

On Characterization of Certain Wright's Generalized Hypergeometric Functions Involving Certain Subclasses of Analytic Functions

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Abstract. This paper presents several results associated with the Wright's generalized hypergeometric function, depicting their interesting characterization properties. Special cases are also pointed out.

Key words: Wright's generalized hypergeometric function, starlike function, convex function, univalent function, Bessel-Maitland and Mittag-Leffler functions.

1. Introduction and Preliminaries

Let E be the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (1.1)$$

which are regular and univalent in the open unit disk $U = \{z: |z| < 1\}$. Let $S(A, B)$ represent the class of functions $f(z) \in E$ satisfying the inequality

$$\left| \frac{(zf'(z)/f(z)) - 1}{A - B(zf'(z)/f(z))} \right| < 1 \quad (z \in U), \quad (1.2)$$

where $-1 \leq B < A \leq 1$, and $-1 \leq B \leq 0$.

Also, a function $f(z) \in E$ is in the class $K(A, B)$, if and only if $zf'(z) \in S(A, B)$. One may refer to Srivastava and Shigeyoshi Owa (1992) for aforesaid definitions and other related details. The Wright's generalized hypergeometric function (Srivastava and

Manocha, 1984) is defined by

$$\begin{aligned} {}_p\Psi_q[z] &= {}_p\Psi_q \left[\begin{matrix} (\lambda_1, A_1), \dots, (\lambda_p, A_p); \\ (\mu_1, B_1), \dots, (\mu_q, B_q); \end{matrix} z \right] \\ &= \sum_{n=0}^{\infty} \left\{ \prod_{i=1}^p \Gamma(\lambda_i + A_i n) \right\} \left\{ \prod_{j=1}^q \Gamma(\mu_j + B_j n) \right\}^{-1} \frac{z^n}{n!}, \end{aligned} \quad (1.3)$$

where the coefficients A_i ($i = 1, \dots, p$) and B_j ($j = 1, \dots, q$) are positive real numbers such that

$$1 + \sum_{j=1}^q B_j - \sum_{i=1}^p A_i \geq 0.$$

The function ${}_p\Psi_q[z]$ reduces to the generalized hypergeometric function ${}_pF_q[z]$ when $A_i = 1$ ($i = 1, \dots, p$) and $B_j = 1$ ($j = 1, \dots, q$) in (1.3).

An integral operator $J_\gamma(f)$ is defined by Bernardi (1969)

$$J_\gamma(f) = \frac{\gamma + 1}{z^\gamma} \int_0^z t^{\gamma-1} f(t) dt \quad (\gamma > 0). \quad (1.4)$$

A generalization of (1.4) was introduced in (Jung *et al.*, 1993) and is defined by

$$\phi_\gamma^\alpha f(z) = \left(\frac{\alpha + \gamma}{\gamma} \right) \frac{\alpha}{z^\gamma} \int_0^z \left(1 - \frac{t}{z} \right)^{\alpha-1} t^{\gamma-1} f(t) dt, \quad (\alpha > 0, \gamma > 0). \quad (1.5)$$

The linear operator $T_C^A(a, c)$ studied by Raina (1997) (which is a generalization of Carlson–Shaffer’s linear operator (Carlson and Shaffer, 1984)) defined by means of the usual convolution $*$ as

$$T_C^A(a, c)f(z) = \theta_C^A(a, c; z) * f(z) \quad (f \in E), \quad (1.6)$$

where

$$\theta_C^A(a, c; z) = \frac{\Gamma(c)}{\Gamma(a)} z {}_2\Psi_1 \left[\begin{matrix} (a, A), (1, 1); \\ (c, C); \end{matrix} z \right].$$

The operator $L(a, c)$ is defined by Carlson and Shaffer (1984)

$$L(a, c)f(z) = \theta_1^1(a, c) * f(z) = T_1^1(a, c)f(z). \quad (1.7)$$

It is easy to verify that the functions $K(A, B)$ and $S(A, B)$ are connected by the relation

$$K(A, B) = T_1^1(1, 2)S(A, B). \quad (1.8)$$

In this paper we present various results involving the Wright’s generalized hypergeometric function and study some interesting characterization properties associated with certain subclasses of analytic functions. Special cases of our results are also pointed out.

2. Characterization Properties

We first state the following results due to Raina (1996), which shall be used in the sequel.

Lemma 1. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition*

$$\left| \frac{{}_p\Psi'_q \left[\begin{matrix} (\lambda_i, A_i)_{1,p}; \\ (\mu_j, B_j)_{1,q}; \end{matrix} z \right]}{{}_p\Psi_q \left[\begin{matrix} (\lambda_i, A_i)_{1,p}; \\ (\mu_j, B_j)_{1,q}; \end{matrix} z \right]} \right| < \frac{A - B}{1 + |B|} \quad (z \in U), \tag{2.1}$$

for $-1 \leq B < A \leq 1$, $-1 \leq B \leq 0$. Then $z \Delta {}_p\Psi_q[z] \in S(A, B)$, where

$$\Delta = \left\{ \prod_{i=1}^p \Gamma(\lambda_i) \right\}^{-1} \left\{ \prod_{j=1}^q \Gamma(\mu_j) \right\}. \tag{2.2}$$

Lemma 2. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition (2.1) for $z \in U$, and $-1 \leq B < A \leq 1$, $-1 \leq B \leq 0$. Then*

$$z \Delta {}_{p+1}\Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \in K(A, B), \tag{2.3}$$

where Δ is given by (2.2).

Lemma 3. *Let $f(z)$ be defined by (1.1), satisfy the condition*

$$\left| \frac{z f'(z)}{f(z)} - 1 \right|^{1-h} \left| \frac{z f''(z)}{f'(z)} \right| < \frac{(A - B)(2 + A + A^2)^h}{(1 + |B|)(1 + A)^{2h}} \quad (z \in U), \tag{2.4}$$

for fixed constants A, B and h , such that $-1 \leq B < A \leq 1$, $-1 \leq B \leq 0$, $h \geq 0$. Then $f(z) \in S(A, B)$.

Now we establish the following results depicting the characterization properties of the function ${}_p\Psi_q[z]$ defined by (1.3).

Lemma 4. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition*

$$\left| \frac{{}_p\Psi''_q \left[\begin{matrix} (\lambda_i, A_i)_{1,p}; \\ (\mu_j, B_j)_{1,q}; \end{matrix} z \right]}{{}_p\Psi'_q \left[\begin{matrix} (\lambda_i, A_i)_{1,p}; \\ (\mu_j, B_j)_{1,q}; \end{matrix} z \right]} \right| < \frac{(A - B)(2 + A + A^2)}{(1 + |B|)(1 + A)^2} \quad (z \in U), \tag{2.5}$$

for $-1 \leq B < A \leq 1$, $-1 \leq B \leq 0$. Then

$$z \nabla_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \in S(A, B), \quad (2.6)$$

where

$$\nabla = \left\{ \prod_{i=1}^p \Gamma(\lambda_i + A_i) \right\}^{-1} \left\{ \prod_{j=1}^q \Gamma(\mu_j + B_j) \right\}. \quad (2.7)$$

Proof. Consider the function $P(z)$ defined by

$$P(z) = \{ \cdot {}_p\Psi_q[z] - \Delta^{-1} \} \nabla.$$

It follows then that $P(z) \in E$ and satisfies

$$\left| \frac{zP''(z)}{P'(z)} \right| = \left| \frac{z {}_p\Psi_q''[z]}{{}_p\Psi_q'[z]} \right| < \frac{(A-B)(2+A+A^2)}{(1+|B|)(1+A)^2} \quad (z \in U).$$

Using Lemma 3, it follows then that $P(z) \in S(A, B)$.

That is

$$\begin{aligned} \int_0^z \frac{p(t)}{t} dt &\in K(A, B) \\ \implies \nabla \int_0^z t^{-1} \{ \cdot {}_p\Psi_q[t] - \Delta^{-1} \} dt &\in K(A, B) \\ \implies T_1^{-1}(1, 2) \left\{ \nabla z_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \right\} &\in K(A, B). \end{aligned}$$

Applying the inverse operator $T_1^{-1}(2, 1)$ on both sides, then by virtue of the relation (1.8), we arrive at the desired result of Lemma 4.

Lemma 5. Let $f(z)$ be defined by (1.1), and $f(z) \in S(A, B)$. Then

$$\phi_\gamma^\alpha f(z) \in S(A, B), \quad \text{for } \alpha > 0, \gamma > -1. \quad (2.8)$$

Proof. Let $f(z) \in S(A, B)$. Then

$$\left| \frac{(zf'(z)/f(z)) - 1}{A - B(zf'(z)/f(z))} \right| < 1.$$

It then follows that

$$\sum_{n=2}^{\infty} \{ (n-1) - (A - Bn) \} a_n \leq (A - B). \quad (2.9)$$

We have from (1.1) and (1.5) that

$$\phi_\gamma^\alpha f(z) = z + \sum_{n=2}^{\infty} \sigma(n) a_n z^n \quad (z \in U),$$

where

$$\sigma(n) = \frac{(1 + \gamma)_{n-1}}{(1 + \alpha + \gamma)_{n-1}} \quad (n \geq 2). \tag{2.10}$$

In order to show that $\phi_\gamma^\alpha f(z) \in S(A, B)$, it is sufficient to show that (in view of (1.2))

$$\left| \frac{\left[\left(z \frac{d}{dz} \phi_\gamma^\alpha f(z) \right) / (\phi_\gamma^\alpha f(z)) \right] - 1}{A - B \left[\left(z \frac{d}{dz} \phi_\gamma^\alpha f(z) \right) / (\phi_\gamma^\alpha f(z)) \right]} \right| < 1.$$

That is

$$\sum_{n=2}^{\infty} \{ (n - 1) - (A - Bn) \} a_n \sigma(n) \leq A - B. \tag{2.11}$$

We observe that for $\alpha > 0, \gamma > -1$, the function $\sigma(n)$ is a decreasing function of n ($n \geq 2$), and it follows that

$$0 < \sigma(n) \leq \sigma(2) < 1. \tag{2.12}$$

The assertion (2.11) follows easily from (2.9) and (2.12), and the proof is complete.

Lemma 6. *Let $f(z)$ be defined by (1.1), and $f(z) \in K(A, B)$. Then*

$$\phi_\gamma^\alpha f(z) \in K(A, B), \text{ for } \alpha > 0, \gamma > -1. \tag{2.13}$$

Proof. Let $f(z) \in K(A, B)$, then $zf'(z) \in S(A, B)$. Using Lemma 5, we have then $\phi_\gamma^\alpha(zf'(z)) \in S(A, B)$. It is easy to verify that

$$\begin{aligned} z \frac{d}{dz} (\phi_\gamma^\alpha f(z)) &= \phi_\gamma^\alpha(zf'(z)) \\ \implies z \frac{d}{dz} (\phi_\gamma^\alpha f(z)) &\in S(A, B) \\ \implies \phi_\gamma^\alpha f(z) &\in K(A, B), \end{aligned}$$

which proves Lemma 6.

3. Further Characterization Properties

Our characterization theorem for a class of Wright's generalized hypergeometric functions is contained in the following.

Theorem 1. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition (2.1) for $z \in U$. Then*

$$z^{\Delta_1} \cdot {}_{p+1}\Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (\gamma + 1, 1); \\ (\mu_j, B_j)_{1,q}, (\gamma + \alpha + 1, 1); \end{matrix} z \right] \in S(A, B) \quad (3.1)$$

for $\alpha > 0$, $\gamma > -1$, where

$$\Delta_1 = \frac{\Delta \Gamma(1 + \alpha + \gamma)}{\Gamma(1 + \gamma)}, \quad (3.2)$$

and Δ is given by (2.2).

Proof. In view of Lemma 1, we have $z \Delta_p \Psi_q[z] \in S(A, B)$. Then, we find that

$$\phi_\gamma^\alpha(z \Delta_p \Psi_q[z]) = \Delta \left(\frac{\alpha + \gamma}{\gamma} \right) \frac{\alpha}{z^\gamma} \int_0^z \left(1 - \frac{t}{z} \right)^{\alpha-1} t^\gamma {}_p\Psi_q[t] dt.$$

Evaluating the integral (by using the definition (1.3)), we obtain

$$\phi_\gamma^\alpha(z \Delta_p \Psi_q[z]) = z^{\Delta_1} \cdot {}_{p+1}\Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (\gamma + 1, 1); \\ (\mu_j, B_j)_{1,q}, (\gamma + \alpha + 1, 1); \end{matrix} z \right],$$

where Δ_1 is given by (3.2). Applying Lemma 5 to $z \Delta_p \Psi_q[z]$, we arrive at once to the assertion (3.1) of Theorem 1.

REMARK 1. If $A = 1 - 2\rho$, and $B = -1$, then the class of functions

$$S(A, B) = S(1 - 2\rho, -1) = S^*(\rho), \quad (3.3)$$

represents the class of functions in E which are starlike of order ρ ($0 \leq \rho < 1$). By setting the parameters in (1.3) as

$$A_i = 1 \quad (i = 1, \dots, p), \quad B_j = 1 \quad (j = 1, \dots, q), \quad (3.4)$$

and noting that

$${}_p\Psi_q \left[\begin{matrix} (\lambda_1, 1), \dots, (\lambda_p, 1); \\ (\mu_1, 1), \dots, (\mu_q, 1); \end{matrix} z \right] = \Delta_p F_q[z], \quad (3.5)$$

where Δ is given by (2.2), then by virtue of the specializations (3.3) with $(\rho = 0)$, (3.4), (3.5) and $\alpha = 1$, Theorem 1 corresponds to the known result (Srivastava and Owa, 1985, p. 200, Theorem 1).

For $\gamma = 0, \alpha = 1$, Theorem 1 gives the following result.

COROLLARY 1. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition (2.1) for $z \in U$. Then*

$$z \Delta_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \in S(A, B), \tag{3.6}$$

where Δ is given by (2.2).

Next we prove the following.

Theorem 2. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition (2.1) for $z \in U$. Then*

$$z \Delta_{1 \cdot p+2} \Psi_{q+2} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (\gamma + 1, 1), (1, 1); \\ (\mu_j, B_j)_{1,q}, (\gamma + \alpha + 1, 1), (2, 1); \end{matrix} z \right] \in K(A, B), \tag{3.7}$$

for $\alpha > 0, \gamma > -1$, where Δ_1 is given by (3.2).

Proof. In view of Lemma 2, we have

$$z \Delta_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \in K(A, B).$$

Then

$$\begin{aligned} & \phi_\gamma^\alpha \left\{ z \Delta_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \right\} \\ &= \Delta \binom{\alpha + \gamma}{\gamma} \frac{\alpha}{z^\gamma} \int_0^z \left(1 - \frac{t}{z}\right)^{\alpha-1} t^\gamma {}_{p+1}\Psi_{q+1}[t] dt. \end{aligned}$$

Evaluating the integral (by using the definition (1.3), we obtain

$$\begin{aligned} & \phi_\gamma^\alpha \left\{ z \Delta_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \right\} \\ &= z \Delta_{1 \cdot p+2} \Psi_{q+2} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (\gamma + 1, 1), (1, 1); \\ (\mu_j, B_j)_{1,q}, (\gamma + \alpha + 1, 1), (2, 1); \end{matrix} z \right], \end{aligned}$$

where Δ_1 is given by (3.2).

Applying Lemma 6 to $z \Delta_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right]$, we are lead to the as-
sertion (3.7) of Theorem 2.

For $\gamma = 1, \alpha = 1$, Theorem 2 gives the following result.

COROLLARY 2. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfies the condition (2.1) for $z \in U$. Then*

$$z \Delta_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i, A_i)_{1,p}, (1, 1); \\ (\mu_j, B_j)_{1,q}, (3, 1); \end{matrix} z \right] \in K(A, B), \quad (3.8)$$

where Δ is given by (2.2).

REMARK 2. In view of the parametric substitutions (3.4), and the fact that for $A = 1 - 2\rho$ and $B = -1$, the class of functions

$$K(A, B) = K(1 - 2\rho, -1) = K^*(\rho), \quad (3.9)$$

represents the class of all convex functions $f(z) \in E$, of order ρ ($0 \leq \rho < 1$), then Corollary 2 on using (3.5) and (3.9) (with $\rho = 0$) corresponds to the known result (Srivastava and Owa, 1985, p. 201).

Finally, we prove the following characterization theorem.

Theorem 3. *Let the Wright's generalized hypergeometric function ${}_p\Psi_q[z]$ defined by (1.3) satisfy the condition (2.5) for $z \in U$. Then*

$$z \nabla_{p \cdot p+2} \Psi_{q+2} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (\gamma + 1, 1), (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (\gamma + \alpha + 1, 1), (2, 1); \end{matrix} z \right] \in S(A, B), \quad (3.10)$$

for $\alpha > 0, \gamma > -1$, where

$$\nabla_1 = \frac{\nabla \Gamma(1 + \alpha + \gamma)}{\Gamma(1 + \gamma)}, \quad (3.11)$$

and ∇ is given by (2.7).

Proof. In view of Lemma 4, we have

$$z \nabla \cdot {}_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \in S(A, B).$$

Then we find that

$$\begin{aligned} & \phi_\gamma^\alpha \left\{ z \nabla_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \right\} \\ &= \nabla \left(\begin{matrix} \alpha + \gamma \\ \gamma \end{matrix} \right) \frac{\alpha}{z^\gamma} \int_0^z \left(1 - \frac{t}{z} \right)^{\alpha-1} t^\gamma {}_{p+1}\Psi_{q+1}[t] dt. \end{aligned}$$

Evaluating the integral (by using the definition (1.3)), we obtain

$$\begin{aligned} & \phi_\gamma^\alpha \left\{ z \nabla_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right] \right\} \\ &= z \nabla_1 \cdot {}_{p+2}\Psi_{q+2} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (\gamma + 1, 1), (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (\gamma + \alpha + 1, 1), (2, 1); \end{matrix} z \right], \end{aligned}$$

where ∇_1 is given by (3.11).

Applying Lemma 5 to the function $z \nabla_{p+1} \Psi_{q+1} \left[\begin{matrix} (\lambda_i + A_i, A_i)_{1,p}, (1, 1); \\ (\mu_j + B_j, B_j)_{1,q}, (2, 1); \end{matrix} z \right]$, we get the assertion (3.10) of Theorem 3.

For $A_i = 1$ ($i = 1, \dots, p$), $B_j = 1$ ($j = 1, \dots, q$) in Theorem 3, and using (2.7) and (3.5), we have the following result.

COROLLARY 3. *Let the generalized hypergeometric function ${}_pF_q[z]$ satisfy the condition*

$$\left| \frac{z {}_pF_q''[z]}{{}_pF_q'[z]} \right| < \frac{A - B}{1 + |B|} \quad (z \in U). \tag{3.12}$$

Then

$$\begin{aligned} & z {}_{p+2}F_{q+2} [\lambda_1 + 1, \dots, \lambda_p + 1, \gamma + 1, 1; \mu_1 + 1, \dots, \mu_q + 1, \gamma + \alpha + 1, 2; z] \\ & \in S(A, B). \end{aligned} \tag{3.13}$$

The special case of Corollary 3 (when $\alpha = 0$; and the class of functions $S(A, B)$ reduces to $S^*(0)$ by taking $A = 1, B = -1$) corresponds to the known result (Srivastava and Owa, 1985, p. 199).

4. Concluding Remarks

Due to the generality of the class of functions ${}_p\Psi_q[z]$, the results obtained in Sections 2–3 can be applied to various special functions, including the functions like the Bessel-

Maitland and Mittag-Leffler functions. By noting the relationships

$${}_1\Psi_1 \left[\begin{matrix} (1, 1); \\ (\beta, \alpha); \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\beta + \alpha n)} = z^{\frac{1-\beta}{\alpha}} E_{\alpha, \beta}(z), \quad (4.1)$$

and

$${}_0\Psi_1 \left[\frac{z^n}{(1 + \nu, \mu);}; z \right] = \sum_{n=0}^{\infty} \frac{z^n}{n! \Gamma(1 + \nu + \mu n)} = J_{\nu}^{\mu}(-z), \quad (4.2)$$

where $E_{\alpha, \beta}(z)$ and J_{ν}^{μ} denote the Mittag-Leffler and Bessel-Maitland functions, respectively, the results presented in Section 2 (Lemmas 4–6), and those in Section 3 (Theorems 1–3) can be applied to these functions defined by (4.1) and (4.2) above. These consequences of Lemmas 4–6 and Theorems 1–3 being straightforward, further results in this regard are hence omitted.

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Raito (Wright's) apibendrintųjų hipergeometrinių funkcijų, turinčių analizinių funkcijų poklasių, kai kurių charakteristikų klausimu

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Nagrinėjamos Raito (Wright's) apibendrintosios hipergeometrinės funkcijos. Įrodomos kai kurių jų charakteristikų savybės. Apibrėžiamos atskiros analizinių funkcijų klasės, sudarančios nagrinėjamų hipergeometrinių funkcijų poklasių.