INFORMATICA, 1997, Vol. 8, No. 2, 265–272

STEP-BY-STEP METHOD OF CONTROLLING OUTPUT FLOW RATE FOR THE QUEUEING SYSTEM WITH VARIABLE LOAD

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Abstract. The paper discusses the peculiarities of the mono-channel normalized queueing system model with the nonstationary arrival rate of customers. To stabilize the output flow rate at the desirable level – the on-line control system of the service intensity has to be organized. This problem can be solved by means of the automatic control theory approach.

Key words: queueing system, input flow, nonstationary, automatic control system, step-by-step.

1. Introduction and problem statement. Many information systems such as computers and computer networks may be simulated by means of queuieng system. In general, queueing systems model is developed assuming the arrival rate and service intensity to be in the equilibrium state. The well-known methods of the queueing system investigation are based on the stationary behaviour of the input flow and service duration (Kleinrok, 1979). Taking into account these characteristics as well as technical-economical criteria, the optimal system performance parameters are determined.

In real conditions the input flow arrival rate is affected by the step-by-step influence and the system state can essentially differ from the desired one. Here we come across the problem of compensating these differences with the purpose of equalizing the real value of output of customers' flow to the desirable one.

The main idea of this work lies in the identification of the queueing system as the control object with the further constructing discrete control closed scheme.

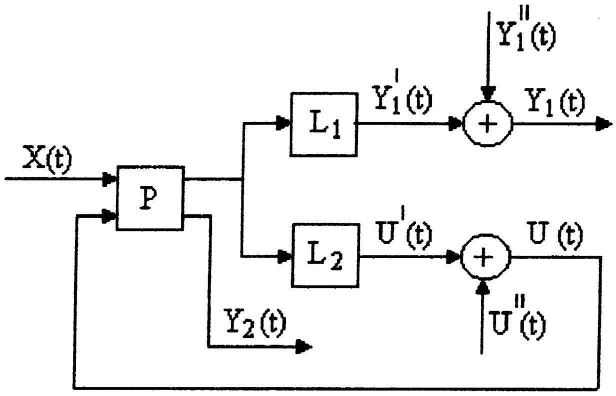

Fig. 1. Scheme of one channel normalized serving system with losses.

2. The object of control identification based on a single-channel queueing system model with losses. Judging from the works of Pugachev (1974) we can draw conclusion that the normalized servicing system without queues may be represented as the scheme (Fig. 1) with the following indications: p – the distributor determining the direction of customers' flow depending on the fact – whether channel is free or busy; L_1, L_2 – linear systems with the weight functions $\varphi(t-\tau)$ and $\psi(t-\tau)$ accordingly. These are $\varphi(\sigma)$ – probability density of service time and $\psi(\sigma)$:

$$\psi(\sigma) = \int_{\sigma}^{\infty} \varphi(\xi)\, d\xi;$$

$X(t)$ – incoming customers' flow, $Y_1(t)$ – served customers' flow, $Y_2(t)$ – rejected customers' flow. In control system $X(t)$, $Y_1(t)$ and $Y_2(t)$ represent sequence of δ – functions, which probably appear. On the other hand $Y_1^{II}(t)$ and $U^{II}(t)$ are inside Gauss system with zero mathematical expectation; $U(t)$ is a binary signal which can get values or being dependent on the channel conditions (free or busy) at the time moment t; $Y_1^{I}(t)$ and $U^{I}(t)$ are inside variables. Often $\varphi(\sigma) = \mu\exp(-\mu\sigma)$; $\psi(\sigma) = \exp(-\mu\sigma)$ take place, where μ is the service intensity.

Then the equation dynamics for the investigated system would be by Pugachev (1974):

$$\frac{dY_1^{|}}{dt} + (m_x + \mu)Y_1^{||} = m_x m_{y_1} - \mu m_x U^{||} + (\mu - m_{y_1})X,$$
$$Y_2 = \frac{m_{y_1}}{\mu}X + \frac{m_x}{\mu}\left(Y_1^{|} - m_y\right) + m_x U^{||},$$
$$Y_1 = Y_1^{|} + Y_2^{||}.$$

We should obtain mathematical expectations $Y_1(t)$ and $Y_2(t)$ from these equalizations:

$$\frac{dm_{y_1}}{dt} = -(m_x + \mu)m_{y_1} + \mu m_x,$$
$$m_{y_2} = \frac{m_{y_1} m_x}{\mu}.$$

Now let's linearize the system of equations regarding mathematical expectations in the neighbourhood of support points μ^0, m_x^0, $m_{y_1}^0$ and $m_{y_2}^0$.

As a result we have:

$$\tau\frac{d\Delta m_{y_1}}{dt} + \Delta m_{y_1} = k_1 \Delta\mu(t) - k_2 \Delta m_x(t),$$

where: $\tau = \dfrac{1}{(\mu^0 + m_x^0)}$, $k_1 = \dfrac{(m_x^0 - m_{y_1}^0)}{(m_x^0 - \mu^0)}$, $k_2 = \dfrac{(m_{y_1}^0 - \mu^0)}{(m_x^0 + \mu^0)}$, $\Delta m_x(t)$ – incoming traffic deviation, $\Delta\mu(t)$ – compensating influence, $\Delta m_{y_1}(t)$ – traffic requirements increase.

3. Synthesis of the service intensity changes law. Let's consider the system performance to be optimal (proper) if we find such intensity control in which influence of "jumping" changes in incoming traffic on the behaviour of $m_{y_1}(t)$ can be eliminated in a minimum time. Let's confine ourselves to research system with step-changes $\Delta\mu(t)$ of service intensity. The determination of $\Delta\mu(t)$ leads to the definition of the controlling sequence in the closed discrete scheme with the zero range extrapolation as well as with the minimum duration of transient. Discrete control system designed structural scheme is represented in Fig. 2. For minimizing equations symbol "increase" (Δ) will further be omitted in Fig. 2.

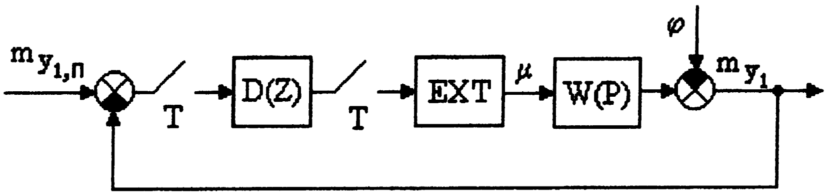

Fig. 2. Control system for intensity of output traffic.

These are some indications:

$m_{y_1,\Pi}$ – the projecting intensity value of output flow requirements which will be taken as constants;

$m_{y_1}(t)$ – the real value of rate in the output flow;

T – step value, in which control value $\mu(t)$ is constant;

$D(Z)$ – z-generating function of on-line control block;

$W(p) = k_1/(\tau p + 1)$ – control object operator;

$\varphi(p) = k_2 m_x(p)/(\tau p + 1)$;

$m_x(p)$ is the deviation influence;

p – the Laplas's form variable;

EXT – zero range extrapoler.

Let's find the best discrete control policy. z-generating function of the received uninterrupted system part

$$W_{\text{np}}(Z) = \frac{k_1 [1 - \exp(-T/\tau)]}{Z - \exp(-T/\tau)}.$$

According to Tsipkin (1977), Nickolsky (1973) one can find z-generating function of on-line block:

$$D(Z) = \frac{[z - \exp(-T/\tau)]\, a}{z - 1}.$$

Taking into account Fig. 2 transmittance in the inclosed system is determined:

$$W(Z) = D(Z) W_{\text{np}}(Z) = \frac{a k_1 [1 - \exp(-T/\tau)]}{Z - 1}.$$

Compose characteristically equalization of closed system:

$$1 + W(Z) = 0, \quad \text{or} \quad Z - 1 + a k_1 [1 - \exp(-T/\tau)] = 0.$$

Taking into account the finite duration of transient one can find:

$$a = \frac{1}{k_1\left[1 - \exp(-T/\tau)\right]}.$$

Determine z-form control influence caused by step-changing $m_{y_1,\Pi}$:

$$\begin{aligned}\delta\mu_1(Z) &= \frac{D(Z)}{1+W(Z)} m_{y_1,\Pi}(Z) \\ &= \frac{1}{k_1\left[1-\exp(-T/\tau)\right]} + \frac{1}{k_1} Z^{-1} + \frac{1}{k_1} Z^{-2} + \ldots.\end{aligned}$$

Find system service intensity increase which is necessary for compensating step changing intensity of input flow in Z-form:

$$\delta\mu_2(Z) = \frac{D(Z)\varphi(Z)}{1+W(Z)} = \frac{k_2 m_x}{Z-1},$$

where m_x = constant.

Compute increasing of service intensity in total:

$$\begin{aligned}\mu(Z) =& \delta\mu_1(Z) + \delta\mu_2(Z) \\ =& \frac{m_{y_1,\Pi}}{k_1\left[1-\exp(-T/\tau)\right]} + \left(k_2 m_x + \frac{m_{y_1,\Pi}}{k_1}\right) Z^{-1} \\ &+ \left(k_2 m_x + \frac{m_{y_1,\Pi}}{k_1}\right) Z^{-2} + \ldots .\end{aligned}$$

From this and (Tsipkin, 1977) one can obtain:

$$\begin{aligned}\mu_0 &= \frac{m_{y_1,\Pi}}{k_1\left[1-\exp(-T/\tau)\right]}, \\ \mu_i &= k_2 m_x + \frac{m_{y_1,\Pi}}{k_1}, \quad i = 1, \ldots, \infty,\end{aligned}$$

where i – number of control step.

The control variable $\mu(t)$ is often limited $|\mu| \leqslant \mu_{\text{доп}}$; where $\mu_{\text{доп}}$ is the admitting value for service productivity. The step value T of discrete control is determined from the condition:

$$\mu_{\text{доп}} = \frac{m_{y_1,\Pi}}{k_1\left[1-\exp(-T/\tau)\right]}.$$

Then T may be obtained:

$$T = \tau \ln \frac{1}{1 - \frac{m_{y_1,\text{П}}}{\mu_{\text{доп}} k_1}}.$$

In terms of $\mu(t)$ after checking one could see that transient is finished during one time spacing and the compensation of the step-by-step influence $m_x(t)$ is carried out. It is the fact – we'd like to prove.

Conclusions. In this article the attention is paid to the normalized queueing system model with the step-by-step service intensity changes. The most original fact of this approach is the presentation of the queueing system as the dynamical object with the corresponding control of influence and then with constructing the closed scheme with operating control. Thus synthesis of control may be treated as algebraic operations making the given approach rather simple for the engineering research (calculations).

REFERENCES

Kleinrok, L. (1979). *Queueing theory.* Mashinostroenie, Moscow. 431 pp. (in Russian).

Nickolsky, V. A., and N.P. Sevastyanov (1973). *K(E)-form grating functions in discrete system research problems. Avtomatika i telemechanika, Collected papers.* Nauka, Moscow. pp. 30–36 (in Russian).

Pugachev, V.S. (1974). Approaching research for stohastical systems by means of normalization method. *Avtomatika i telemechanika,* 5, 74–81 (in Russian).

Tsipkin, J.Z. (1977). *Control System Theory.* Nauka, Moscow. 559 pp. (in Russian).

Received December 1996

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MASINIO APTARNAVIMO SISTEMOS SU KINTAMU KRŪVIU IŠĖJIMO SRAUTO VALDYMO ŽINGSNINIS METODAS

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Straipsnyje nagrinėjamos normalizuotos vienakanalės masinio aptarnavimo sistemos modelio ypatybės, kai atvykstančių vartotojų srautas yra nestacionarus. Norint stabilizuoti išėjimo srauto greitį norimame lygyje, organizuojama aptarnavimo intensyvumo valdymo tiesioginė tarnyba. Ši problema išspręsta naudojant automatinio valdymo teorijos būdus.