, u + r); \qquad (16)\\
E_2 &= E\{X_t|x_{t-1},\dots,x_{t-p_2}\} = \\
&= E\Big\{-\sum_{i=1}^{p_2} a_i^{(2)} x_{t-i} + b_2 V_t\Big\} = \\
&= \sum_{i=1}^{p_2} a_i^{(2)} x_{t-i} (t = u + r + 1, \dots, n - u_r); \qquad (17)\\
D_1 &= E\{(X_t|x_{t-1},\dots,x_{t-p_1}) - E_1\}^2 = \\
&= E\{b_1 V_t\}^2 = b_1^2 (t = n_0 + p_1 + 1, \dots, u); \qquad (18)\\
D_{12} &= E\{(X_t|x_{t-1},\dots,x_{t-p_2}) - E_{12}\}^2 = \\
&= E\{\beta V_t\}^2 = \beta^2 (t = u + 1, \dots, u + r); \qquad (19)\\
D_2 &= E\{(X_t|x_{t-1},\dots,x_{t-p_2}) - E_2\}^2 = \\
&= E\{b_2 V_t\}^2 = b_2^2 (t = u + r + 1, \dots, n - r). \qquad (20)
\end{aligned}$$

Using formulas (6), ..., (20) one can write

$$p(y_{n_0} + p_1 + 1, \dots, y_n|u) =$$

$$= (2\pi)^{-\frac{n-r-n_0-p_1}{2}} b_1^{-u+n_0+p_1} \beta^{-r} b_2^{-n+u+2r} \times$$

$$\times \exp\Big\{ -\frac{1}{2b_1^2} \sum_{t=n_0+p_1+1}^{u} \Big[y_t + \sum_{i=1}^{p_1} a_i^{(1)} x_{t-i} \Big]^2 -$$

$$-\frac{1}{2\beta^2} \sum_{t=u+1}^{u+r} \Big[y_t + \sum_{i=1}^{p_2} \alpha_i x_{t-i} \Big]^2 -$$

$$-\frac{1}{2b_2^2} \sum_{t=u+r+1}^{n-r} \Big[y_t + \sum_{i=1}^{p_2} a_i^{(2)} x_{t-i} \Big]^2 \Big\}. \tag{21}$$

Taking into consideration (5), (11), (21), the logarithm of the likelihood function can be expressed in such a way:

$$\ln f(u|x_{n_0+1}, \ldots, x_n) =$$

$$= -\frac{p_1}{2} \ln(2\pi) - \frac{1}{2} \det K_1 - \frac{1}{2} \sum_{i,j=n_0+1}^{n_0+p_1} \varkappa_{ij}^{(1)} x_i x_j -$$

$$-\frac{n-r-n_0-p_1}{2} \ln(2\pi) - (u - n_0 - p_1) \ln b_1 - r \ln \beta -$$

$$-(n-u-2r) \ln b_2 - \frac{1}{2b_1^2} \sum_{t=n_0+p_1+1}^{u} \Big(\sum_{i=0}^{p_1} a_i^{(1)} x_{t-i} \Big)^2 -$$

$$-\frac{1}{2\beta^2} \sum_{t=u+1}^{u+r} \Big(\sum_{i=0}^{p_2} \alpha_i x_{t-i} \Big)^2 -$$

$$-\frac{1}{2b_2^2} \sum_{t=u+r+1}^{n-r} \Big(\sum_{i=0}^{p_2} a_i^{(2)} x_{t-i} \Big)^2. \tag{22}$$

Then formula (4) for the calculation of the most probabler time moment of slowly changing by the linear law properties of random sequences can be written like this:

$$u^* = \mathop{\arg\max}_{n_0+p_1+1 \leqslant u \leqslant n-r} \Big\{ -\frac{p_1}{2} \ln(2\pi) - \frac{1}{2} \det K_1 -$$

$$-\frac{1}{2}\sum_{i,j=n_0+1}^{n_0+p_1}\varkappa_{ij}^{(1)}x_ix_j-\frac{n-r-n_0-p_1}{2}\ln(2\pi)-$$
$$-(u-n_0-p_1)\ln b_1-r\ln\beta-(n-u-2r)\ln b_2-$$
$$-\frac{1}{2b_1^2}\sum_{t=n_0+p_1+1}^{u}\Big(\sum_{i=0}^{p_1}a_i^{(1)}x_{t-i}\Big)^2-$$
$$-\frac{1}{2\beta^2}\sum_{t=u+1}^{u+r}\Big(\sum_{i=0}^{p_2}\alpha_ix_{t-i}\Big)^2-$$
$$-\frac{1}{2b_2^2}\sum_{t=u+r+1}^{n-r}\Big(\sum_{i=0}^{p_2}a_i^{(2)}x_{t-i}\Big)^2+\ln p(u)\Big\}. \tag{23}$$

In the solution of practical problems $p_1 << n-r-n_0$, as a rule. In this case the magnitude of the first three components of expression (23) comprises a negligible part of values of the function $\ln f(u|x_{n_0+1},\dots,x_n)$ (Jenkins and Watts 1971, 1972). Omission of these components actually has no effect upon the results of calculations but it simplifies the calculations. Therefore, the values of u^* can be calculated by the formula

$$u^*\simeq u'=\underset{n_0+p_1+1\leqslant u\leqslant n-x}{\arg\max}\big\{\ln f'(u|x_{n_0+1},\dots,x_n)+$$
$$+\ln p(u)\big\}, \tag{24}$$

where

$$\ln f'(u|x_{n_0+1},\dots,x_n)=$$
$$=-\frac{n-r-n_0-p_1}{2}\ln(2\pi)-(u-n_0-p_1)\ln b_1-r\ln\beta-$$
$$-(n-u-2r)\ln b_2-\frac{1}{2b_1^2}\sum_{t=n_0+p_1+1}^{u}\Big(\sum_{i=0}^{p_1}a_i^{(1)}x_{t-i}\Big)^2-$$

$$-\frac{1}{2\beta^2}\sum_{t=u+1}^{u+r}\Big(\sum_{i=0}^{p_2}\alpha_i x_{t-i}\Big)^2-$$

$$-\frac{1}{2b_2^2}\sum_{t=u+r+1}^{n-r}\Big(\sum_{i=0}^{p_2}a_i^{(2)}x_{t-i}\Big)^2+\ln p(u).$$

5. An example. Let us consider an example to illustrate the possibilities of practical application of the developed method for detection of slowly changing properties of random processes. Imagine such a situation.

We have a stochastic dynamic system. Up till a random time moment $t = U$ this system is functioning in a stationary condition. Let us call it condition 1. It can be, for instance, a normal working condition. After a random moment $t = U$ the functioning of the stochastic dynamic system begins changing. It can be, for instance, the appearance of undesirable failure in the system functioning. Let's call that condition 2. At the beginning of condition 2 there is a transition process. The transition process of disorder in the functioning of the stochastic dynamic system lasts up till the time moment $t = U + r$. In the sequel the system continues working in the settled condition 2.

An output coordinate $x_t(t = n_0+1,\dots,n)$ of the stochastic dynamic system is being observed. It is necessary to detect a most probable instant $t = u'$ of the appearance of disorder in the functioning of the stochastic dynamic system in condition 1.

The output coordinate of the stochastic dynamic system represents a random sequence $X_t(t = n_0+1,\dots,n)$, described by equation (1). In the interval $t = n_0+1,\dots,u$ its properties are determined by the parameters p_1; $a_1^{(1)}$; $a_2^{(1)}$; b_1 while the stochastic dynamic system is functioning in condition 1.

During working condition 2 the situation is as follows. In the interval $t = u+1,\dots,u+r$ the properties of the random sequences are described by the parameters p_2; $\alpha_1 = a_1^{(1)}(a_1^{(2)} - a_1^{(1)})(i-u)r^{-1}$; $\alpha_2 = a_2^{(1)}(a_2^{(2)} - a_2^{(1)})(i-u)r^{-1}$; $\beta = b_1 + (b_2 - b_1)(i-u)r^{-1}$; $a_1^{(2)}$; $a_2^{(2)}$; b_2. After the instant $t = u + r$ and up to the end of observation $t = n$ the properties of the random process are defined by the parameters p_2; $a_1^{(2)}$; $a_2^{(2)}$; b_2. There are no a priori data on the random instant $t = U$ of the emergence of changes in the working condition of the stochastic dynamic system. Therefore, it is assumed that the emergence of such an phenomenon during the interval of observation is equiprobable, i.e., $p(u) = \text{const}$.

The estimates of the most probable instant $t = u'$, when the properties of the random sequence x_t begin slowly changing and $p_1 = 2$, can be defined according formula (24) in the following way

$$u' = \underset{n_0+3 \leqslant u \leqslant n-r}{\arg\max} \; L_1(u), \tag{25}$$

where

$$\begin{aligned} L_1(u) = {} & \frac{n-r-n_0-2}{2} \ln(2\pi) - (u - n_0 - 2)\ln b_1 - \\ & - r\ln\beta - (n-u-2r)\ln b_2 - \\ & - \frac{1}{2b_1^2} \sum_{t=n_0+3}^{u} \left(x_t + a_1^{(1)} x_{t-i} + a_2^{(1)} x_{t-2}\right)^2 - \\ & - \frac{1}{2\beta^2} \sum_{t=u+1}^{u+r} \left(x_t + \alpha_1 x_{t-1} + \alpha_2 x_{t-2}\right)^2 - \\ & - \frac{1}{2b_2^2} \sum_{t=u+r+1}^{n-r} \left(x_t + a_2^{(1)} x_{t-i} + a_2^{(2)} x_{t-2}\right)^2. \end{aligned} \tag{26}$$

As the first example in figure 1 the results of an experiment are presented, when the properties of the random process are expressed like this: $p_1 = 2$; $a_1^{(1)} = -0,75$; $a_2^{(1)} = 0,5$; $b_1 = 1$; $p_2 = 2$; $a_1^{(2)} = 0$; $a_2^{(2)} = 0,5$; $b_2 = 1$; $n_0 = 0$; $r = 200$;

$n = 1000$. And slow changing in the properties starts at the instant $t = u = 400$.

$K_1(T)$ and $K_2(T)$ illustrate the correlation functions of the random sequence X_t in the intervals $1 \leqslant t \leqslant u$ and $u + r + 1 \leqslant t \leqslant n$, respectively. $x_t(t = 1, \ldots, 1000)$ presented the sample of the random process $X_t(t = 1, \ldots, 1000)$ described by equation (1) with the parameters given above. $L_1(u)$ represents function (26), according to the maximum of which the most probable instant u' of the start of a gradual linear change in the properties of the random sequence is defined. $L_2(u)$ represents the function which would be obtained using the methods for detection of most probable changes $t = u''$ in the properties of jumpwise changes in random sequences (Telksnys, 1975).

As the second example in fig. 2 the results of experiment for an analogous case are presented, when the properties of the random process are expressed like this: $p_1 = 2; a_1^{(1)} = -0,9$; $a_2^{(1)} = -0,54$; $b_1 = 1$; $p_2 = 2$; $a_2^{(1)} = 0,203$; $a_2^{(1)} = 0,073$; $b_2 = 1$.

The estimates of instants u' of the heginning of linear changes in the properties of random processes are stochastic. That is why it is desirable to determine the laws of probability distributions of such estimates. Unfortunately, to solve this problem in an analytical way was not a success as yet. In order to evaluate the probability distribution laws for estimation of the instants of changes in the properties of random processes computerised means of simulation have been constructed.

Fig. 3 illustrates the histograms $\widehat{h}_1(u)$, $\widehat{h}_2(u)$ of the estimates of u' and u'' for the cases when random processes are described in the same way as in example 1. The number of experiments to obtain the estimates in simulation $M = 1000$. Analogous data are given in fig. 4 for the case when random processes are expressed in the same way as in example 2. The

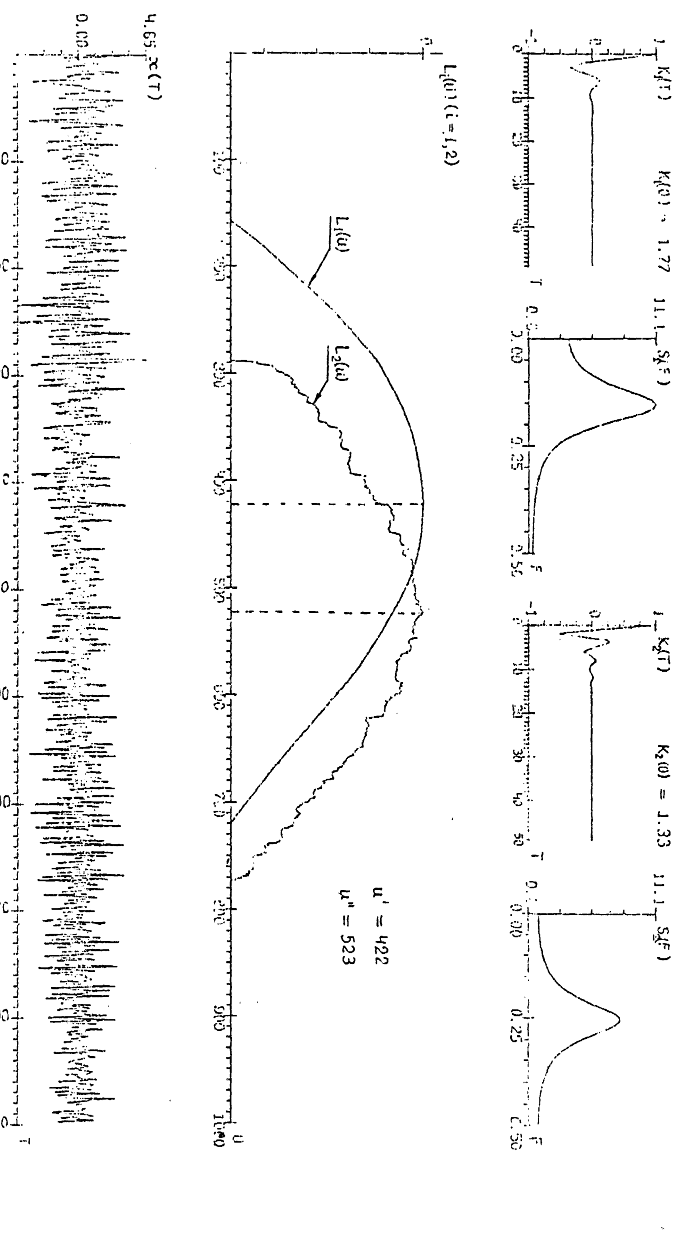

Fig. 1. Results of an experiment 1.

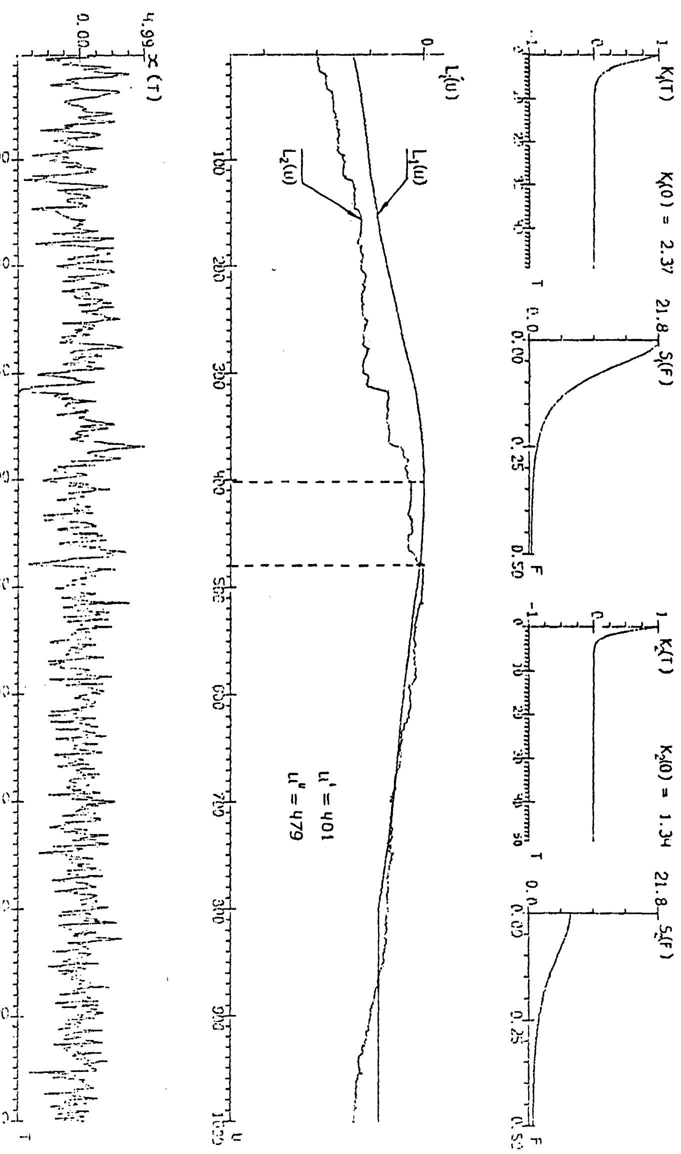

Fig. 2. Results of an experiment 2.

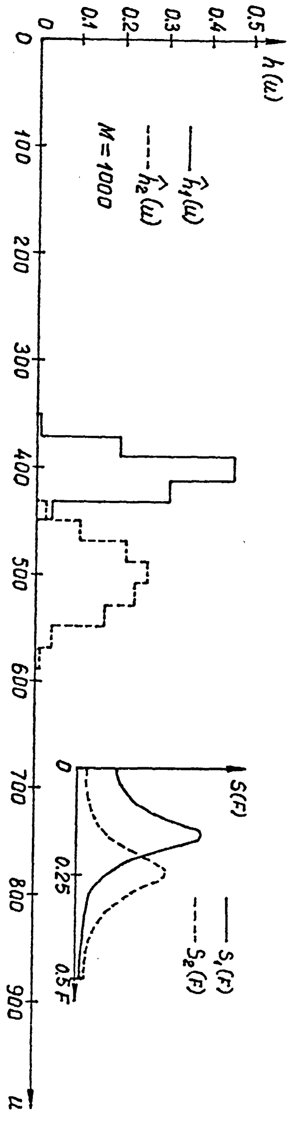

Fig.3. Simulation results. Example 1.

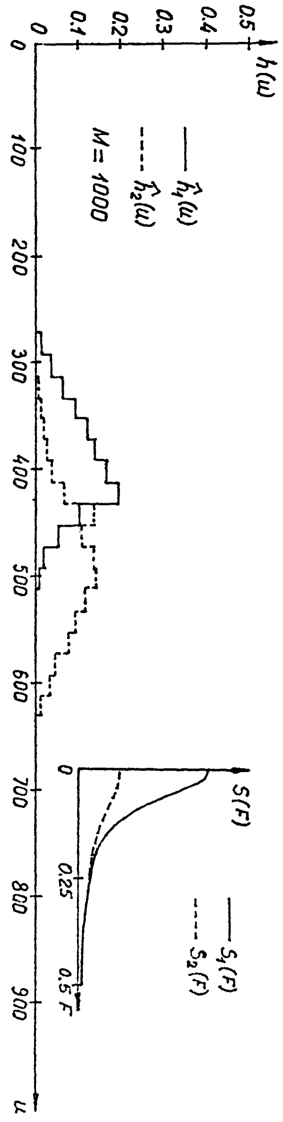

Fig. 4. Simulation results. Example 2.

presented results show that usage of the algorithm considering the presence of a gradual change in the properties of a random sequence enables us to detect the emergence of the change point in the properties more precisely.

6. Conclusions. The above statements show that the described method for detection of slow linear changes in the properties of random processes allows us to determine the presence of changes in the properties more exactly than the known methods for detecting jumpwise changes in the properties of random properties. This method can be realised conveniently by means of computers and applied to solve practical problems. It can be modified for the case when parameters of the system are unknown and the properties of random processes change according to the laws different from linear ones.

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