

GENERAL ESTIMATION OF STATIC MODEL PARAMETERS AND SYSTEMATIC MEASUREMENT ERRORS

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Abstract. The present paper considers the problem of general estimation of static model parameters and systematic measurement errors. The general estimation algorithm is based on static model linearization and on the least-squares method. The efficiency of this algorithm is illustrated by means of computer-aided digital simulation. The obtained equations and the algorithm of general estimation of static model parameters and systematic measurement errors can be applied for the solution of different practical problems. Estimability conditions must be satisfied in all cases.

Key words: model linearization, least-squares estimates, recursive estimation.

1. Introduction. Estimation tasks usually consider the observation errors as a sequence of independent random variables with zero mean and a certain covariation matrix (Bard, 1970; Brandt, 1975; Demidenko, 1981). However, in numerous applications the zero-mean requirement for the random error sequence is not satisfied. This fact results in a supplementary bias of estimates. Thus it is necessary to obtain the estimates of a static system model by shifted observations,

i.e. by observations with a systematic error. A certain approach to the estimation of static-model vector parameters and systematic measurement errors was proposed in Nemura and Spečiūnas (1989).

The aim of this paper is to investigate the problem of general estimation of systematic measurement errors and static model parameters, and to design an algorithm of general estimation, based on the linearization and least-squares methods.

2. Algorithm for general estimation of model parameters and systematic measurement errors. Mathematical model of the multivariable nonlinear static system being considered can be defined by a following system of nonlinear equations:

$$y_{1k} = f_1(x_k, c_1) + v_{1k}, \quad (1)$$

$$y_{2k} = f_2(x_k, c_2) + v_{2k}, \quad (2)$$

$$\begin{bmatrix} v_{1k} \\ v_{2k} \end{bmatrix} = v_k = \Phi_k \alpha + e_k, \quad (3)$$

where

$$\Phi_k \alpha = \delta_k, \quad \delta_k = \begin{bmatrix} \delta_{1k} \\ \delta_{2k} \end{bmatrix}, \quad e_k = \begin{bmatrix} e_{1k} \\ e_{2k} \end{bmatrix},$$

c_1, c_2 - estimated vectors of r_1 and r_2 measure correspondingly, $y_{1k}, y_{2k}(k = \overline{1, s})$ - m -measured sequences of the measurement results for the m -measured nonlinear functions $f_1(x_k, c_1)$ and $f_2(x_k, c_2)$; $x_k(k = \overline{1, s})$ - m -measured deterministic input sequence; v_{1k} and $v_{2k}(k = \overline{1, s})$ - m -measured sequences of random variables representing the measurement errors for the nonlinear functions $f_1(x_k, c_1), f_2(x_k, c_2)$. Sequences of m -measured vector random variables v_{1k} and $v_{2k}(k = \overline{1, s})$ have non-zero mean values δ_{1k} and $\delta_{2k}(k = \overline{1, s})$ and corresponding covariation matrices Ω_1 and Ω_2 , and they can be defined by the equation (3), where $e_{1k}, e_{2k}(k = \overline{1, s})$

are sequences of independent m -measured random variables with zero mean and covariation matrices Ω_1 and Ω_2 correspondingly.

A following problem is being considered. Let us assume that the following values are known: observances y_{1k}, y_{2k} , analytical expressions $f_1(x_k, c_1)$ and $f_2(x_k, c_2)$, certain initial values \hat{c}_{1o} and \hat{c}_{2o} for the vector parameters c_1 and c_2 and the covariation matrix Ω of the vector of random measurement errors $e_k^T = [e_{1k}^T, e_{2k}^T]$. It is necessary to estimate c_1, c_2 and $\delta_k (k = \overline{1, s})$.

In order to obtain necessary relationships, linearization of the equations (1), (2) is accomplished in the vicinity of the working point, defined by estimates

$$\hat{c}_\nu = \begin{bmatrix} \hat{c}_{1\nu} \\ \hat{c}_{2\nu} \end{bmatrix}, \quad \hat{\delta}_\nu = \begin{bmatrix} \hat{\delta}_{1\nu} \\ \hat{\delta}_{2\nu} \end{bmatrix}, \quad (4)$$

obtained in the previous step ν of the estimation process:

$$y_{1k} = f_1(\hat{x}_{k\nu}, \hat{c}_{1\nu}) + L_{1k\nu}(x_k - \hat{x}_{k\nu}) + F_{1k\nu}(c_1 - \hat{c}_{1\nu}) + v_{1k\nu}, \quad (5)$$

$$y_{2k} = f_2(\hat{x}_{k\nu}, \hat{c}_{2\nu}) + L_{2k\nu}(x_k - \hat{x}_{k\nu}) + F_{2k\nu}(c_2 - \hat{c}_{2\nu}) + v_{2k\nu}, \quad (6)$$

where

$$\hat{x}_{k\nu} = f_1^{-1}(y_{1k} - \hat{\delta}_{1k\nu}, \hat{c}_{1\nu}) \text{ or } \hat{x}_{k\nu} = f_2^{-1}(y_{2k} - \hat{\delta}_{2k\nu}, \hat{c}_{2\nu}),$$

$$L_{1k\nu} = \left. \frac{\partial f_1(\cdot)}{\partial x_k} \right|_{\substack{x_k = \hat{x}_{k\nu} \\ c_1 = \hat{c}_{1\nu}}}, \quad F_{1k\nu} = \left. \frac{\partial f_1(\cdot)}{\partial c_1} \right|_{\substack{x_k = \hat{x}_{k\nu} \\ c_1 = \hat{c}_{1\nu}}},$$

$$L_{2k\nu} = \left. \frac{\partial f_2(\cdot)}{\partial x_k} \right|_{\substack{x_k = \hat{x}_{k\nu} \\ c_2 = \hat{c}_{2\nu}}}, \quad F_{2k\nu} = \left. \frac{\partial f_2(\cdot)}{\partial c_2} \right|_{\substack{x_k = \hat{x}_{k\nu} \\ c_2 = \hat{c}_{2\nu}}}.$$

Let us introduce the following notations:

$$q_{1k\nu} = y_{1k} - f_1(\hat{x}_{k\nu}, \hat{c}_{1\nu}) + L_{1k\nu}\hat{x}_{k\nu} + F_{1k\nu}\hat{c}_{1\nu}, \quad (7)$$

$$q_{2k\nu} = y_{2k} - f_2(\hat{x}_{k\nu}, \hat{c}_{2\nu}) + L_{2k\nu}\hat{x}_{k\nu} + F_{2k\nu}\hat{c}_{2\nu}. \quad (8)$$

Then the equations (5), (6) can be rewritten as

$$q_{1k\nu} = L_{1k\nu}x_k + F_{1k\nu}c_1 + v_{1k\nu}, \quad (9)$$

$$q_{2k\nu} = L_{2k\nu}x_k + F_{2k\nu}c_2 + v_{2k\nu}. \quad (10)$$

Considering the equations (9), (10) simultaneously, we can eliminate the unknown vector x_k . That leads to a following relationship:

$$L_{1k\nu}^{-1}F_{1k\nu}c_1 - L_{2k\nu}^{-1}F_{2k\nu}c_2 + L_{1k\nu}^{-1}v_{1k\nu} - L_{2k\nu}v_{2k\nu} = z_{k\nu}, \quad (11)$$

where

$$z_{k\nu} = L_{1k\nu}^{-1}q_{1k\nu} - L_{2k\nu}^{-1}q_{2k\nu}. \quad (12)$$

Equation (11) can be rewritten in a more compact form:

$$\left[L_{1k\nu}^{-1}F_{1k\nu}; -L_{2k\nu}^{-1}F_{2k\nu}; L_{1k\nu}^{-1}; -L_{2k\nu}^{-1} \right] \begin{bmatrix} c_1 \\ c_2 \\ v_{1k\nu} \\ v_{2k\nu} \end{bmatrix} = z_{k\nu}. \quad (13)$$

Taking into account the relationship (3), the latest matrix equation can be rewritten as

$$\begin{aligned} & \left[L_{1k\nu}^{-1}F_{1k\nu}; -L_{2k\nu}^{-1}F_{2k\nu}; \left(L_{1k\nu}^{-1}; -L_{2k\nu}^{-1} \right) \Phi_k \right] \begin{bmatrix} c_1 \\ c_2 \\ \alpha \end{bmatrix} + \\ & + \left[L_{1k\nu}^{-1}; -L_{2k\nu}^{-1} \right] \begin{bmatrix} e_{1k\nu} \\ e_{2k\nu} \end{bmatrix} = z_{k\nu}. \end{aligned} \quad (14)$$

By introducing the notation

$$\lambda^T = (c_1^T, c_2^T, \alpha^T), \quad (15)$$

$$G_{k\nu} = \begin{bmatrix} L_{1k\nu}^{-1}; & -L_{2k\nu}^{-1} \end{bmatrix}, \quad (16)$$

$$D_{k\nu} = \begin{bmatrix} L_{1k\nu}^{-1} F_{1k\nu}; & -L_{2k\nu}^{-1} F_{2k\nu}; & G_{k\nu} \Phi_k \end{bmatrix}, \quad (17)$$

instead of (14) we get

$$D_{k\nu} \lambda + G_{k\nu} e_{k\nu} = z_{k\nu}. \quad (18)$$

By minimizing the function

$$Q_\nu(\lambda) = \sum_{k=1}^n (D_{k\nu} \lambda - z_{k\nu})^T (G_{k\nu} \Omega G_{k\nu}^T)^{-1} (D_{k\nu} \lambda - z_{k\nu}) \quad (19)$$

according to all the components of vector λ , we obtain the necessary relationships for the estimation of λ , $V_{\hat{\lambda}}$ and δ :

$$V_{\hat{\lambda}_{\nu+1}} = \left[\sum_{k=1}^s D_{k\nu}^T (G_{k\nu} \Omega G_{k\nu}^T)^{-1} D_{k\nu} \right]^{-1}, \quad (20)$$

$$\hat{\lambda}_{\nu+1} = V_{\hat{\lambda}_{\nu+1}} \sum_{k=1}^s D_{k\nu}^T (G_{k\nu} \Omega G_{k\nu}^T)^{-1} z_k, \quad (21)$$

$$\hat{\delta}_{k,\nu+1} = \Phi_k \hat{\alpha}_{\nu+1}. \quad (22)$$

The recursive estimation process begins with $c = \hat{c}_0$, $\hat{\delta}_0 = 0$ and continues until the value

$$\Delta_{\nu+1} = \max_j (\hat{\lambda}_{j\nu+1} - \hat{\lambda}_{j\nu}); \quad (j = \overline{1, r_1 + r_2 + p}) \quad (23)$$

becomes sufficiently small. The whole estimation algorithm can be defined by the equations (16), (17), (20)–(22) and (23).

The estimatibility conditions can be obtained from the requirement for the matrix

$$A = \left[\sum_{k=1}^s D_{k\nu}^T (G_{k\nu} \Omega G_{k\nu}^T)^{-1} D_{k\nu} \right], \quad (24)$$

i.e. $\det \neq 0$ must be satisfied; besides, initial estimates \widehat{c}_{10} and \widehat{c}_{20} must be sufficiently close to the genuine values c_1^* and c_2^* .

3. Computer-aided simulation of the general parameter and systematic errors estimation algorithm.

The efficiency of the obtained general estimation algorithm can be illustrated by the results of computer-aided numerical solution of a simple problem.

Let us consider a model, describing a static system and observations, in the following form:

$$y_{1k} = x_k + \delta_1, \quad (25)$$

$$y_{2k} = ax_k^2 + \delta_2 \quad (k = \overline{1, s}), \quad (26)$$

where a is an unknown parameter, that needs to be estimated, δ_1 and δ_2 are systematic measurement errors, that must also be estimated. We assume, that there are no random measurement errors. It is necessary to estimate the scalar parameter a and systematic errors δ_1 and δ_2 by the observations listed in the Table 1.

Table 1. Signal measurements

k	1	2	3	4	5
y_{1k}	2	3	4	5	6
y_{2k}	3	6	11	18	27

The following relations were obtained for the calculation of estimates $\widehat{\delta}_{1\nu}$ and $\widehat{\delta}_{2\nu}$:

$$L_{2k\nu} = 2\widehat{a}_\nu \widehat{x}_{k\nu}, \quad (27)$$

$$\widehat{x}_{k\nu} = y_{1k} - \widehat{\delta}_{1\nu}, \quad \widehat{\delta}_{10} = 0, \quad \widehat{x}_{k0} = y_{1k}, \quad (28)$$

$$z_{k\nu} = y_{1k} - y_{2k}/2\hat{a}_\nu \hat{x}_{k\nu} - \left(1 - \frac{1}{2a_\nu}\right) \hat{x}_{k\nu}, \quad (29)$$

$$G_{k\nu} = \left[1 - \frac{1}{2\hat{a}_\nu \hat{x}_{k(\nu-1)}}\right] \quad (k = \overline{1, s}; \nu = 0, 1, \dots). \quad (30)$$

The estimate of the parameter a can be obtained via least-squares method according to the equation:

$$\hat{a}_\nu = \frac{\sum_{k=1}^s y_{2k} \hat{x}_{k(\nu-1)}^2 - \hat{\delta}_{2\nu-1} \sum_{k=1}^s \hat{x}_{k\nu}^2}{\sum_{k=1}^s \hat{x}_{k\nu}^4} \quad (\nu = 1, 2, \dots). \quad (31)$$

The genuine values in this case are: $a^* = 1$, $\delta_1^* = 1$, $\delta_2^* = 2$.

Investigation of two component estimation algorithms was accomplished. Algorithm No 1: in the first step the parameter a is being estimated according to (31) with $\hat{a}_0 = 0$ and $\hat{\delta}_{10} = \hat{\delta}_{20} = 0$, then δ_1 and δ_2 are being estimated, using the already obtained estimate \hat{a}_1 ; in the second step first of all the parameter a is being estimated, using estimates $\hat{\delta}_{11}$ and $\hat{\delta}_{21}$, and then the estimates $\hat{\delta}_{12}$ and $\hat{\delta}_{22}$ are obtained, using the estimate \hat{a}_2 , and so on. Algorithm No 2: in the first step a_1 , δ_1 and δ_2 are being estimated, using the initial values \hat{a}_0 , $\hat{\delta}_{10}$ and $\hat{\delta}_{20}$; in the second step a , δ_1 , δ_2 are being estimated, using recently obtained estimates \hat{a}_1 , $\hat{\delta}_{11}$ and $\hat{\delta}_{21}$ and so on.

The resulting estimates are listed in the Table 2. The obtained results prove, that algorithm No 1 is much more effective than the algorithm No 2. Besides the initial value \hat{a}_0 necessary for the first algorithm, while for the

Table 2. Estimates of the parameter a and systematic errors δ_1 and δ_2

Algo- rithm N°	Estimate values in the first 6 steps of the i estimation process						i	
	0	1	2	3	4	5	6	
1	\hat{a}_ν	-	0.7317	1.1248	1.0113	1.0001	1.0000	1.0000
	$\hat{\delta}_{1\nu}$	0	1.3666	11.041	1.0004	1.0000	1.0000	1.0000
	$\hat{\delta}_{2\nu}$	0	3.0000	2.1344	2.0016	2.0000	2.0000	2.0000
2	\hat{a}_ν	0.5	0.7317	1.5649	0.6805	0.958	0.980	1.0007
	$\hat{\delta}_{1\nu}$	0	2.0000	0.5654	1.090	0.9577	1.0018	0.99926
	$\hat{\delta}_{2\nu}$	0	3.0000	3.0000	3.5435	2.008	2.0018	2.0000
3	\hat{a}_ν	1.5	0.7317	0.8115	1.0518	0.986	0.9993	0.9999
	$\hat{\delta}_{1\nu}$	0	0.6666	1.122	0.9716	0.9986	1.0002	1.0000
	$\hat{\delta}_{2\nu}$	0	3.0000	2.111	2.014	2.0008	2.0000	2.0000

second algorithm it must be sufficiently close to the genuine value (e.g. with $\hat{a}_0 \leq 0.3$ the estimation process is unstable).

4. Conclusions

1. The necessary equations for the calculation of parameter and systematic measurement error estimates were obtained on the basis of the least-squares method and on the model linearization principle.

2. The efficiency of the obtained estimation algorithm was illustrated by means of digital computer-aided simulation.

3. Algorithm of the general estimation of model parameters and systematic measurement errors can be applied in the solution of different practical tasks of joint estimation, when matrix (24) is non singular.

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