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THE UNIQUE SOLVABILITY OF NON-MIGRATING LIMITED PANMICTION POPULATION EVOLUTION PROBLEM

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Abstract. This paper is devoted to the consideration of the evolution of the non-migrating limited panmiction population taking into account the size, sex and age structure, pregnancy and females restoration period after delivery. The unique solvability of this model and the condition for the population to vanishe is obtained.

Key words: limited population, panmiction mating, population evolution, gestation and restoration period.

1. Introduction. In this paper the evolution of one sexual population will be considered. Dynamic models of such populations are well known (see Gimelfarb *et al.*, 1974; Poluektov *et al.*, 1980; Svirezhev and Pasekov, 1982). There are few works that are devoted to a solvability of limited population models. Gurtin and MacCamy (1974), Griffel (1976) and Matsenko (1981) dealt with unique solvability of non-linear models taking into account age of individuals. Sowunmi (1976) took into consideration age and sex of individuals. Swick (1977) took into account age and a lag between conception and birth. Bulanzhe (1988) studied the solution structure of community model taking into consideration age of individuals of limited population, which, moreover, interacts with parasite population. The deterministic model, developed by Skakauskas (1994), includes: age and sex of individuals, pregnancy of females, possible destruction of the foetus (abortions), organism restoration periods after abortions and delivery, panmiction mating of the sexes. This model allows us to obtain densities of interacting groups such that: males, single and fecundated females and females after abortions and delivery. In the steady case of our model we observed a possible nonmonotonic decrease of numbers density of single females as age increases in the reproductive interval. In the case, when abortions and restoration period after delivery are ignored, Skakauskas (1995) proved the unique solvability of this model for limited population. The goal of this paper is to prove the unique solvability of our model (1994) for limited population taking into account the size, age structure, pregnancy and females restoration period after delivery. We do not discuss the advantages of our model and do not comparise it with the known ones.

2. Problem formulation. Suppose that:

$n(t)$ is total population density and $y(t,\tau_y)$, $x(t,\tau_x)$, $z(t,\tau_y,\tau_x,\tau_z)$, $v(t,\tau_y,\tau_x,\tau_v)$ are densities of numbers of males, single and fecundated females and females from restoration interval, respectively, where τ_y, τ_x, τ_z are ages of males, single females and embriou's, t is time and τ_v is time passed after delivery;

$p(t,\tau_y,\tau_x,n)$ is fecundation rate and $\nu^y(t,\tau_y,n)$, $\nu^x(t,\tau_x,n)$, $\nu^z(t,\tau_y,\tau_x,\tau_z,n)$, $\nu^v(t,\tau_y,\tau_x,\tau_v,n)$, where $n=n(t)$, are death rates of males, single and fecundated females and females from restoration interval, respectively;

$y^0(\tau_y)$, $x^0(\tau_x)$, $z^0(\tau_y,\tau_x,\tau_z)$, $v^0(\tau_y,\tau_x,\tau_v)$ are initial functions for y, x, z and v, respectively;

$\sigma_{xz}(\tau_z)=(\tau_{1x}+\tau_z,\tau_{2x}+\tau_z]$, $\sigma_{xv}(\tau_v)=(\widetilde{\tau}_{1x}+\tau_v,\widetilde{\tau}_{2x}+\tau_v]$, $\widetilde{\tau}_{kx}=\tau_{kx}+T_z$, $k=1,2$;

$\sigma_y=(\tau_{1y},\tau_{2y}]$ and $\sigma_{xz}(0)$ are reproductive intervals of males and females, respectively, $\sigma_z=(0,T_z]$ and $\sigma_v=(0,T_v]$ are gestation and restoration intervals; $\sigma=\sigma_y\times\sigma_{xz}(T_z)$, $E^{0z}=\{(\tau_y,\tau_x,\tau_z)\in\sigma_y\times\sigma_{xz}(\tau_z)\times\sigma_z\}$, $E^{0v}=\{(\tau_y,\tau_x,\tau_v)\in\sigma_y\times\sigma_{xv}(\tau_v)\times\sigma_v\}$, $I=(0,\infty)$, $\overline{I}=[0,\infty)$, $E^y=\{(t,\tau_y)\in I\times I\}$, $E^x=\{(t,\tau_x)\in I\times(I\setminus\bigcup_{i=1}^{4}\tau_i)$, $\tau_i=\tau_{ix}$, $\tau_{i+2}=\tau_{ix}+T$, $T=T_z+T_v$, $i=1,2\}$, $E^z=\{(t,\tau_y,\tau_x,\tau_z)\in I\times\sigma_y\times\sigma_{xz}(\tau_z)\times\sigma_z\}$, $E^v=\{(t,\tau_y,\tau_x,\tau_v)\in I\times\sigma_y\times\sigma_{xv}(\tau_v)\times\sigma_v\}$;

$[x(t,\tau_i)]$ is a jump of the function x at the line $\tau_x=\tau_i$;

$b^y(t,\tau_y,\tau_x,n(t-T_z))$ and $b^x(t,\tau_y,\tau_x,n(t-T_z))$ are the birth rates of males and females offsprings, respectively;

$2^{-1/2}D^y y$, $2^{-1/2}D^x x$, $3^{-1/2}D^z z$, $3^{-1/2}D^v v$ represent directional derivatives along the positive direction of characteristics of operators

$$L^y=\frac{\partial}{\partial t}+\frac{\partial}{\partial\tau_y},\quad L^x=\frac{\partial}{\partial t}+\frac{\partial}{\partial\tau_x},\quad L^z=L^x+\frac{\partial}{\partial\tau_z},\quad L^v=L^x+\frac{\partial}{\partial\tau_v},$$

respectively.

The system (see Skakauskas, 1994)

$$D^y y = -y\nu^y \quad \text{in} \quad E^y, \tag{1}$$

$$D^x x = -x d^x + X \quad \text{in} \quad E^x, \tag{2}$$

$$D^z z = -z\nu^z \quad \text{in} \quad E^z, \tag{3}$$

$$D^v v = -v\nu^v \quad \text{in} \quad E^v, \tag{4}$$

$$d^x = \nu^x + \begin{cases} 0, & \tau_x \notin \sigma_{xz}(0), \\ n_y^{-1} \int\limits_{\sigma_y} yp\, d\tau_y, & \tau_x \in \sigma_{xz}(0), \end{cases} \tag{5a}$$

$$n_y = \int\limits_{\sigma_y} y\, d\tau_y, \tag{5b}$$

$$X = \begin{cases} 0, & \tau_x \notin \sigma_{xv}(T_v), \\ \int\limits_{\sigma_y} v\big|_{\tau_v=T_v} d\tau_y, & \tau_x \in \sigma_{xv}(T_v), \end{cases} \tag{6}$$

$$n = \int\limits_0^\infty x\, d\tau_x + \int\limits_0^\infty y\, d\tau_y + \int\limits_{E^{oz}} z\, d\tau_y\, d\tau_x\, d\tau_z + \int\limits_{E^{ov}} v\, d\tau_y\, d\tau_x\, d\tau_v, \tag{7}$$

subject to conditions

$$y\big|_{t=0} = y^0, \quad x\big|_{t=0} = x^0, \quad z\big|_{t=0} = z^0, \quad v\big|_{t=0} = v^0, \tag{8a – d}$$

$$y\big|_{\tau_y=0} = \int\limits_\sigma b^y z\big|_{\tau_z=T_z} d\tau_y\, d\tau_x, \quad x\big|_{\tau_x=0} = \int\limits_\sigma b^x z\big|_{\tau_z=T_z} d\tau_y\, d\tau_x, \tag{9a, b}$$

$$z\big|_{\tau_z=0} = n_y^{-1} xyp, \quad v\big|_{\tau_v=0} = z\big|_{\tau_z=T_z}, \tag{10a, b}$$

$$\left[x\big|_{\tau_x=\tau_i}\right] = 0, \quad i = \overline{1,4}, \tag{11}$$

$$n(t) = \omega(t), \quad t \in [-T_z, 0] \tag{12}$$

governs the evolution of the population. t, τ_x are the arguments of functions d^x and X. The non-negative demographic functions ν^y, ν^x, ν^z, ν^v, p, b^x, b^y and initial functions y^0, x^0, z^0, v^0, ω are given. It is also assumed, that functions x^0, y^0, z^0, v^0, ω satisfy reconcilable conditions, i.e., conditions (7) – (12) for

$t = 0$. As it follows from the biological meaning, unknown functions $y,\ x,\ z$ and v also must be non-negative.

3. Problem solvability. We shall study the existence and uniqueness of non-negative solution of the problem (1) – (12) in two following cases.

3.1. The case when the fecundation rate and the fecundated females death rate depend on age of males. This means that $\partial f/\partial\tau_y \neq 0,\ f = p, \nu^z$. We shall consider the case $h^x = \tau_{2x} - \tau_{1x} < T$. The opposite case can be studied in the same way. Let us denote

$$\begin{aligned}
&\overline{x}(t) = x(t,0), \quad \overline{y}(t) = y(t,0),\\
&\overline{z}(t,\tau_y,\tau_x) = z\big|_{\tau_z=0}, \quad \overline{v}(t,\tau_y,\tau_x) = v\big|_{\tau_v=0},\\
&d^s = \nu^s,
\end{aligned} \tag{13a – e}$$

$$\begin{aligned}
F_1(\gamma,n) =&\gamma(r_0^\gamma)\exp\Big\{-\int_0^t d^\gamma(r_\eta^\gamma,n)\,d\eta\Big\}\\
&+\int_0^t X(r_\alpha^\gamma)\exp\Big\{-\int_\alpha^t d^\gamma(r_\eta^\gamma,n)\,d\eta\Big\}\,d\alpha,
\end{aligned} \tag{14}$$

$$\begin{aligned}
F_2(\gamma,\mu,n) =&\gamma(h_\mu^\gamma)\exp\Big\{-\int_\mu^{\tau_\gamma} d^\gamma(h_\eta^\gamma,\ n)\,d\eta\Big\}\\
&+\int_\mu^{\tau_\gamma} X(h_\alpha^\gamma)\exp\Big\{-\int_\alpha^{\tau_\gamma} d^\gamma(h_\eta^\gamma,n)\,d\eta\Big\}\,d\alpha,
\end{aligned} \tag{15}$$

where

$$s = y,z,v \quad\text{and}\quad X(r_\alpha^\gamma) = 0, \qquad X(h_\alpha^\gamma) = 0 \quad\text{for}\quad \gamma = y,z,v.$$

Then from Eqs. (1) – (4), (8), (11) and (13) we obtain formal integral representations of functions $y,\ z,\ x,\ v$:

$$y = \begin{cases} F_1(y,n), & y(r_0^y) = y^0(\tau_y - t),\\ & 0 \leqslant t \leqslant \tau_y,\\ F_2(y,0,n), & y(h_0^y) = \overline{y}(t-\tau_y),\\ & 0 \leqslant \tau_y < t, \end{cases} \tag{16}$$

$$v = \begin{cases} F_1(v,n), & v(r_0^v) = v^0(\tau_y, \tau_x - t, \tau_v - t), \\ & 0 \leqslant t \leqslant \tau_v, \\ F_2(v,0,n), & v(h_0^v) = \overline{v}(t - \tau_v, \tau_y, \tau_x - \tau_v), \\ & 0 \leqslant \tau_v < t, \end{cases} \tag{17}$$

$$z = \begin{cases} z_1(t,\tau_y,\tau_x,\tau_z) = F_1(z,n), & z(r_0^z) = z^0(\tau_y, \tau_x - t, \tau_z - t), \\ & 0 \leqslant t \leqslant \tau_z, \\ z_2(t,\tau_y,\tau_x,\tau_z) = F_2(z,0,n), & z(h_0^z) = \overline{z}(t - \tau_z, \tau_y, \tau_x - \tau_z), \\ & 0 \leqslant \tau_z < t, \end{cases} \tag{18a, b}$$

$$x = \begin{cases} F_1(x,n), & x(r_0^x) = x^0(\tau_x - t), \\ & 0 \leqslant t \leqslant \tau_x - \tau_i, \quad \tau_x \in (\tau_i, \tau_{i+1}], \\ F_2(x,\tau_i,n), & t > \tau_x - \tau_i, \tau_x \in (\tau_i, \tau_{i+1}], \\ & x(h_0^x) = \overline{x}(t - \tau_x), \quad t > \tau_x \in [0, \tau_1], \end{cases} \tag{19a, b}$$

here $i = \overline{0,4}$, $\tau_0 = 0$, $\tau_5 = \infty$ and $r_\eta^s = (\eta, \eta+\tau_s-t)$, $h_\eta^s = (\eta+t-\tau_s, \eta)$, $s = x,\ y$, $r_\eta^s = r_\eta(t,\tau_s) = (\eta, \tau_y, \eta + \tau_x - t, \eta + \tau_s - t)$, $h_\eta^s = h_\eta(t,\tau_s) = (\eta + t - \tau_s, \tau_y, \eta + \tau_x - \tau_s, \eta)$, $s = z, v$ are sets of arguments written in brackets. In formulas (14), (15) the argument of n coincides with the first argument of the neighbouring set r_η^γ, or h_η^γ, respectively.

Using Eqs. (16) – (19), (13) – (15) and (9) – (10) we obtain

$$n = n^y + n^x + n^z + n^v, \tag{20}$$

where

$$n^y = \int_0^t F_2(y,0,n)\, d\tau_y + \int_t^\infty F_1(y,n)\, d\tau_y, \tag{21}$$

$$n^x = \sum_{i=0}^4 \left\{ \int_{\tau_i}^{\alpha_i(t)} F_2(x,\tau_i,n)\, d\tau_x + \int_{\alpha_i(t)}^{\tau_{i+1}} F_1(x,n)\, d\tau_x \right\}, \tag{22}$$

$$\begin{aligned} n^z = & \int_0^{\beta_z(t)} d\tau_z \int_{\sigma_y \times \sigma_{xz}(\tau_z)} F_2(z,0,n)\, d\tau_y\, d\tau_x \\ & + \int_{\beta_z(t)}^{T_z} d\tau_z \int_{\sigma_y \times \sigma_{xz}(\tau_z)} F_1(z,n)\, d\tau_y\, d\tau_x, \end{aligned} \tag{23}$$

$$n^v = \int\limits_0^{\beta_v(t)} d\tau_v \int\limits_{\sigma_y \times \sigma_{xv}(\tau_v)} F_2(v,0,n)\, d\tau_y\, d\tau_x + \int\limits_{\beta_v(t)}^{T_v} d\tau_v \int\limits_{\sigma_y \times \sigma_{xv}(\tau_v)} F_1(v,n)\, d\tau_y\, d\tau_x, \tag{24}$$

here $\alpha_i(t) = \min(t + \tau_i, \tau_{i+1})$, $\beta_s(t) = \min(t, T_s)$, $s = z, v$. Using step by step method we shall prove, that Eq. (20) is an integral equation for n.

Using Eqs. (13d), (10b) and (18) we can write the first term in the right-hand side of Eq. (24) in the form

$$\int\limits_\sigma d\tau_y\, d\tau_x \int\limits_{t-\beta_v(t)}^{t} \overline{v}(\xi, \tau_y, \tau_x) \exp\left\{ -\int\limits_\xi^t \nu^v\left(r_\eta(\xi,0), n\right) d\eta \right\} d\xi,$$

or as

$$\begin{cases} \int\limits_{t-\beta_v(t)}^{t} f_1\, d\xi, & 0 \leqslant t \leqslant T_z, \\ \int\limits_{t-T_v}^{T_z} f_1\, d\xi + \int\limits_{T_z}^{t} f_2\, d\xi, & T_z < t \leqslant T, \\ \int\limits_{t-T_v}^{t} f_2\, d\xi, & t > T, \end{cases} \tag{25a – c}$$

if $T_v \leqslant T_z$, and as

$$\begin{cases} \int\limits_0^t f_1\, d\xi, & 0 \leqslant t \leqslant T_z, \\ \int\limits_0^{T_z} f_1\, d\xi + \int\limits_{T_z}^{t} f_2\, d\xi, & T_z < t \leqslant T_v, \\ \int\limits_{t-T_v}^{T_z} f_1\, d\xi + \int\limits_{T_z}^{t} f_2\, d\xi, & T_v < t \leqslant T, \\ \int\limits_{t-T_v}^{t} f_2\, d\xi, & t > T, \end{cases} \tag{25d – g}$$

if $T_v > T_z$. Here

$$f_1(\xi, t) = \int\limits_\sigma z^0(\tau_y, \tau_x - \xi, T_z - \xi) \exp\left\{ -\int\limits_0^\xi \nu^z\left(r_\eta(\xi, T_z), n\right) d\eta - \right.$$

$$- \int_{\xi}^{t} \nu^{v} \left(r_{\eta}(\xi, 0), n\right) \, d\eta \Big\} \, d\tau_y \, d\tau_x,$$

$$f_2(\xi, t) = \int_{\sigma} \overline{z}(\xi - T_z, \tau_y, \tau_x - T_z) \exp\Big\{ - \int_{0}^{T_z} \nu^{z} \left(h_{\eta}(t, T_z), n\right) \, d\eta$$
$$- \int_{\xi}^{t} \nu^{v} \left(r_{\eta}(\xi, 0), n\right) \, d\eta \Big\} \, d\tau_y \, d\tau_x.$$

Using Eqs. (13d), (10b) and (18) we can also rewrite functions X and $\overline{x}$, $\overline{y}$ as follows

$$X = \begin{cases} \int\limits_{\sigma_y} v^0(\tau_y, \tau_x - t, T_v \\ \quad -t) \exp\Big\{ - \int\limits_0^t \nu^v \left(r_\eta(t, T_v), n\right) \, d\eta \Big\} \, d\tau_y, & 0 \leqslant t \leqslant T_v, \\ \int\limits_{\sigma_y} z_1(t - T_v, \tau_y, \tau_x \\ \quad -T_v, T_z) \exp\Big\{ - \int\limits_0^{T_v} \nu^v \left(h_\eta(t, T_v), n\right) \, d\eta \Big\} \, d\tau_y, & t \in (T_v, T], \\ \int\limits_{\sigma_y} z_2(t - T_v, \tau_y, \tau_x \\ \quad -T_v, T_z) \exp\Big\{ - \int\limits_0^{T_v} \nu^v \left(h_\eta(t, T_v), n\right) \, d\eta \Big\} \, d\tau_y, & t > T, \end{cases} \tag{26a – c}$$

$$\overline{\rho} = \begin{cases} \int\limits_{\sigma} b^\rho z^0(\tau_y, \tau_x - t, T_z \\ \quad -t) \exp\Big\{ - \int\limits_0^t \nu^z \left(r_\eta(t, T_z), n\right) \, d\eta \Big\} \, d\tau_y \, d\tau_x, & 0 \leqslant t \leqslant T_z, \\ \int\limits_{\sigma} b^\rho \overline{z}(t - T_z, \tau_y, \tau_x \\ \quad -T_z) \exp\Big\{ - \int\limits_0^{T_z} \nu^z \left(h_\eta(t, T_z), n\right) \, d\eta \Big\} \, d\tau_y \, d\tau_x, & t > T_z, \end{cases} \tag{27a, b}$$

where $\rho = x,\ y$ and

$$z_1(t - T_v, \tau_y, \tau_x - T_v, T_z)$$
$$= z^0(\tau_y, \tau_x - t, T - t) \exp\Big\{ - \int_0^{t - T_v} \nu^z \left(r_\eta(t, T), n\right) \, d\eta \Big\}, \tag{28a}$$

$$z_2(t - T_v, \tau_y, \tau_x - T_v, T_z)$$
$$= \overline{z}(t - T, \tau_y, \tau_x - T) \exp\left\{ - \int_0^{T_z} \nu^z\left(h_\eta(t, T), n\right) d\eta \right\}. \qquad (28b)$$

We define $I_i = ((i-1)\widetilde{T}, i\widetilde{T}]$, $\widetilde{T} = \min(T_z, T_v)$, $i = 1, 2, \ldots$.

Let $t \in I_1$. From Eqs. (24) and (25a) or (25d) we obtain the function $n^v(n)$. Similarly from Eqs. (26a) and (27a) we get $X(n)$, $\overline{x}(n)$, $\overline{y}(n)$. Taking into account these functions from Eqs. (5b), (5a), (16) and (19) we get functions $y(n)$, $n_y(n)$, $d^x(n)$ and $x(n)$. Then we substitute $y(n)$, $x(n)$ into Eqs. (10a), (13c), use Eqs. (18b), (15) and obtain $z = z_2 = z(n)$ for $t \leqslant \tau_z + \widetilde{T}$. Finally, knowing functions $x(n)$, $y(n)$, $z(n)$, from Eqs. (21) – (23) we obtain $n^y(n)$, $n^x(n)$, $n^z(n)$, substitute them into Eq. (20) and obtain an integral equation $n = V_1(n)$. Here $z(n)$, $\overline{x}(n)$, $\overline{y}(n)$, $n_y(n)$, $d^x(n)$, $X(n)$, $x(n)$, $y(n)$, $n^y(n)$, $n^x(n)$, $n^z(n), n^v(n)$, $V_1(n)$ are right-hand sides of (18), (27a) for $\rho = x$, y, (5b), (5a), (26a), (19), (16), (21) – (24), (20), respectively.

Let $t \in I_2$. We know functions n, x, y, z, v for $t \in I_1$ and relation $z = z_2 = z(n)$ for $t \leqslant \tau_z + \widetilde{T}$. That allows us to repeat analogous argumentation and to obtain an integral equation $n = V_2(n)$ and so on.

As a result of our argumentation we get an integral equation $n = V(n)$ for $t \in (0, \infty)$.

Let's define:

$$E_1 = \overline{I} \times \sigma_y \times \overline{I}, \qquad E_2 = \sigma_{xz}(\tau_z) \times \sigma_z,$$
$$E_3 = \sigma_{xv}(\tau_v) \times \sigma_v, \quad E_4 = \overline{I} \times \sigma_y \times \sigma_{xz}(0) \times \overline{I},$$

$$n_x(t) = \int_{\sigma_{xz}(0)} x \, d\tau_x, \quad n^{y0} = \int_0^\infty y^0 \, d\tau_y,$$

$$n^{x0} = \int_0^\infty x^0 \, d\tau_x, \quad n^{z0} = \int_{E^{0z}} z^0 \, d\tau_y \, d\tau_x \, d\tau_z, \quad n^{v0} = \int_{E^{0v}} v^0 \, d\tau_y \, d\tau_x \, d\tau_v,$$

$$a = \sup_{[0,\tau_4]} x^0, \quad p^* = \sup_{E_4} p, \quad \nu_*^x = \inf_{\overline{I} \times \overline{I} \times \overline{I}} \nu^x, \quad \nu_*^y = \inf_{\overline{I} \times \overline{I} \times \overline{I}} \nu^y,$$

$$\nu_*^z = \inf_{E^z \times \overline{I}} \nu^z, \quad \nu_*^v = \inf_{E^v \times \overline{I}} \nu^v, \quad \nu^{y*} = \sup_{\overline{I} \times \overline{I} \times \overline{I}} \nu^y,$$

$$B^* = \max\left\{ \int\limits_{\sigma_{xv}(0)} \sup_{(t,\tau_y,n)\in E_1} b^x \, d\tau_x, \int\limits_{\sigma_{xv}(0)} \sup_{(t,\tau_y,n)\in E_1} b^y \, d\tau_x \right\},$$

$$q = (B^*/a) \max\left\{ \int\limits_{\sigma_y} \sup_{(\tau_x,\tau_z)\in E_2} z^0 \, d\tau_y, \int\limits_{\sigma_y} \sup_{(\tau_x,\tau_v)\in E_3} v^0 \, d\tau_y \right\},$$

$$\gamma^y = \max\left(n^{y0}, aq/\nu_*^y\right),$$

$$\gamma^x = ah^x + \max\left(a(1+q)/\nu_*^x,\ n^{x0} - \int\limits_{\sigma_{xv}(0)} x^0 \, d\tau_x \right),$$

$$\gamma^z = ah^x p^* T_z \max(1, q/(B^*p^*)), \quad \gamma^v = ah^x q T_v/B^*,$$

$$\gamma = \gamma^x + \gamma^y + \gamma^z + \gamma^v.$$

Denoting $C_\gamma = \left\{f(t) : f \in C(\overline{I}),\ 0 \leqslant f \leqslant \gamma,\ \| f \| = \sup_{\overline{I}} | f | \right\}$ we shall prove the following theorem.

Theorem 1. *Assume that:* 1) ω, p *and* s^0, ν^s, *where* $s = x, y, z, v$, *are given non-negative continuous functions;* b^x *and* b^y *are bounded non-negative continuous in* $\xi = (t, n)$ *and piecewise continuous functions in* $\tau = (\tau_y, \tau_x)$; 2) a, p^* *and* ν^{y*}, n^{s0}, ν_*^s, *where* $s = x, y, z, v$, *are given positive constants;* 3) $B^* p^* \exp\{-T_z \nu_*^z\} \leqslant q \leqslant \min(1, B^* \nu_*^x)$; 4) *functions* $x^0, y^0, z^0, v^0, \omega$ *satisfy the reconcilable conditions. Then operator* V *acts in* C_γ *and the following estimates are valid:*

$$\max\left(\sup_{t\in\overline{I}} \overline{x}(n),\ \sup_{t\in\overline{I}} \overline{y}(n) \right) \leqslant aq^{k+1}, \quad k\tau_4 < t \leqslant (k+1)\tau_4, \tag{29}$$

$$0 \leqslant y(n) \leqslant \begin{cases} y^0(\tau_y - t)\exp\{-t\nu_*^y\}, & 0 < t \leqslant \tau_y < \infty, \\ aq^{k+1}\exp\{-\tau_y \nu_*^y\}, & k\tau_4 < t - \tau_y \leqslant (k+1)\tau_4, \\ & \tau_y \in I, \end{cases} \tag{30}$$

$$0 \leqslant x(n) \leqslant \begin{cases} x^0(\tau_x - t) \exp\{-t\nu_*^x\}, & 0 < t \leqslant \tau_x \in (0, \tau_3], \\ a, & 0 < t \leqslant \tau_x \in (\tau_3, \tau_4], \\ x^0(\tau_x - t) \exp\{-t\nu_*^x\}, & 0 < t \leqslant \tau_x - \tau_4, \\ & \tau_x \in (\tau_4, \infty), \\ aq^{k+1} \exp\{-\tau_x \nu_*^x\}, & k\tau_4 < t - \tau_x \leqslant (k+1)\tau_4, \\ & \tau_x \in (0, \tau_3], \\ aq^{k+1}, & k\tau_4 < t - \tau_x \leqslant (k+1)\tau_4, \\ & \tau_x \in (\tau_3, \tau_4], \\ aq^k \exp\{-(\tau_x - \tau_4)\nu_*^x\}, & (k-1)\tau_4 < t - \tau_x \leqslant k\tau_4, \\ & \tau_x \in (\tau_4, \infty), \end{cases} \tag{31}$$

$$\begin{aligned} &0 \leqslant n^x(n) \leqslant \gamma^x, \quad 0 < n^y(n) \leqslant \gamma^y, \\ &0 \leqslant n^z(n) \leqslant \gamma^z, \quad 0 < V(n) \leqslant \gamma. \end{aligned} \tag{32}$$

Here $k = 0, 1, \ldots$.

Proof. The construction of operator V and assumptions of our theorem show that $V(n) \in C(\bar{I})$, if $n \in C_\gamma$. The estimation (30) follows from relations (16) and estimate (29), while the estimation (31) for $\tau_x \in (0, \tau_3]$ we obtain from Eq. (19) and inequality (29). Similarly for $\tau_x \in (\tau_4, \infty)$ we get it from formula (19) and estimate (31) for $\tau_x \in (\tau_3, \tau_4]$. Thus, we must obtain the estimates (29) and (31) for $\tau_x \in (\tau_3, \tau_4]$. We derive these estimates for $t \in I_{k+2}$ using Gronwal's lemma, the assumptions of our theorem and already proved estimate (31) for $\tau_x \in (0, \tau_3]$ from Eqs. (28), (26) and (19) for $t \in ((k+1)\widetilde{T} - T, (k+1)\widetilde{T}]$, $(k+1)\widetilde{T} - T \geqslant 0$. That should be performed by successive cosideration. Note, that similar estimates in detail were performed by Skakauskas (1995). Finally, using relations (29) – (31), from Eqs. (21) – (24) we obtain the following estimates:

$$n^{y0} \exp\{-t\nu^{y*}\} \leqslant n^y(n) \leqslant \int_0^t \bar{y}(t - \tau_y) \exp\{-\tau_y \nu_*^y\}\, d\tau_y + \int_t^\infty y^0(\tau_y - t)\, d\tau_y \exp\{-t\nu_*^y\} \leqslant \gamma^y,$$

$$n^x(n) \leqslant \int_0^{\tau_3} x\, d\tau_x + \int_{\tau_3}^{\tau_4} x\, d\tau_x + \int_{\tau_4}^\infty x\, d\tau_x \leqslant$$

$$\leqslant \int\limits_0^{\min(t,\tau_3)} \overline{x}(t-\tau_x)\exp\{-\tau_x\nu_*^x\}\,d\tau_x$$

$$+ \int\limits_{\min(t,\tau_3)}^{\tau_3} x^0(\tau_x - t)\,d\tau_x \exp\{-t\nu_*^x\} + ah^x$$

$$+ \int\limits_{\tau_4}^{t+\tau_4} a\exp\{-\nu_*^x(\tau_x-\tau_4)\}\,d\tau_x + \int\limits_{t+\tau_4}^{\infty} x^0(\tau_x - t)\,d\tau_x\exp\{-t\nu_*^x\} \leqslant \gamma^x,$$

$$n^z(n) \leqslant aq(B^*)^{-1}h^x\max(T_z - t, 0) + h^x ap^*\beta_z(t) \leqslant \gamma^z,$$

$$n^v(n) \leqslant aqh^x(B^*)^{-1}\max(T_v - t, 0)$$

$$+ \left\{\begin{array}{ll} aqh^x\beta_v(t)/B^*, & 0 \leqslant t \leqslant T_z \\ (T-t)aqh^x/B^* + (t - T_z)ah^xp^*\exp\{-T_z\nu_*^z\}, & T_z < t \leqslant T \\ T_vp^*ah^x\exp\{-T_z\nu_*^z\}, & t > T \end{array}\right\} \leqslant \gamma^v.$$

Hence $0 < V(n) \leqslant \gamma$ and the proof of Theorem 1 is complete.

Theorem 2. *Assume that:* 1) *the assumptions of Theorem 1 hold;* 2) *functions* ν^x, ν^y, ν^z, ν^v *and* p *are Lipshitz continuous in* n *with constants* κ^x, κ^y, κ^z, κ^v, *respectively;* 3) $f(t) = n_x/n_y \leqslant f_0 = \text{const}$. *Then* $\| V_i(n_2) - V_i(n_1) \| \leqslant \kappa_*\varepsilon \| n_2 - n_1 \|$, *where* $n_s \in C_\gamma$, $s = 1,2$; $\kappa_* = \max(\kappa^x, \kappa^y, \kappa^z, \kappa^v)$ *and* $\varepsilon = \varepsilon(\nu_*, a, q, T_z, T_v, h^x, B^*, p^*, n^{x0}, n^{y0}, n^{z0}, n^{v0})$ *is a positive function monotonically decreasing to zero as* $\nu_* = \min(\nu_*^y, \nu_*^x, \nu_*^z, \nu_*^v) \to \infty$.

Proof. Let $n_s \in C_\gamma$, $s = 1,2$. Assume, that $\widetilde{C}$ is a positive constant, independent of κ_* and ν_*^s, $s = x,y,z,v$. Let's denote $g_s = g(n_s)$, $\Delta g = g_2 - g_1$, $P_{\xi s}^{u,\gamma} = \exp\{-\int\limits_\xi^u \nu_s^\gamma d\eta\}$, $\nu_s^\gamma = \nu^\gamma(l_\eta^\gamma, n_s)$, $l_\eta^\gamma = r_\eta^\gamma, h_\eta^\gamma$, where $s = 1,2$, and the argument of the function n_s is the same as the first argument of the collection l_η^γ.

Note that the function $\overline{z}^{T_z} = \overline{z}(t - T_z, \tau_y, \tau_x - T_z)$ for $t \in I_i$ we can express by values of functions n, x, y for $t \in ((i-1)\widetilde{T} - T_z, i\widetilde{T} - T_z]$ that should be found from equations $n = V_k(n)$ for $k < i$. Therefore, when we consider the solvability of the equation $n = V_i(n)$, functions $\overline{z}^{T_z}$ and $n(t-T_z)$ are assumed to be given. Similarly, the function $\overline{z}^T = \overline{z}(t-T, \tau_y, \tau_x - T)$ for $t \in I_i$ can be expressed by values of functions x, y, n for $t \in ((i-1)\widetilde{T} - T,\ i\widetilde{T} - T]$ and is assumed to be known in the equation $n = V_i(n)$.

We shall get the estimation of the norm $\| V_i(n_2) - V_i(n_1) \|$ for $t \in I_i$. In the rest of this paper we shall use the estimation

$$\left|\Delta P_\xi^{u,\gamma}\right| = \left|P_{\xi 2}^{u,\gamma} - P_{\xi 1}^{u,\gamma}\right| \leqslant \exp\{-(u-\xi)\nu_*^\gamma\}(u-\xi)\kappa^\gamma \| \Delta n \| \leqslant \kappa^\gamma \varepsilon(\nu_*^\gamma, .) \| \Delta n \|,$$

$$\varepsilon(\nu_*^\gamma, .) = 1/e\nu_*^\gamma, \quad \xi \leqslant u,$$

without refering to it. The point-argument of ε represents the other arguments of this function.

From Eq. (28) we have the following estimation

$$| \Delta \overline{\rho} | \leqslant \left\{ \begin{array}{ll} \int\limits_\sigma b^\rho z^0 \left|\Delta P_0^{t,z}\right| d\tau_y\, d\tau_x, & 0 \leqslant t \leqslant T_z \\ \int\limits_\sigma b^\rho\, \overline{z}^{T_z} \left|\Delta P_0^{T_z,z}\right| d\tau_y\, d\tau_x, & t > T_z \end{array} \right\} \leqslant aq\kappa^z T_z \| \Delta n \|, \quad \rho = y, x. \tag{33}$$

Similarly, from Eqs. (27) and (26) we get

$$| \Delta X | \leqslant \left\{ \begin{array}{l} \int\limits_{\sigma_y} v^0 \left|\Delta P_0^{t,v}\right| d\tau_y, \quad 0 \leqslant t \leqslant T_v \\ \int\limits_{\sigma_y} z^0 \left\{ \left|\Delta P_0^{t-T_v,z}\right| P_{02}^{T_v,v} + P_{01}^{t-T_v,z} \left|\Delta P_0^{T_v,v}\right| \right\} d\tau_y, \\ \qquad T_v < t \leqslant T \\ \int\limits_{\sigma_y} \overline{z}^T \left\{ \left|\Delta P_0^{T_z,z}\right| P_{02}^{T_v,v} + P_{01}^{T_z z} \left|\Delta P_0^{T_v,v}\right| \right\} d\tau_y, \quad t > T \end{array} \right\}$$

$$\leqslant \kappa_* \widetilde{C} \| \Delta n \|, \quad \widetilde{C} = aqT/B^*. \tag{34}$$

Using relations (16), (21), (30) and (33) we can obtain

$$| \Delta n^y | \leqslant \int\limits_0^\infty y^0 \mid \Delta P_0^{t,y} \mid d\tau_y + \int\limits_0^t \{\overline{y}_1 \mid \Delta P_0^{\tau_y,y} \mid + P_{02}^{\tau_y,y} \mid \Delta \overline{y} \mid\} d\tau_y \leqslant \kappa_* \varepsilon \| \Delta n \| . \tag{35}$$

We shall evaluate the function $| \Delta n^x | = | \sum_{i=0}^{4} \int_{\tau_i}^{\tau_{i+1}} \Delta x \, d\tau_x |$. For $\tau_x \in (0, \tau_1]$ from Eq. (19) we obtain the inequality

$$| \Delta x | \leqslant \begin{Bmatrix} x^0(\tau_x - t) \mid \Delta P_0^{t,x} \mid, & 0 \leqslant t \leqslant \tau_x \\ \overline{x}_2 \mid \Delta P_0^{\tau_x,x} \mid + P_{01}^{\tau_x,x} \mid \Delta \overline{x} \mid, & t > \tau_x \end{Bmatrix} \tag{36}$$

$$\leqslant Q(t, \tau_x) \parallel \Delta n \parallel,$$

$$Q(t, \tau_x) = \begin{Bmatrix} x^0(\tau_x - t)\kappa^x / e\nu_*^x, & 0 \leqslant t \leqslant \tau_x \\ aq \exp\{-\tau_x \nu_*^x\}(\kappa^x \tau_x + \kappa^z T_z), & t > \tau_x \end{Bmatrix} \leqslant \kappa_* \varepsilon.$$

Thus

$$\int_0^{\tau_1} | \Delta x | \, d\tau_x \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel . \tag{37}$$

Now we evaluate $\int_{\tau_1}^{\tau_2} | \Delta x | \, d\tau_x$. Using Eqs. (2) and (11) for Δx we obtain the estimation

$$| \Delta x | \leqslant \begin{cases} \int_0^t S^x(t,\xi) \left[(x_1 \, |\Delta d^x|) \, \big| r_\xi^x \right] d\xi, & 0 \leqslant t \leqslant \tau_x - \tau_1, \\ S^x(\tau_x, \tau_1) \left[| \Delta x | \, \big| h_{\tau_1}^x \right] & \\ + \int_{\tau_1}^{\tau_x} S^x(\tau_x, \xi) \left[(x_1 \, |\Delta d^x|) \, \big| h_\xi^x \right] d\xi, & t > \tau_x - \tau_1. \end{cases} \tag{38}$$

Here and in the rest of this paper $S^x(t,\xi) = \exp\{-\nu_*^x(t-\xi)\}$, $[g|r_\xi^x] = g(r_\xi^x)$, $[g|h_\xi^x] = g(h_\xi^x)$. By inequality (35) and assumption 2) of our theorem from Eq. (5a) we get inequality

$$| \Delta d^x | \leqslant \kappa_* \left(\widetilde{C} + n_{y1}^{-1} \varepsilon \right) \parallel \Delta n \parallel, \tag{39}$$

which, together with relations (38) and (31) leads to the estimate

$$| \Delta x | \leqslant \parallel \Delta n \parallel \kappa_* \varepsilon \begin{cases} 1 + \varphi(t, \tau_x), & 0 \leqslant t \leqslant \tau_x - \tau_1, \\ 1 + \psi(t, \tau_x) + F(t, \tau_x), & t > \tau_x - \tau_1, \end{cases} \tag{40}$$

where

$$\varphi(t,\tau_x) = \int_0^t S^x(t,\xi)\left[\left(x_1(n_{y1})^{-1}\right)\left|r_\xi^x\right]\, d\xi,$$

$$\psi(t,\tau_x) = \int_{\tau_1}^{\tau_x} S^x(\tau_x,\xi)\left[\left(x_1(n_{y1})^{-1}\right)\left|h_\xi^x\right]\, d\xi,$$

$$\varepsilon F(t,\tau_x) = S^x(\tau_x,\tau_1)Q(h_{\tau_1}^x) \leqslant S^x(\tau_x,\tau_1)\kappa_*\varepsilon.$$

Using the estimate (40), we can show, that

$$\int_{\tau_1}^{\tau_2} |\Delta x|\, d\tau_x \leqslant \|\ \Delta n\ \|\ \kappa_*\varepsilon\left\{1 + \int_{\tau_1}^{u} (F(t,\tau_x) + \psi(t,\tau_x))\, d\tau_x \right.$$

$$\left. + \int_u^{\tau_2} \varphi(t,\tau_x)\, d\tau_x \right\},$$

where $u = \min(t+\tau_1, \tau_2)$. The following inequalities are valid:

$$H_1 = \int_{\tau_1}^{u} \psi(t,\tau_x)\, d\tau_x = \int_{\tau_1}^{u} d\tau_x \int_{\tau_1}^{\tau_x} S^x(\tau_x,\xi)x(\xi+t-\tau_x,\xi)/n(\xi+t-\tau_x)\, d\xi$$

$$= \int_{\tau_1}^{u} d\xi \int_{\xi}^{u} S^x(\tau_x,\xi)x(\xi+t-\tau_x,\xi)/n(\xi+t-\tau_x)\, d\tau_x$$

$$\leqslant \int_{\tau_1}^{u} d\xi \int_{\xi+t-u}^{t} S^x(t,\eta)x(\eta,\xi)/n(\eta)\, d\eta \leqslant \int_{\tau_1}^{u} d\xi \int_0^t S^x(t,\eta)x(\eta,\xi)/n(\eta)\, d\eta$$

$$\leqslant \int_{\tau_1}^{\tau_2} d\xi \int_0^t S^x(t,\eta)x(\eta,\xi)/n(\eta)d\eta = \int_0^t S^x(t,\eta) \int_{\tau_1}^{\tau_2} x(\eta,\xi)d\xi/n(\eta)d\eta$$

$$\leqslant \int_0^t S^x(t,\eta)f(\eta)\, d\eta,$$

and

$$H_2 = \int_u^{\tau_2} \varphi(t, \tau_x) d\tau_x = \int_u^{\tau_2} d\tau_x \int_0^t S^x(t, \xi) x(\xi, \xi + \tau_x - t)/n(\xi) d\xi$$

$$= \int_0^t d\xi S^x(t, \xi) \int_u^{\tau_2} x(\xi, \xi + \tau_x - t) d\tau_x / n(\xi)$$

$$= \int_0^t d\xi S^x(t, \xi) \int_{\xi+u-t}^{\xi+\tau_2-t} x(\xi, \eta) d\eta / n(\xi)$$

$$\leqslant \int_0^t d\xi S^x(t, \xi) \int_{\tau_1}^{\tau_2} x(\xi, \eta)\, d\eta / n(\xi) = \int_0^t S^x(t, \xi) f(\xi) d\xi, \quad \text{when} \quad t < h^x,$$

$H_2 = 0$, when $t \geqslant h^x$. Then, from these estimates and from the assumption 3) of our theorem, we obtain

$$\int_{\tau_1}^{\tau_2} |\, \Delta x \,|\, d\tau_x \leqslant \|\, \Delta n \,\|\, \kappa_* \varepsilon \Big\{ 1 + 2 \int_0^t S^x(t, \xi) f(\xi)\, d\xi + \int_{\tau_1}^{u} F\, d\tau_x \Big\} \leqslant \kappa_* \varepsilon \,\|\, \Delta n \,\|\, . \tag{41}$$

Let's consider $\int_{\tau_2}^{\tau_3} |\, \Delta x \,|\, d\tau_x$. Using Eq. (19) and estimate (30) we can write the inequality

$$|\, \Delta x \,| \leqslant \begin{cases} x^0(\tau_x - t) \left|\Delta P_0^{t,x}\right| \leqslant \kappa_* \varepsilon\, x^0(\tau_x - t) \,\|\, \Delta n \,\|, \\ \qquad 0 \leqslant t \leqslant \tau_x - \tau_2, \\ S^x(\tau_x, \tau_2) \left[|\, \Delta x \,|\, \left|h^x_{\tau_2}\right] + a \left|\Delta P_{\tau_2}^{\tau_x, x}\right| \right., \\ \qquad t > \tau_x - \tau_2, \end{cases} \tag{42}$$

which shows that

$$\int_{\tau_2}^{\tau_3} |\, \Delta x \,|\, d\tau_x \leqslant \kappa_* \varepsilon \,\|\, \Delta n \,\| + \int_m^t S^x(t, \rho) \,|\, \Delta x \,|\, \big|_{(\rho, \tau_2)}\, d\rho,$$

$$m = \tau_2 + t - \min(t + \tau_2, \tau_3).$$

This estimate together with inequality (40) enables us to obtain the estimate

$$\int_{\tau_2}^{\tau_3} |\, \Delta x \,|\; d\tau_x \leqslant \kappa_* \varepsilon (1 + \widetilde{R}_1) \, \| \, \Delta n \, \|,$$

where

$$\widetilde{R}_1 = \int_m^t S^x(t,\rho) \begin{Bmatrix} \varphi(\rho,\tau_2), & 0 \leqslant \rho \leqslant h^x \\ \psi(\rho,\tau_2) + F(\rho,\tau_2), & \rho > h_x \end{Bmatrix} d\rho \leqslant \varepsilon + R_1,$$

$$R_1 = \int_m^t S^x(t,\rho) \begin{Bmatrix} \varphi(\rho,\tau_2), & 0 \leqslant \rho \leqslant h^x \\ \psi(\rho,\tau_2), & \rho > h^x \end{Bmatrix} d\rho.$$

To estimate R_1 we consider two cases: $h_1 = T - h^x \leqslant h^x$ and $h_1 > h^x$.

If $h_1 \leqslant h^x$, then $R_1 = R_{11}, R_{12}, R_{13}, R_{14}$ for $t \in [0, h_1]$, $(h_1, h^x]$, $(h^x, T]$, (T, ∞), respectively, where

$$R_{11} = \int_0^t \widetilde{\varphi}\, d\rho, \qquad R_{12} = \int_{t-h_1}^t \widetilde{\varphi}\, d\rho,$$

$$R_{13} = \int_{t-h_1}^{h^x} \widetilde{\varphi}\, d\rho + \int_{h^x}^t \widetilde{\psi}\, d\rho, \quad R_{14} = \int_{t-h_1}^t \widetilde{\psi}\, d\rho;$$

here and later $\widetilde{\varphi} = \varphi(\rho,\tau_2) S^x(t,\rho)$, $\widetilde{\psi} = \psi(\rho,\tau_2) S^x(t,\rho)$. By the same way, that we used to obtain the estimates of H_1, H_2, we can get

$$\max\{R_{11}, R_{12}, 1/2 R_{13}, R_{14}\} \leqslant \int_0^t S^x(t,\rho) f(\rho)\, d\rho \leqslant (\nu_*)^{-1} f_0,$$

and $R_1 \leqslant \varepsilon$.

If $h_1 > h^x$, then $R_1 = R_{11}, R_{12}, R_{13}, R_{14}$ for $t \in [0, h^x]$, $(h^x, h_1]$, $(h_1, T]$, (T, ∞), respectively, where

$$R_{11} = \int_0^t \widetilde{\varphi}\, d\rho, \qquad R_{12} = \int_0^{h^x} \widetilde{\varphi}\, d\rho + \int_{h^x}^t \widetilde{\psi}\, d\rho,$$

$$R_{13} = \int_{t-h_1}^{h^x} \widetilde{\varphi}\, d\rho + \int_{h^x}^t \widetilde{\psi}\, d\rho, \quad R_{14} = \int_{t-h_1}^t \widetilde{\psi}\, d\rho.$$

As $f \leqslant f_0$, then, acting as above, we get

$$\max\left(R_{11}, R_{14}, 1/2R_{12}, 1/2R_{13}\right) \leqslant \int\limits_0^t S^x(t,\rho) f(\rho)\, d\rho \leqslant (\nu_*)^{-1} f_0$$

and $R_1 \leqslant \varepsilon$.

Thus

$$\int\limits_{\tau_2}^{\tau_3} \mid \Delta x \mid d\tau_x \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel . \tag{43}$$

Now we evaluate $\int\limits_{\tau_3}^{\tau_4} \mid \Delta x \mid d\tau_x$. From the relations (19), (34) we can obtain the estimates:

$$\mid \Delta x \mid \leqslant a \mid \Delta P_0^{t,x} \mid + \int\limits_0^t \left\{ X_1 \mid \Delta P_\xi^{t,x} \mid + P_{\xi 2}^{t,x} \mid \Delta X \mid \right\} d\xi \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel$$

for $0 \leqslant t \leqslant \tau_x - \tau_3$;

$$\mid \Delta x \mid \leqslant a \left| \Delta P_{\tau_3}^{\tau_x, x} \right| + S^x(\tau_x, \tau_3) \left[\mid \Delta x \mid \left| h_{\tau_3}^x \right] + \int\limits_{\tau_3}^{\tau_x} \left\{ S^x(t,\xi) \mid \Delta X \mid \right.\right.$$

$$\left. + X_1 \left| \Delta P_\xi^{\tau_x, x} \right| \right\} d\xi \leqslant S^x(\tau_x, \tau_3) \left[\mid \Delta x \mid \left| h_{\tau_3}^x \right] + \kappa_* \varepsilon \parallel \Delta n \parallel \right.$$

for $t > \tau_x - \tau_3$.

Thus

$$\mid \Delta x \mid \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel + \begin{cases} 0, & 0 \leqslant t \leqslant \tau_x - \tau_3, \\ S^x(\tau_x, \tau_3) \left[\mid \Delta x \mid \left| h_{\tau_3}^x \right] \right., & t > \tau_x - \tau_3, \end{cases} \tag{44}$$

$$\int\limits_{\tau_3}^{\tau_4} \mid \Delta x \mid d\tau_x \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel + \int\limits_{\tau_3}^{u} S^x(\tau_x, \tau_3) \left[\mid \Delta x \mid \left| h_{\tau_3}^x \right] d\tau_x \right.$$

$$\leqslant \kappa_* \varepsilon \parallel \Delta n \parallel + \widetilde{R}_2,$$

$$\widetilde{R}_2 = \int\limits_m^t S^x(t,\rho) \mid \Delta x \mid \Big|_{(\rho, \tau_3)} d\rho,$$

where $m = \tau_3 + t - u,\ u = \min(t + \tau_3, \tau_4)$. Taking into account the estimates (42) and (40) we get

$$\widetilde{R}_2 \leqslant \| \Delta n \| \ \kappa_* \varepsilon + \int\limits_{m-h_1}^{t-h_1} S^x(t,\eta) \left\{ \begin{array}{ll} 0, & -h_1 \leqslant \eta \leqslant 0 \\ | \Delta x | \,|_{(\eta,\tau_2)}, & \eta > 0 \end{array} \right\} d\eta$$

$$\leqslant \kappa_* \varepsilon (1 + R_2) \ \| \Delta n \|,$$

$$R_2 = \int\limits_{m-h_1}^{t-h_1} S^x(t,\eta) \left\{ \begin{array}{ll} 0, & -h_1 \leqslant \eta \leqslant 0 \\ \varphi(\eta, \tau_2), & 0 \leqslant \eta \leqslant h^x \\ \psi(\eta, \tau_2), & \eta > h^x \end{array} \right\} d\eta.$$

But $R_2 = 0,\ R_{21},\ R_{22},\ R_{23}$ for $t \in [0, h_1],\ (h_1, T],\ (T, T + h^x],\ (T + h^x, \infty)$, respectively, where $R_{21} = \int\limits_0^{t-h_1} \widetilde{\varphi} d\rho,\ R_{22} = \int\limits_{t-T}^{h^x} \widetilde{\varphi} d\rho + \int\limits_{h^x}^{t-h_1} \widetilde{\psi} d\rho,\ R_{23} = \int\limits_{t-T}^{t-h_1} \widetilde{\psi} d\rho$.

Acting as above, we get

$$R_{2k} \leqslant \int\limits_0^{t-h_1} S^x(t,\xi) f(\xi)\, d\xi, \quad k = 1, 3,$$

$$R_{22} \leqslant \int\limits_0^{h^x} S^x(t,\xi) f(\xi) d\xi + \int\limits_0^{t-h_1} S^x(t,\xi) f(\xi) d\xi,$$

and $R_2 \leqslant \varepsilon$. Hence

$$\int\limits_{\tau_3}^{\tau_4} | \Delta x | \, d\tau_x \leqslant \kappa_* \varepsilon \ \| \Delta n \| \,. \tag{45}$$

At last we evaluate $\int\limits_{\tau_4}^{\infty} | \Delta x | \, d\tau_x$. Using relations (19) and (44) we get

$$| \Delta x | \leqslant \left\{ \begin{array}{ll} x^0(\tau_x - t) \left| \Delta P_0^{t,x} \right|, & 0 \leqslant t \leqslant \tau_x - \tau_4, \\ a \left| \Delta P_{\tau_4}^{\tau_x, x} \right| + S^x(\tau_x, \tau_4) \left[| \Delta x | \ \left| h_{\tau_4}^x \right] \right., & t > \tau_x - \tau_4, \end{array} \right.$$

$$\int_{\tau_4}^{\infty} \mid \Delta x \mid d\tau_x \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel + \widetilde{R}_3,$$

$$\widetilde{R}_3 = \int_{-h^x}^{t-h^x} S^x(t,\eta) \left\{ \begin{array}{ll} 0, & -h^x \leqslant \eta \leqslant 0, \\ \mid \Delta x \mid \big|_{(\eta,\tau_3)}, & \eta > 0 \end{array} \right\} d\eta.$$

Taking into account estimates (42) and (40) we obtain inequalities:

$$\widetilde{R}_3 \leqslant \begin{cases} 0, \quad 0 \leqslant t \leqslant h^x, \\ \kappa_* \varepsilon \parallel \Delta n \parallel + \int\limits_{-h_1}^{t-T} S^x(t,\rho) \left\{ \begin{array}{l} 0, -h_1 \leqslant \rho \leqslant 0 \\ \mid \Delta x \mid \big|_{(\rho,\tau_2)}, \rho > 0 \end{array} \right\} d\rho, \ t > h^x, \end{cases}$$

$$\widetilde{R}_3 \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel (1 + R_3),$$

$$R_3 = \begin{cases} 0, \ 0 \leqslant t \leqslant T \\ \int\limits_0^{t-T} S^x(t,\rho) \left\{ \begin{array}{l} \varphi(\rho,\tau_2), \ 0 \leqslant \rho \leqslant h^x \\ \psi(\rho,\tau_2), \ \rho > h^x \end{array} \right\} d\rho, \ t > T. \end{cases}$$

But $R_3 = 0, \ R_{31}, \ R_{32}$ for $t \in [0,T], \ (T, T+h^x], \ (T+h^x, \infty)$, respectively, where

$$R_{31} = \int_0^{t-T} \widetilde{\varphi}\, d\rho, \quad R_{32} = \int_0^{h^x} \widetilde{\varphi} d\rho + \int_{h^x}^{t-T} \widetilde{\psi}\, d\rho.$$

Acting as above we get

$$R_{31} \leqslant \int_0^{t-T} S^x(t,\xi)\, f(\xi)\, d\xi,$$

$$R_{32} \leqslant \int_0^{h^x} S^x(t,\xi)\, f(\xi) d\xi + \int_0^{t-T} S^x(t,\xi) f(\xi)\, d\xi$$

and $R_3 \leqslant \varepsilon$. Thus

$$\int_{\tau_4}^{\infty} \mid \Delta x \mid d\tau_x \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel . \tag{46}$$

From the estimates (37), (41), (43), (45) and (46) we can obtain the following estimation

$$\mid \Delta n^x \mid \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel . \tag{47}$$

Now we evaluate $|\Delta n^z|$. From Eq. (23) we get $|\Delta n^z| \leqslant J_1 + J_2$, where

$$J_1 = \int_{\beta_z(t)}^{T_z} d\tau_z \int_{\sigma_{xz}(\tau_z)\times\sigma_y} |\Delta F_1| \, d\tau_x \, d\tau_y,$$

$$J_2 = \left| \int_0^{\beta_z(t)} d\tau_z \int_{\sigma_{xz}(\tau_z)\times\sigma_y} \Delta F_2 \, d\tau_x d\tau_y \right|.$$

Here $J_1 \leqslant \kappa_* \varepsilon \, \| \Delta n \|$. Denoting $u = t - \min(t, T_z)$, using assumptions of our theorem and estimates for $|\Delta x|$, $|\Delta y|$, $|\Delta n^y|$ obtained above we get

$$\begin{aligned}
J_2 = J_2^{12} \leqslant \int_u^t d\xi \int_{\sigma_y} d\tau_y \int_{\sigma_{xz}(0)} \Big\{ & |x_1 - x_2| \, y_1 p_1 n_{y1}^{-1} P_{\xi 1}^{t,z} \\
& + x_2 p_1 n_{y1}^{-1} P_{\xi 1}^{t,z} \, |y_1 - y_2| + x_2 y_2 n_{y1}^{-1} P_{\xi 1}^{t,z} \, |p_1 - p_2| \\
& + x_2 y_2 p_2 n_{y1}^{-1} \left| P_{\xi 1}^{t,z} - P_{\xi 2}^{t,z} \right| \\
& + x_2 y_2 p_2 n_{y1}^{-1} n_{y2}^{-1} P_{\xi 2}^{t,z} \, |n_{y1} - n_{y2}| \Big\} d\tau_x \\
\leqslant \int_u^t \Big\{ p_* & \int_{\sigma_{xz}(0)} |x_1 - x_2| \, d\tau_x \\
& + p_* \int_{\sigma_y} |y_1 - y_2| \, d\tau_y n_{y1}^{-1} \int_{\sigma_{xz}(0)} x_2 d\tau_x \\
& + \kappa^p \, \| n_1 - n_2 \| \, a h^x n_{y1}^{-1} \int_{\sigma_y} y_2 d\tau_y \\
& + \kappa^z p_* \, \| n_1 - n_2 \| \, (t - \xi) a h^x n_{y1}^{-1} \int_{\sigma_y} y_2 d\tau_y \\
& + p_* n_{y1}^{-1} \int_{\sigma_{xz}(0)} x_2 d\tau_x \, \| n_1^y - n_2^y \| \Big\} S^x(t, \xi) d\xi \\
\leqslant \int_u^t \Big\{ & \left(1 + n_{y2} n_{y1}^{-1}\right) \kappa_* \varepsilon + \kappa^p a h^x n_{y2} n_{y1}^{-1} \\
& + \kappa^z p_* a h^x (t - \xi) n_{y2} n_{y1}^{-1} + \kappa_* \varepsilon n_{y2} n_{y1}^{-1} \Big\} S^x(t, \xi) d\xi \, \| n_1 - n_2 \| \leqslant
\end{aligned}$$

$$\leqslant \kappa_* \varepsilon \left(1 + \sup_t (n_{y2} n_{y1}^{-1})\right) \| n_1 - n_2 \| .$$

Here $P_{\xi k}^{t,z} = \exp\{-\int_\xi^t \nu^z(\eta, \tau_y, \eta + \tau_x - \xi, \eta - \xi, n_k(\eta))d\eta\}, \quad k = 1, 2.$

From the equality $J_2^{12} = J_2^{21}$ we get another estimate

$$J_2^{12} \leqslant \kappa_* \varepsilon \left(1 + \sup_t (n_{y1} n_{y2}^{-1})\right) \| n_1 - n_2 \| .$$

These estimates show, that $J_2 \leqslant \kappa_* \varepsilon \| \Delta n \|$. Hence

$$| \Delta n^z | \leqslant \kappa_* \varepsilon \| \Delta n \| . \tag{48}$$

Now we evaluate $| \Delta n^v |$. Using Eq. (24) we get $| \Delta n^v | \leqslant J_3 + J_4$, where

$$J_3 = \int_{\beta_v(t)}^{T_v} d\tau_v \int_{\sigma_y \times \sigma_{xv}(\tau_v)} | \Delta F_1 | \, d\tau_y d\tau_x,$$

$$J_4 = \int_0^{\beta_v(t)} d\tau_v \int_{\sigma_y \times \sigma_{xv}(\tau_v)} | \Delta F_2 | \, d\tau_y d\tau_x .$$

From the Eq. (17) we have $| \Delta F_1 | \leqslant v(r_0^v) \, | \Delta P_0^{tv} |$. Therefore $J_3 \leqslant \kappa_* \varepsilon \|\Delta n\|$ for $t \in [0, T_v]$ and $J_3 = 0$ for $t > T_v$. Using Eqs. (25a – d) and the fact, that functions z_1 and $\bar{z}^{T_z}$ are known, we have

$$J_4 \leqslant \int_{t-\beta_v(t)}^{t} | \Delta f | \, d\xi$$

$$\leqslant \int_{t-\beta_v(t)}^{t} d\xi \int_\sigma z^0(\tau_y, \tau_x - \xi, T_z - \xi) \Big\{ \left|\Delta P_0^{\xi,z}\right| P_{\xi 2}^{t,v}$$

$$+ P_{01}^{\xi,z} \left|\Delta P_\xi^{t,v}\right| \Big\} d\tau_y \, d\tau_x \leqslant \kappa_* \varepsilon \| \Delta n \|$$

for $t \in [0, T_z], \ T_v \leqslant T_z$;

$$J_4 \leqslant \int_{t-T_v}^{T_z} | \Delta f_1 | \, d\xi + \int_{T_z}^{t} | \Delta f_2 | \, d\xi \leqslant$$

$$\leqslant \int_{t-T_v}^{T_z} z_1(\xi,\tau_y,\tau_x,T_z)\left|\Delta P_\xi^{t,v}\right| d\xi$$

$$+\int_{T_z}^{t} d\xi \int_\sigma \overline{z}^{T_z}\left\{\left|\Delta P_0^{T_z,z}\right| P_{\xi 2}^{t,v} + P_{01}^{T_z,z}\left|\Delta P_\xi^{t,v}\right|\right\} d\tau_y\, d\tau_x$$

$$\leqslant \kappa_* \varepsilon \parallel \Delta n \parallel$$

for $t \in (T_z, T]$, $T_v \leqslant T_z$;

$$J_4 \leqslant \int_{t-T_v}^{t} \mid \Delta f_2 \mid d\xi$$

$$\leqslant \int_{t-T_v}^{t} d\xi \int_\sigma \overline{z}^{T_z}\left\{\left|\Delta P_0^{T_z,z}\right| P_{\xi 2}^{t,v} + P_{01}^{T_z,z}\left|\Delta P_\xi^{t,v}\right|\right\} d\tau_y\, d\tau_x$$

$$\leqslant \int_{T_z}^{t} S^z(t,\xi)\, d\xi \int_\sigma \overline{z}^{T_z}\, d\tau_y\, d\tau_x \kappa_* \varepsilon \parallel \Delta n \parallel$$

$$\leqslant \kappa_* \varepsilon \parallel \Delta n \parallel$$

for $t > T_z$, $T_v \leqslant T_z$ and similarly

$$J_4 \leqslant \left\{\begin{array}{l} \int_0^t \mid \Delta f_1 \mid d\xi, \quad t \in [0, T_z] \\ \int_0^{T_z} \mid \Delta f_2 \mid d\xi + \int_{T_z}^{t} \mid \Delta f_2 \mid d\xi, \quad t \in (T_z, T_v] \\ \int_{t-T_v}^{T_z} \mid \Delta f_1 \mid d\xi + \int_{T_z}^{t} \mid \Delta f_2 \mid d\xi, \quad t \in (T_v, T] \\ \int_{t-T_v}^{t} \mid \Delta f_2 \mid d\xi, \quad t > T \end{array}\right\} \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel$$

for $T_v > T_z$. Here arguments of function $P_{\xi k}^{i,j}$ are obvious. Therefore

$$\mid \Delta n^v \mid \leqslant \kappa_* \varepsilon \parallel \Delta n \parallel . \tag{49}$$

Finally, from the estimates (35), (47), (48) and (49) we obtain the inequality $| V_i(n_2) - V_i(n_1) | \leqslant \kappa_* \varepsilon \| \Delta n \|$, which completes the proof of our theorem. The concrete form of the function ε is simple but rather cumbrous and so we do not represent it here. It is not difficult to note that our $\varepsilon \to 0$ as $\nu_* \to \infty$.

Note. Assumption 3) of Theorem 2 holds f.e. if

1) $\nu^x(t,\tau,n(t)) \geqslant \nu^y(t,\tau,n(t)) \; \forall \; (t,\tau,n(t)) \in I \times [0,\tau_{2x}] \times I$,
2) $b^x(t,\tau_y,\tau_x,n(t-T_z)) \leqslant b^y(t,\tau_y,\tau_x,n(t-T_z)) \; \forall \; (t,\tau_y,\tau_x,n(t-T_z)) \in I \times \sigma_y \times \sigma_{xz}(T_z) \times I$,
3) $x^0(\tau) \leqslant y^0(\tau) \; \forall \; \tau \in [0,\tau_{2x}]$

in the case $h^x < T$;

and if

1) $\nu^x(t,\tau,n(t)) \geqslant \nu^y(t,\tau,n(t)) \; \forall \; (t,\tau,n(t)) \in I \times [0,\tau_{2x}] \times I$,
2) $\nu^z(t,\tau_y,\tau_x,\tau_z,n(t)) \geqslant \nu^x(t,\tau_x,n(t)) \; \forall \; (t,\tau_y,\tau_x,\tau_z,n(t)) \in E^z \times I$,
3) $\nu^v(t,\tau_y,\tau_x,\tau_v,n(t)) \geqslant \nu^x(t,\tau_x,n(t)) \; \forall \; (t,\tau_y,\tau_x,\tau_v,n(t)) \in E^v \times I$,
4) $b^x(t,\tau_y,\tau_x,n(t-T_z)) \leqslant b^y(t,\tau_y,\tau_x,n(t-T_z)) \; \forall \; (t,\tau_y,\tau_x,n(t-T_z)) \in I \times \sigma_y \times \sigma_{xz}(T_z) \times I$,
5) $y^0(\tau_x) \geqslant x^0(\tau_x) + F^z(z^0) + F^v(v^0) \; \forall \; \tau_x \in [0,\tau_{2x}+T]$,

where

$$F^z(z) = \begin{cases} 0, & \tau_x \notin (\tau_{1x},\tau_{2x}+T], \\ \int\limits_{\sigma_y} d\tau_y \int\limits_{\omega^z(\tau_x)} z \, d\tau_z, & \tau_x \in (\tau_{1x},\tau_{2x}+T_z], \end{cases}$$

$$F^v(v) = \begin{cases} 0, & \tau_x \notin (\tau_{1x}+T_z,\tau_{2x}+T], \\ \int\limits_{\sigma_y} d\tau_y \int\limits_{\omega^v(\tau_x)} v \, d\tau_v, & \tau_x \in (\tau_{1x}+T_z,\tau_{2x}+T], \end{cases}$$

$$\omega^z(\tau_x) = [\max(\tau_x - \tau_{2x}, 0), \; \min(\tau_x - \tau_{1x}, T_z)] \, ,$$

$$\omega^v(\tau_x) = [\max(\tau_x - \tau_{2x} - T_z, 0), \; \min(\tau_x - \tau_{1x} - T_z, T_v)] \, ,$$

in the case $h^x \geqslant T$.

The statement of Note in the case $h^x < T$ is obvious because $x(t,\tau) \leqslant y(t,\tau) \; \forall \tau \in [0,\tau_{2x}]$. The statement of Note in the case $h^x \geqslant T$ should be proved. From Eqs. (17), (18) and (19) we can obtain the equation

$$D^x \left(x + F^z(z) + F^v(v)\right) = -\nu^x x - F^z(\nu^z z) - F^v(\nu^v v). \qquad (50)$$

By using assumptions 1) – 3) of Note from Eq. (50) we obtain the inequality $D^x(x + F^z(z) + F^v(v)) \leqslant -\nu^y(t,\tau_x)(x + F^z(z) + F^v(v))$, which together with assumptions 4) and 5) of Note shows that $x + F^z(z) + F^v(v) \leqslant y(t, \tau_x)$. This result proves the statement of our Note.

The following theorem is valid.

Theorem 3. *Assume that:* 1) *assumptions of Theorem* 2 *are satisfied,* 2) $\kappa_* \varepsilon(\nu_*, a, q, T_z, T_v, h^x, B^*, p^*, n^{x0}, n^{y0}, n^{z0}, n^{v0}) < 1,\ \nu_* < \nu^{y*}$. *Then the equation* $n = V(n)$ *has unique positive solution in* C_γ.

Proof. If $\kappa_* \varepsilon(\nu_*, .) < 1$, then the operator V_i is contractive. This inequality can be satisfied by appropriate choosing the parameters κ_* and ν_* (or even parameter κ_* only) for a given other arguments. Since V acts in C_γ and ε is independent of i, the successive consideration of equations $n = V_i(n),\ i = 1, 2, \ldots$, proves Theorem 3.

Corollary. If assumptions of Theorem 3 are satisfied, then problem (1) – (12) has unique non-negative continuous solution, such that estimates (29) – (32) hold and functions $D^y y,\ D^x x,\ D^z z,\ D^v v$ are continuos in $E^y,\ E^x, E^z$, E^v, respectively.

3.2. The case, when the fecundation rate and the fecundated females death rate are independent of age of males. This means that $\partial f / \partial \tau_y = 0,\ f = p, \nu^z$. We consider the case $h^x \geqslant T$. The opposite case we can consider in the similar way. To prove the solvability of system (1) – (12) we use the same method as above. In our case we have $\tau_3 < \tau_2$. Hence we must write Eq. (19) for $\tau_x \in [0, \tau_1],\ (\tau_1, \tau_3],\ (\tau_3, \tau_2],\ (\tau_2, \tau_4],\ (\tau_4, \infty)$. Acting as above we prove Theorem 1. Hence the estimates (29) – (32) are valid.

The function d^x in this case is

$$d^x = \nu^x + \begin{cases} 0, & \tau_x \notin \sigma_{xz}(0), \\ p, & \tau_x \in \sigma_{xz}(0). \end{cases}$$

Hence $| \Delta d^x | \leqslant \kappa_* \widetilde{C} \| \Delta n \|$. This enables us to write the estimate $| \Delta n^x | \leqslant \kappa_* \varepsilon \| \Delta n \|$ without consideration of the functions φ, ψ (see estimates of H_i and R_{sk}).

The integral $J_2 = \left| \int_0^{\beta_z} d\tau_z \int_{\sigma_{xz}(\tau_z) \times \sigma_y} \Delta F_2 d\tau_x d\tau_y \right|$, where $F_2 = F_2(z, 0, n)$, $\beta_z = \min(t, T_z)$, we can rewrite as

$$J_2 = \left| \int_{t-\beta_z}^{t} d\xi \int_{\sigma_{xz}(0)} \left(x_2 p_2 P_{\xi 2}^{t,z} - x_1 p_1 P_{\xi 1}^{t,z} \right) d\tau_x \right| \leqslant \kappa_* \varepsilon \| \Delta n \|;$$

here $P_{\xi k}^{t,z} = \exp\{-\int\limits_{\xi}^{t} \nu^z(\eta, \eta+\tau_x-\xi, \eta-\xi, n(\eta))d\eta\}$. Hence inequality (48) is valid. The estimates for $|\Delta n^y|$, $|\Delta n^v|$ remain the same as above. Therefore Theorem 2 in this case is valid without condition 3) which is essential in the case 3.1. Theorem 3 is also valid.

Conclusions. Taking into account the size, age structure, pregnancy and females restoration period after delivery the unique solvability of the model describing the evolution of non-migrating limited panmiction population, composed of two sexes, is proved.

As it follows from estimates (29) – (32) the population is bounded if $q \leqslant 1$ and vanishes if $q < 1$ as time increases.

If the assumption 3) of Theorem 2 does not hold and if fecundation rate and death rate of fecundated females depend on age of males and total population density the demographic functions probably can not be given a priori for all time.

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NEMIGRUOJANČIOS LIMITUOTOS PANMIKSINĖS POPULIACIJOS EVOLIUCIJOS PROBLEMOS VIENINTELIS IŠSPRENDŽIAMUMAS

Vladas SKAKAUSKAS

Nagrinėjama nemigruojančios limituotos populiacijos evoliucija, kai demografinės funkcijos priklauso nuo populiacijos dydžio. Populiaciją sudaro dvi lytys. Be to, priimamas dėmesin individų amžius. Patelės skirstomos į tris klases: nepastojusias, pastojusias ir pateles iš reabilitacijos intervalo po gimdymo. Nepaisoma vaisiaus žuvimo. Reproduktyvieji patelių ir patinėlių amžiaus intervalai laikomi baigtiniais, o patelė gali susilaukti baigtinį skaičių palikuonių vadų. Evoliucijos modelį sudaro integrodiferencialinių lygčių sistema trūkiais koeficientais su integralinėmis sąlygomis. Kai demografinės (mirtingumo, gimstamumo ir apsivaisinimo) funkcijos tenkina specialias sąlygas, įrodytas vienintelio klasikinio sprendinio egzistavimas bei gauti sprendinio įverčiai.