

## PROCESSING OF SEQUENTIAL INFORMATION OF COMPLICATED DYNAMIC SYSTEM STATES

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**Abstract.** A method for processing sequential information of states of technological processes or other complicated dynamic systems and for sequential detection of many abrupt or slow changes in several unknown states is considered. The method is based on a sequential nonlinear mapping of many-dimensional vectors of parameters (collection of which describes the present state of dynamic systems) into two-dimensional vectors in order to reflect the states and their changes on the PC screen. The mapping error function is chosen and expressions for sequential nonlinear mapping are obtained. The mapping preserves the inner structure of distances among the vectors. An example is given. A theoretical minimum amount of parameter vectors mapped simultaneously at the very beginning is obtained.

**Key words:** states of a dynamic system, sequential information of states, sequential nonlinear mapping, control.

**1. Introduction.** Technological processes or other complicated dynamic systems (DS) are described by many parameters that define the present state of DS. The purpose of this paper is to present a new method for discrete sequential identification of states of complicated DS for their control in real time. While watching a technological process or other complicated DS there is a necessity to identify the DS states or detect their abrupt or slow changes. As the state of DS changes,  $L$  parameters (of any physical nature) describing the state change as well. If the DS state is described by a random process generated by this object, then the state is described by the  $L$  parameters characterizing the random process. The DS can have several unknown states and we need to observe the states and detect their changes sequentially and independently of the history. It is convenient to observe the DS states and their changes marking them by some mark on the PS screen. According to the mark position we can make a

decision on the DS state and its change, if the mark position changes.

To solve these problems it is necessary to have a method for sequential detection of many changes in several unknown properties of a random process. There are many methods of detection of changes in the properties of random processes in the scientific publications (Kligiené and Telksnys, 1984; Basseville and Benveniste, 1986; Nikiforov, 1983), but there are no methods to solve the above mentioned problems.

In the paper, we present a method for discrete sequential identification of states and for detection of many abrupt or slow changes in several unknown states of DS, based on sequential nonlinear mapping onto the plane of vectors of the  $L$  parameters given by DS. The mapping error function is chosen and expressions for sequential nonlinear mapping are presented along with some experimental results. A theoretical minimum amount of vectors mapped simultaneously at the very beginning is obtained.

The technique described in this paper can be useful in industry, at the control desk, in identification and control of technological processes, and in many regions where it is possible to get parameters describing the states of some objects. The experiment given below illustrates that.

**2. Problem formulation.** Let a DS be in any state  $s_i$  of the set of possible states:  $s_i \in S$ . We can observe the vector of  $L$  parameters at the output of the DS. By the way, these parameters can be of any physical nature (then we must introduce scale coefficients for each parameter). At the output of the DS we can watch a random process, too. The process may be described by a proper mathematical model, e.g., autoregressive sequence. Then the DS state is defined by a vector consisting of all  $L$  autoregressive parameters.

For identification of states of the DS it is necessary, at discrete time moments, to map the  $L$ -dimensional vectors sequentially and nonlinearly into two-dimensional vectors (preserving the inner structure of distance among the vectors) in order to represent the present state by some mark on the PC screen and, having in mind the existence of particular states, to identify the current state, a deviation from it or a transition to another state when the mark changes its position.

**3. Sequential mapping algorithm.** The sequential nonlinear mapping requires for the existence of earlier mapped vectors, so at the very beginning we

simultaneously have to carry out the nonlinear mapping of  $M$  vectors ( $M \geq 2$ ). We shall use for that the expressions in Sammon (1969). Afterwards, we need to map sequentially and nonlinearly the received parameter vectors and, in such a way, to identify the present state, its changes and deviations from it for a practically unlimited time. In order to formalize the method we denote by  $N$  this practically unlimited number of appearing vectors.

Thus, let us have  $M + N$  vectors in the  $L$ -hyperspace. We denote them by  $X_i$ ,  $i = 1, \dots, M$ ;  $X_j$ ,  $j = M + 1, \dots, M + N$ . The  $M$  vectors are already simultaneously mapped into two-dimensional vectors,  $Y_i$ ,  $i = 1, \dots, M$ , using the expressions in Sammon (1969). Now we need to sequentially map the  $L$ -dimensional vectors  $X_j$  into two-dimensional vectors  $Y_j$ ,  $j = M + 1, \dots, M + N$ . Here the nonlinear mapping expressions will change into sequential nonlinear mapping expressions, respectively (Montvilas, 1993). First, before performing iterations it is expedient to put the two-dimensional vectors being mapped in the same initial conditions, i.e.,  $y_{jk} = C_k$ ,  $j = M + 1, \dots, M + N$ ;  $k = 1, 2$ . Note, that in the case of simultaneous mapping of the first  $M$  vectors, the initial conditions are chosen in a random way (Sammon, 1969). Let the distance between the vectors  $X_i$  and  $X_j$  in the  $L$ -hyperspace be defined by  $d_{ij}^X$  and on the plane – by  $d_{ij}^Y$ , respectively. This algorithm uses the Euclidean distance measure, because, if we have no a priori knowledge concerning the data, we would have no reason to prefer any metric to the Euclidean metric.

For computing the mapping error of distances  $E$  we can find at least three expressions:

$$E_1 = \frac{1}{\sum_{i=1}^M (d_{ij}^X)^2} \sum_{i=1}^M (d_{ij}^X - d_{ij}^Y)^2, \quad j = M + 1, \dots, M + N; \quad (1)$$

function  $E_1$  reveals the largest errors independently of magnitudes of  $d_{ij}^X$ ; but if  $d_{ij}^X$  is small, then the mapping error can be comparable with the same distance;

$$E_2 = \sum_{i=1}^M \left( \frac{d_{ij}^X - d_{ij}^Y}{d_{ij}^X} \right)^2, \quad j = M + 1, \dots, M + N; \quad (2)$$

function  $E_2$  reveals the largest partial errors independently of magnitudes of  $|d_{ij}^X - d_{ij}^Y|$ ; but, in this case, big distances will have rather a great mapping error;

$$E_3 = \frac{1}{\sum_{i=1}^M d_{ij}^X} \sum_{i=1}^M \frac{(d_{ij}^X - d_{ij}^Y)^2}{d_{ij}^X}, \quad j = M+1, \dots, M+N; \quad (3)$$

function  $E_3$  is a useful compromise and reveals the largest product of the error and partial error. So we choose the third expression for computing the mapping error of distances  $E$ .

For correct mapping we have to change the positions of vectors  $Y_j$ ,  $j = M+1, \dots, M+N$  on the plane so that the error  $E$  be minimal. This is achieved by using the steepest descent procedure. After the  $r$ -th iteration the error of distances will be

$$E_j(r) = \frac{1}{\sum_{i=1}^M d_{ij}^X} \sum_{i=1}^M \frac{[d_{ij}^X - d_{ij}^Y(r)]^2}{d_{ij}^X}, \quad j = M+1, \dots, M+N; \quad (4)$$

here

$$d_{ij}^Y(r) = \sqrt{\sum_{k=1}^2 [y_{ik} - y_{jk}(r)]^2}, \quad i = 1, \dots, M; \quad j = M+1, \dots, M+N. \quad (5)$$

During the  $r+1$  - iteration co-ordinates of the mapped vectors  $Y_j$  will be

$$y_{jk}(r+1) = y_{jk}(r) - F \cdot \Delta_{jk}(r), \quad j = M+1, \dots, M+N; \quad k = 1, 2; \quad (6)$$

where

$$\Delta_{jk}(r) = \frac{\partial E_j(r)}{\partial y_{jk}(r)} \bigg/ \left| \frac{\partial^2 E_j(r)}{\partial y_{jk}^2(r)} \right|, \quad (7)$$

$F$  is the coefficient for correction of the coordinates and it is defined empirically to be  $F = 0.35$ ,

$$\frac{\partial E_j}{\partial y_{jk}} = H \sum_{i=1}^M \frac{D \cdot C}{d_{ij}^X \cdot d_{ij}^Y}, \quad (8)$$

$$\frac{\partial^2 E_j}{\partial y_{jk}^2} = H \sum_{i=1}^M \frac{1}{d_{ij}^X \cdot d_{ij}^Y} \left[ D - \frac{C^2}{d_{ij}^Y} \left( 1 + \frac{D}{y_{ij}^Y} \right) \right], \quad (9)$$

$$H = -\frac{2}{\sum_{i=1}^M d_{ij}^X}; \quad D = d_{ij}^X - d_{ij}^Y; \quad C = y_{jk} - y_{ik}.$$

For  $E(r) < \varepsilon$ , where  $\varepsilon$  can be taken arbitrarily small, the iteration process is over and the result is shown on the PC screen. In fact, it is enough  $\varepsilon = 0.01$ . In order to have an equal computing time for each mapping we can do constant number of iterations  $R$ . In practice, it is enough  $R = 30$ .

**4. Experimental results.** Let a technological process or DS be described by  $L = 5$  parameters defining states and have any state  $s_i$  of the set of possible states:  $s_i \in S$ . Let  $S = 4$ , and we detect the states of DS(4) at  $M + N = 18$  time moments. Let us take the case when the number of initial simultaneously mapped vectors is  $M = 2$  and does not involve all the possible states of DS(4). Afterwards we detect the states of DS(4) at the time moments  $N = 3 \div 18$  sequentially. A priori the states of DS(4) are known at the time moments (see Table 1).

**Table 1.** The states of DS(4) at the time moments  
 $M + N = 2 + 16 = 18$ .

MAPPING	SIMULT (i)		SEQUENTIAL (j)															
MARK	×		+															
TIME MOMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
STATE	1	2	3	4	2	1	3	3-4	4	4	2	2-3	3	1	2	1	4	3

In Fig. 1 the mapping results are presented, where at the first  $M = 2$  time moments state vectors mapped simultaneously are denoted by mark  $\times$  with an index that means the time moment number, and the state vectors mapped sequentially are denoted by mark  $+$  with the respective index.

At the time moment number 8 and number 12 there were slow changes in the states of DS(4) occurring between the states number 3 and number 4 and between the states number 2 and number 3, respectively. In Fig. 1 this situation is clear.

*Now about the border between different states.* It depends on a concrete situation, on the character of a dynamic system to be identified or controlled. The borders can touch one another, if DS can not have spoiled states in fact, and they can have distances among them, if the DS can have untechnological states

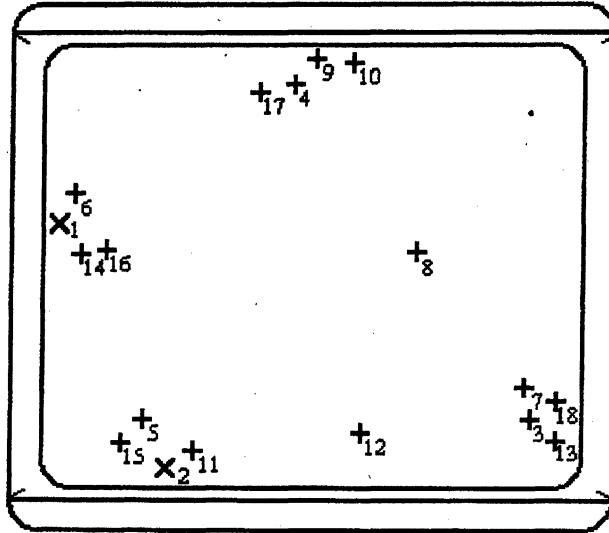


Fig. 1. The view on the PC screen of the end of mapped vectors of DS(4) for 18 time moments.

or can be broken. Real regions of states can be found for a concrete object by additional investigations having all data of states. The main importance of the method is that it is possible to begin work even when the states are *unknown yet*, and while working one can accumulate information and use it for determination of states and borders among them.

**5. Minimum value of  $M$ .** In all generated situations, when the amount of initial simultaneously mapped vectors of parameters of DS states was taken  $M = 2$ , at each time moment the marks of DS states got into their right places on the PC screen and the states were identified correctly (Montvilas, 1992; 1993; 1994). Even marks of the states which were not involved into  $M$  initial vectors of parameters had got their own places on the PC screen and at each time moment the places of marks on the screen corresponded to the right DS states entirely.

However, theoretically such cases are possible, when the points (ends of vectors) being in different places of the  $L$ -hyperspace can be mapped into one point on the plane, because these points can have the same distances as  $M = 2$

simultaneously mapped points. By way of illustration, let us map points from a three-dimensional space ( $L = 3$ ) onto the plane. Let the initial simultaneously mapped points be  $A$  and  $B$  (see Fig. 2). Let them be on the axis of a cylinder. Then the points  $C$ ,  $D$ , and  $E$  being on circle, which is on the surface of the cylinder, have equal distances as that of  $A$  and  $B$ :  $d_{AC}^X = d_{AD}^X = d_{AE}^X$  and  $d_{BC}^X = d_{BD}^X = d_{BE}^X$ . Thus, the points  $C$ ,  $D$  and  $E$ , under the same initial conditions, will be mapped onto the plane into the same point. If these points mean different states of DS then we shall have a mistake.

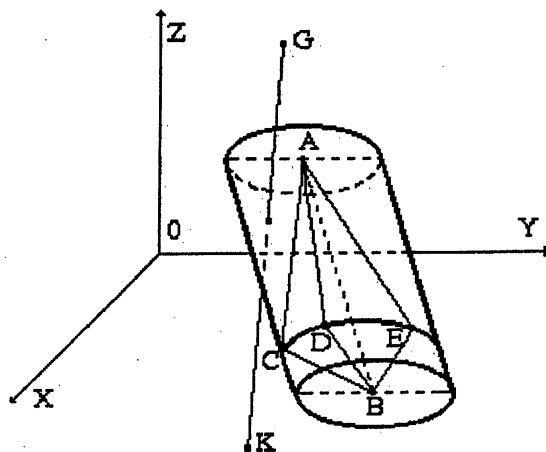


Fig. 2. The case of a three-dimensional space ( $L = 3$ ).

Now let us have three points  $M = 3$  as the initial points for simultaneous mapping:  $A$ ,  $B$ , and  $C$ . Then one can draw any straight line orthogonal to the plane  $ABC$ . Any two points of the line  $G$  and  $K$  being on different sides of the plane  $ABC$  have equal distances to that of points  $A$ ,  $B$ , and  $C$ :  $d_{AG}^X = d_{AK}^X$ ,  $d_{BG}^X = d_{BK}^X$  and  $d_{CG}^X = d_{CK}^X$ . So in this case, the points  $G$  and  $K$  can be mapped into one point as well.

Taking  $M = 4$  points ( $A$ ,  $B$ ,  $C$  and  $D$ ) so that the fourth  $D$  point be not on the plane  $ABC$  and all the  $M = 4$  points form a three-dimensional space, we have a situation when even theoretically one cannot find any two points which have equal distances to that of all simultaneously mapped  $M = 4$  points.

Thus, having the analysis of various possible situations at diverse  $L$  and  $S$

values, we can draw a conclusion that for initial simultaneous mapping we need to take  $M = \min(L + 1, S)$ ,  $S$  being known, or  $M = L + 1$   $S$  being unknown, or a dynamic system can have indeterminate or spoiled states, besides, these  $M = L + 1$  points have to form an  $L$ -dimensional space.

In practice, as it was mentioned above, it is enough to have  $M = 2$  vectors of DS states for the initial simultaneous mapping, because the cases considered here can take place only under coincidence of unexpectedness.

However, in order to avoid only theoretically possible complications, we have to do the following: after having simultaneous mapping of  $M = 2$  and sequential mapping of  $L - 1$  vectors we have already got  $L + 1$  vectors. Then we have to map simultaneously the available  $L + 1$  vectors again and afterwards to map sequentially the received later vectors with respect to the initial  $L + 1$  vectors.

**6. Conclusions.** The described method enables us to sequentially identify the dynamic system states, their abrupt or slow changes and to watch the situation on the PC screen.

Before sequential identification of the states, it is sufficient to map simultaneously only  $M = L + 1$ , where  $L$  is the dimensionality of parameter vectors which describe the dynamic system states. These  $M = L + 1$  points have to form an  $L$ -dimensional space.

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**SUDĖTINGŲ DINAMINIŲ SISTEMŲ BŪSENŲ  
NUOSEKLIOS INFORMACIJOS APDOROJIMAS****Algirdas Mykolas MONTVILAS**

Nagrinėjamas technologinių procesų ar kitokių sudėtingų dinaminių sistemų būsenų nuosekliai gaunamos informacijos apdorojimo bei nuoseklus staigių arba lėtų keletos nežinomų sistemos būsenų pasikeitimų nustatymo uždavinys. Metodas grindžiamas daugelio parametru, aprašančių dinaminės sistemos būsenas, vektorių nuosekliu netiesiniu atvaizdavimu į plokštumą, pateikiant po to sistemos būsenas ir jų pasikeitimus PC ekrane. Pateiktas pavyzdys.