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ADAPTIVE ROBUST CONTROL FOR A CLASS OF SYSTEMS WITH POINT DELAYS

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Abstract. This paper presents a direct adaptive, control algorithm, based on a σ-modification rule, which is robust respect to additive and multiplicative plant unmodelled dynamics for plants involving both internal (i.e., in the state) and external (i.e., in the output or input) known point delays. Several adaptive controller structures are given and analyzed for the case of plants with unknown parameters while being assumed that the nominal plant is of known order and relative order. The parametrized parts of two of the controller structures involve delays while those of the two remaining controllers are delay-free. However, auxiliary compensating signals which weight the plant input and output integrals are incorporated in all the controller structures for stabilization purposes. It is proved that, if the unmodelled dynamics is sufficiently small at low frequencies, then the adaptive algorithm guarantees boundedness of all the signals in the closed-loop system.

Key words: adaptive control, delayed systems, internal and external delays, robust control.

1. Introduction. In the last years, a number of papers have dealt with the problem of presence of delays in the controlled plant and the related properties of controllability, observability and stabilizability have been investigated in Agathoklis and Foda (1989), Amemiya *et al.* (1988), De la Sen (1988a), De la Sen (1988b), De la Sen (1992), De la Sen (1993a), Ichikawa (1989), Manitius (1984), Olbrot (1978), Watanabe *et al.* (1984), Watanabe (1988), Watanabe *et al.* (1992) and Kamen *et al.* (1985), including adaptive stabilization in the case of plant external delay only or by using a time varying controller consisting of a set of gains switched at certain time instants (De la Sen, 1994). The stabilizability for known plants by using matrix Lyapunov equations and some delay-independent stabilization results have been addressed in Agathoklis and Foda (1989) and Amemiya *et al.* (1988). Also, the relationships between the stabilization of systems with point delays have been studied in De la Sen (1992) and De la Sen (1993a) by establishing an equivalent model, subject to point delays only, for the class of systems originally possessing exponentially distributed delays. The spectrum assignability has been investigated in Manitius (1984), Olbrot (1978), Watanabe *et al.* (1984), Watanabe (1988) and Watanabe *et al.* (1992) for systems with commensurate and noncommensurate delays. A system is finite spectrum assignable if and only if it is spectrally controllable, (Olbrot, 1978; Watanabe *et al.*, 1984), being the spectral controllability equivalent to weak controllability, (Morse, 1976). Also, if it is reachable then a closed-loop finite-spectrum can be achieved with a control function based on polynomials on the delay operator (Morse, 1976; Watanabe *et al.*, 1992).

Despite the importance given to the various classes of delayed systems in the last years, the application of adaptive control to such systems has not been exhaustively investigated. A globally stable adaptive algorithm for systems involving input delays has been proposed with the control input incorporating signals of finite Laplace transforms consisting of weighted time-integrals of the input over a time period equal to the delay. In such a scheme, the weighting functions are on-line calculated by the adaptation mechanism together with the parameters of the parametrized part of the controller. However, the more serious stabilizability problems arising from the presence of delays are caused by internal delays, Ichikawa (1989), since unsuitable infinite closed-loop spectra can be generated even if they are not established as a control objective.

This paper focuses the adaptive control of plants possessing simultaneously internal and external known finite point delays. The plant is assumed to be subject to additive and multiplicative unmodelled dynamics and the adaptive controller parametrization is based on the nominal plant dynamics according to the guidelines of Ioannou and Tsakalis (1986) for the delay-free case. Four robust adaptive controller structures are given being extensions of those proposed in Ioannou and Tsakalis (1986), Narendra and Annaswamy (1989) and De la Sen (1986) for delay-free plants. However, no positive realness condition either on a reference model transfer function or on an extended model including zeros is needed since, contrarily to Narendra and Annaswamy (1989) for the delay-free case, Yakubovitch's lemma is not used in the stability analysis. The time-invariant controller structures for the nominal known plants are first studied. They consist of a parametrized part plus a memory-type signal, which involve weighted integrals on the plant input and output. In the scheme's adaptive version, the associate weighting functions are calculated as a part of the estimation scheme. One of the main characteristics of the design in the case of known parameters asociated to the adaptive version proposed in this paper, is that some of the proposed controller structures (namely, the so-called Controller Structures I and II) be able to achieving prefixed (internal delay-dependent) infinite or (delay-independent) finite closed-loop spectra indistinctly. The particular objective depends on the particular controller parametrization, in the nominal situation of known parameters and perfectly modelled dynamics. The nominal closed-loop system can also match reference models whose zeros include those unstable ones of the plant. The contribution of this paper related to previous work, (Ichikawa, 1989; Manitius, 1984; Olbrot, 1978; Watanabe *et al.*, 1984, Watanabe, 1988, Watanabe *et al.*, 1992), is the following:

(a) Both single-internal and single-external point delay are considered in the plant state-space description. The control problem can be considered as non restrictive in the sense that n delays, being integer multiple of the internal delay, (n being the plant order) are automatically generated in the plant transfer function and, on the other hand, it has been proved in the literature that some distributed-delay systems with exponential distribution can be described through equivalent point delay systems (De la Sen, 1992; De la Sen, 1993a). On the other hand, it has been proved that the stabilization of open-loop stabilizable systems subject to internal delays is ensured by the use of distributed-delay controllers even if the plant possesses point delays only (see Manitius, 1984; Olbrot, 1978; Watanabe *et al.*, 1984; Watanabe, 1988; Watanabe *et al.*, 1992 and De la Sen, 1993b).

(b) The considered class of unmodelled dynamics includes both additive and multiplicative model disturbances being eventually subject to internal and external delays in the same way as the nominal plant.

The paper is organized as follows. Section 2 deals with the input/output nominal descriptions of the plant and the model plant. In Section 3, several controller structures are proposed as well as the statement and conditions of achievement of the control objective. The control objective is the adaptive stabilization under stable plant uncertainties of a given class. Section 4 is devoted to the adaptive algorithm and related stability and robustness properties in presence of unmodelled dynamics of the above mentioned class. Section 5 deals with further comments and some numerical simulations to evaluate the perfomance of the algorithms given in the previous sections. A comparative example including the performance of the various adaptive controller acting on the same plant is presented. In Section 6, conclusions end the paper. Finally, the mathematical derivations associated with the stability and robustness of the proposed adaptive scheme are given in the Appendices A–C.

Notation

- The Laplace transform of $f(t)$ is denoted by $f(s)$ and the Laplace transform of $f(-t)$ for $t > 0$ is denoted by $\bar{f}(s)$.
- $\deg_\mu[p(\mu, s)]$ and $\deg_s[p(\mu, s)]$ stand for the degrees of the quasipolynomial (or two-variable polynomial) $p(\mu, s)$ with respect to μ and s respectively. If both degrees are identical or the polynomial is of one variable, then subindices are not used.
- $\det(\cdot)$ and $Adj(\cdot)$ stand for the determinant and Adjoint of the $(\cdot)$ matrix.
- C denotes the set of complex numbers. C^+ and C^- are, respectively, the open left-half plane and its complement in C. $C_v^+ = z \in C : Re(z) \leqslant -v$, and C_v^- is the complement of C_v^+ in C for any real constant v. R and R^+ denote, respectively, the set of real and positive real numbers and $R_0^+ = R^+ \cup \{0\}$.
- Transfer functions involving internal and external delays h and h' are denoted by $G(s) = G(\mu, \mu', s)$ where $\mu = e^{(-hs)}$ and $\mu' = e^{(-h's)}$. The equivalent input-output differential-difference description is $y(t) = G(D, q^{-1}, q'^{-1})u(t)$ with $D = \frac{d}{dt}$, q^{-1} and q'^{-1} being, respectively, the differential and the internal and external delay operator; i.e., $\dot{z} = Dz(t)$, $z(t - h) = q^{-1}z(t)$ and $z(t - h) = q^{-1}z(t)$ for any signal $z(t)$.
- Polynomials denoted $F(\mu', \mu, s)$ and $F(\mu, s)$ are multivariable polynomials defined by $F(\mu, \mu', s) = F_0(\mu, s) + \mu' F_1(\mu, s)$ and $F(\mu, s) = F_0(s) + F_1(\mu, s) = \sum_{i=0}^{n_F} F_i(s)\mu^i$, respectively.

Note that the use of multivariable polynomials and transfer functions has the only purpose of writing compactly the equations, consisting of the polynomials and rational functions describing the model only dependent on the Laplace variable s.

2. Plant and model

2.1. Nominal plant. Consider the SISO plant

$$\frac{y(s)}{u(s)} = G(s) = G_*(s)\left[1+\rho\Delta_2(s)\right] + \rho\Delta_1(s), \tag{1}$$

where $G(s) = k_p B(\mu,\mu',s)/A(\mu,s)$; $G_*(s) = G_*(e^{(-hs)}, e^{(-h's)}, s) = G_*(\mu,\mu',s) = k_p B_*(\mu,\mu',s)/A_*(\mu,s)$ and $k_p > 0$, $\Delta_1(s) = \Delta_1(\mu,\mu',s)$ and $\Delta_2(s) = \Delta_2(\mu,\mu',s)$ are, respectively, the transfer function of the modelled part of the plant (i.e., the nominal plant transfer function, obtained when $\rho = 0$ in (1), and transfer functions of additive and multiplicative plant perturbations, respectively. For clarity of presentation and without loss of generality, both $\Delta_1(s)$ and $\Delta_2(s)$ are rated by the same positive scalar parameter ρ. The notation $\mu = e^{(-hs)}$, $\mu' = e^{(-h's)}$ is used for simplicity in the exposition while having in mind that the various transfer functions are only dependent on a unique variable s, $h \geqslant 0$ and $h' \geqslant 0$ being the single internal and external delays.

2.2. Reference model. A reference model defining the suitable behaviour for the plant is defined by the transfer function

$$G_m(s) = G_m(\mu,\mu',s) = k_m \frac{B_m(\mu,\mu',s)}{A_m(\mu,s)}. \tag{2}$$

The following assumptions are used through the next sections for the plant, unmodelled dynamics and reference model.

ASSUMPTIONS

1. (a) Both the internal and external delays, are known and the plant is strictly proper (i.e., $n = \deg_s(A(\mu,s)) > m = \deg_s(B(\mu,\mu',s))$) and stabilizable.

(b) The nominal plant $G_*(s)$ is strictly proper. It is also exponentially stable if Controllers II or IV are used.

2. The reference model is exponentially stable and realizable so that all the roots of $A_m(\mu,s) = 0$ are in $Re(s) < 0$ and $n_m = \deg_s(A_m(\mu,s)) \geqslant m_m = \deg_s(B_m(\mu,\mu',s))$. Furthermore, $n_m - m_m \geqslant n - m$ (thus, the controller is strictly proper from Assumption 1(a)).

3. Assumptions $1-2$ hold for the nominal transfer function of the plant and reference model with the particular specifications $n_m = n$, $m_m \leqslant m$ and

$A_*(s) = A_*(\mu, s) = s^n + \bar{A}_*(\mu, s)$ for some $\bar{A}_*(\mu, s)$ with $\deg_s(\bar{A}_*(\mu, s) = n - 1)$ (i.e., $A_*(s)$ is monic in s).

4. m and n and the sign of k_p (i.e., the high-frequency gain of the nominal plant) are known. Without loss of generality, it is assumed $k_p > 0$.

5. All the unstable zeros of the nominal plant (if any) are known and included within those of the reference model (2).

6. $\Delta_1(s)$ and $\Delta_2(s)$ are, respectively, strictly proper and exponentially stable, and exponentially stable transfer functions and a positive lower-bound p_0 on the stability margin $p > 0$ for which the poles of $\Delta_1(s-p)$ and $\Delta_1(s-p)$ are exponentially stable is known.

The stabilizability Assumption 1(a) is necessary for stabilization purposes. It is guaranteed by the stronger one of plant spectral controllability. The second part of Assumption 1(b) is stronger that Assumption 1(a) and it will be motivated by the special structure of Controllers II and IV. Assumptions 3, 4 and 6 are standard in the adaptive control literature for the delay-free case. In particular, note that $G_*(s)$ and $\Delta_1(s)$ being strictly proper from Assumptions 1(b) and 6 implies through (1) that the current transfer function $G(s)$ is also strictly proper. The factorizations of the stable and unstable zeros of the numerator and denominator of complex functions $F(s, e^{(-hs)})$ required by Assumption 5 follows directly according to Lemma 2.1 below. Assumption 6 is usually replaced by the more restrictive one of the nominal plant being inverse stable (Ioannou and Tsakalis, 1986; Narendra and Annaswamy, 1989). As discussed in Ioannou and Tsakalis (1986) for the delay-free case, $G(s)$ is not required to be stable inverse or of known order or relative degree. Also, $|\rho\Delta_2(j\omega)|$ may be large at high frequencies even when ρ is very small, since it is allowed to be improper. From the singular perturbation theory, if the slow ("dominant") eigenvalues are $O(1)$, then modes are of at least $O(1/\rho)$ so that ρ is not an artificial parameter and p is of $O(1/\rho)$ when $0 < \rho \ll 1$ (Ioannou and Tsakalis 1986) and in Assumption 6 can be easily found. The constraint $0 < \rho \ll 1$ is not required in the stability and robustness proofs of this section.

The following result extends a well-known one for delay-free systems (see, for instance, Narendra and Annaswamy, 1989) and establishes the fact that any plant transfer function numerator can be factorized into unique (except for a constant) complex-variable functions having their zeros within preassigned (disjoint) stability and instability subsets of C. Such a result is a consequence of the Weiestrass factorization theorem for entire functions, (De la Sen and Jugo, 1994). This property allows addressing the incorporation of the unstable plant zeros to the model zeros in both cases, namely that of known parameters and the adaptive case in order to desing controller allowing model matching, when the set of those zeros is a finite set or it can be described by a multivariable polynomial $B^-(s) = B(s, \mu, \mu')$ (see Notation). This restriction it is motivated by implementability reasons.

Lemma 2.1 (De la Sen and Jugo 1994). *Define* $C_p^+ = \{z \in C : Re(z) \leqslant -p\}$ *and* C_p^- *as being the complement of* C_p^+ *in* C *for any given real* $p' \geqslant 0$. *A unique (except for a nonzero constant) factorization* $B(s) = B(\mu, \mu', s) = B_p^+(s)B_p^-(s)$ *exists where the zeros of the complex variable functions* $B_p^+(s)$ *and* $B_p^-(s)$ *are in* C_p^+ *and* C_p^- , *respectively.*

3. Structures for the adaptive controller. Four adaptive controller structures valid for the stabilization of plants matching a reference model are now described. In the case of nominal plants of known parameters, the constant parametrizations of such controllers allow additionally the achievement of either infinite (delay-dependent) or finite (delay-independent) closed loop spectrum (see De la Sen and Josu Jugo, 1994).

General framework. Four parametrized adaptive controller structures are presented in this section being generically described as $\Sigma_c(\hat{\theta}, \hat{\Lambda}_\omega(\hat{\theta}, t))$, where the "hat"-symbol stands for estimated values, within the general framework proposed in Morse (1990) for delay-free systems, where $\hat{\theta}$ is a estimated parameter vector of dimension n_c and $\hat{\Lambda}_\omega(\hat{\theta}, t) = \{r(t), \omega(h, h', \hat{\theta}, t), \bar{\lambda}(h, h', \hat{\theta}, t)\}$ is an extended regressor parametrized also in $\hat{\theta} \in R^{n_c}$. The signal $r(t)$ is the uniformly bounded external reference which will be taken as the input to an explicit reference model, $\omega(h, h', \hat{\theta}, t)$ is the regressor of the parametrized controller given by a delayed (or undelayed in the case of finite spectra objectives) differential system, also, $\bar{\lambda}(h, h', \hat{\theta}, t) = \{\hat{c}(h, h', \hat{\theta}, t), \lambda(h, \hat{\theta}, t)\}$ with $\lambda(h, \hat{\theta}, t) = \{\hat{\lambda}(h, \hat{\theta}, t), \lambda_i(h, \hat{\theta}, t), \lambda_j'(h, \hat{\theta}, t)\}$; $i = p_1 + 1, p_1 + 2, \ldots, 2n - 1$; $j = p_2 + 1, p_2 + 2, \ldots, 2n - 1$; $p_1 \geqslant 0$; $p_2 \geqslant 0$ is a set of auxiliary weighting functions which are used to compensate for the transmission of unsuited delays through the loop. Such a strategy is adopted since, apart from the original internal delay together with their combinations with the external delay are transmitted through the feedback loop. In particular, the $\hat{c}(\cdot)$-function weights the reference signal related to the external delay and the $\hat{\lambda}_{(\cdot)}(\cdot)$-functions weight the plant input related to the transmission of the internal delay and its integer multiples while the $\hat{\lambda}(\cdot)$ and $\hat{\lambda}'_{(\cdot)}(\cdot)$-functions are used to compensate for the combined effects of the internal and external delays. In the auxiliary Λ-set, the nonnegative integers p_1 and p_2 are chosen so that all the powers of the internal delay greater than p_1 and all their combinations with the external delay being greater than p_2 are cancelled by the $\hat{\lambda}_{(\cdot)}(\cdot)$ and $\hat{\lambda}'_{(\cdot)}(\cdot)$, respectively while such a combined effect for powers of $e^{(-hs)}$ less than p_2 are cancelled by $\hat{\lambda}(h, \hat{\theta}, t)$. The reason for a separate choice of the $\hat{\lambda}(\cdot)$ and the set $\hat{\lambda}'_{(\cdot)}(\cdot)$ is that if the $\hat{\lambda}'(\cdot)$ were omitted, then the unsuitable powers of $e^{(-hs)}$ in the infinite spectra objective could not be zeroed what would introduce 'a priori' constrains in the choice of the reference model. Thus, p_1 and p_2 are chosen by the designer with $p = \max(p_1, p_2) \leqslant 2n - 1$ and are related to the suited maximum power of the internal delay in the closed-loop characteristic two-variable polynomials. Note that ω and the elements of $\bar{\Lambda}$ are parametrized by $\hat{\theta}$. The functions of the set $\bar{\Lambda}$ will be estimated, together $\hat{\theta}$ within the estimation scheme. Thus, the control law to be adopted has the generic form which includes an auxiliary signal $v(\cdot)$, as follows

$$u(t) = \hat{\theta}^T(t)\omega\left(h, h', \hat{\theta}(t), t\right) + v\left(\hat{\Lambda}_\omega(h, h', \hat{\theta}(t), t))\right). \qquad (3)$$

Four controllers within the above framework are now described. The two first structures allow the achievement of both infinite and finite closed-loop spectra in the above mentioned case of nominal plants of known parameters. The maximum power of the internal delay in the objective is chosen by the designer by the choice of p_1 and p_2 in the $\hat{\Lambda}$-set and the regressor is the state of a differential system involving the internal delays of the plant and $(n-1)$ of its integer multiples. The two last controller structures can only be used for (delay-independent) finite spectra control objectives in the same nominal situation.

Controller structure I. The particular control law of (3) is given by

$$u(t) = \hat{\theta}^T(t)\omega(t) + \left[\int_{-h'}^{0} \hat{\lambda}(t,\tau)u(t+\tau)d\tau + \sum_{i=p_1+1}^{2n-1} \cdot \int_{-ih}^{0} \hat{\lambda}_i(t,\tau)u(t+\tau)d\tau\right.$$

$$+ \sum_{i=p_2+1}^{n+m-1} \int_{-(ih+h')}^{0} \hat{\lambda}'_i(t,\tau)u(t+\tau)d\tau + \hat{c}_0(t)r(t) +$$

$$+\int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau\Bigg], \tag{4}$$

where $v(\cdot)$ in (3) is the signal in brackets, $\hat{c}_0$ is a parameter estimate, being unity when the static gain of the plant and the reference model are identical, $\hat{c}_1(\tau)$ is a reference weighting function, and

$$\hat{\theta} = \left[\hat{\bar{\theta}}^T, \hat{\theta}'_1\right]^T, \qquad \hat{\bar{\theta}} = \left[\hat{\bar{\theta}}^{(0)T}, \hat{\bar{\theta}}^{(1)T}, \ldots, \hat{\bar{\theta}}^{(n-1)T}\right], \tag{5}$$

$$\omega(t) = \left[\bar{\omega}^T(t), \bar{\omega}^T(t-h), \ldots, \bar{\omega}^T(t-(n-1)h), y(t)\right]^T, \tag{6}$$

$$\bar{\omega}(t) = \left[\bar{\omega}^{(1)T}(t), \bar{\omega}^{(2)T}(t)\right], \qquad \hat{\bar{\theta}}^{(i)}(t) = \left[\hat{\bar{\omega}}^{(i_1)T}(t), \hat{\bar{\theta}}^{(i_2)T}(t)\right], \quad (i = 0, 1, \ldots, n-1) \tag{7}$$

$$\dot{\bar{\omega}}(t) = \bar{F}\bar{\omega}(t) + \sum_{i=1}^{n-1} \bar{F}_i\bar{\omega}(t-ih) + \bar{Q}\bar{u}(t), \qquad \bar{\omega}(t) = 0, \quad t \in [(n-1)h, 0], \tag{8}$$

$$\bar{F} = \mathrm{Diag}(F;F), \quad \bar{F}_i = \mathrm{Diag}(F_i;F_i) \quad (i = 1, 2, \ldots, n-1), \tag{9}$$

$$\bar{u}(t) = [u(t), y(t)]^T, \quad \bar{Q} = \left[\tilde{Q}|\tilde{Q}\right]^T; \quad \tilde{Q} = \mathrm{Diag}(q_1, q_2), \tag{10}$$

$$F = \begin{pmatrix} O & I_{n-2} \\ -f_{n-2,0}, -f_{n-1,0}, \ldots, -f_{00} \end{pmatrix}, \qquad F_i = \begin{pmatrix} O_{(n-2)x(n-1)} \\ -f_{n-2,i}, -f_{n-1,i}, \ldots, -f_{0i} \end{pmatrix}, \quad (i = 1, 2, \ldots, n-1), \tag{11}$$

where q, $\bar{\omega}^{(l)}(t)$ and $\hat{\bar{\theta}}^{(il)}$ $(i = 0, 1, \ldots, n-1;\ l = 1, 2)$ are $(n-1)$-vectors and F and $\bar{F}$ are $(n-1) \times (n-1)$-matrices. Thus, the number of design parameters in $\hat{\theta}$ is $2n(n-1)+1$ and the numbers of $\hat{\lambda}_{(\cdot)}(\cdot)$ and $\hat{\lambda}'_{(\cdot)}(\cdot)$-functions are, respectively, $(2n - p_1 - 1)$ and $(n + m - p_2 - 1)$. Note that the first right-hand-side of (4) is the output of a general system involving internal delays only driven by $\bar{u}(t) = [u(t), y(t)]^T$ and whose dynamics is given by (5)–(11).

Controller structure II. The control law is now

$$u(t) = \hat{\theta}^T(t)\omega(t) + \left[\int_{h'}^{t} \hat{\lambda}(t,\tau)u(t-\tau)d\tau + \sum_{i=p_1+1}^{2n-1} \int_{ih}^{t} \hat{\lambda}_i(t,\tau)u(t-\tau)d\tau \right.$$
$$+ \sum_{i=p_2+1}^{n+m-1} \int_{ih+h'}^{t} \hat{\lambda}'_i(t,\tau)u(t-\tau)d\tau$$
$$\left. + \hat{c}_0(t)r(t) + \int_{h'}^{t} \hat{c}_1(t,\tau)r(t-\tau)d\tau \right], \qquad (12)$$

subject to (5) – (10). Note that Controller II, compared to Controller I contains convolution integral-type terms constructed with the elements of the $\hat{\Lambda}$-set.

The two next structures are particularizations of Controllers I and II, respectively, and they involve delay-free controller dynamics. It will be then seen that closed-loop, internal delay-dependent dynamics is unachievable by the use of those controllers.

Controller structure III. The control law is

$$u(t) = \hat{\theta}^T(t)\omega(t) + \left[\sum_{i=1}^{n} \int_{-ih}^{0} \hat{\lambda}_i(t,\tau)u(t+\tau)d\tau \right.$$
$$+ \sum_{i=0}^{m} \int_{-(ih+h')}^{0} \hat{\lambda}'_i(t,\tau)u(t+\tau)d\tau$$
$$\left. + \hat{c}_0(t)r(t) + \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau \right], \qquad (13)$$

where

$$\hat{\theta} = \left[\hat{\bar{\theta}}^{(1)T}, \hat{\bar{\theta}}^{(2)T}, \hat{\theta}'_1 \right]^T, \qquad \omega = \left[\bar{\omega}^{(1)T}(t), \bar{\omega}^{(2)T}(t), y(t) \right]^T, \qquad (14)$$

$$\dot{\bar{\omega}} = \bar{F}\bar{\omega}(t) + \bar{Q}\bar{u}(t); \qquad \bar{\omega} = \left[\bar{\omega}^{(1)T}(t), \bar{\omega}^{(2)T}(t) \right]^T, \qquad (15)$$

with $\bar{F}$, $\bar{Q}$ and $\bar{u}$ being defined as for Controllers I – II and $\hat{\theta}'_l$ ($l = 1, 2$) being zero or nonzero scalars (see (9) to (11). In this case, the dimension of n_c is

$2n-1$, n_c being the dimension of $\hat{\theta}$ and $\omega(t)$ and there are n $\hat{\lambda}_{(\cdot)}$-function and m $\hat{\lambda}'_{(\cdot)}$-functions.

Controller structure IV. The control law is

$$u(t) = \hat{\theta}^T(t)\omega(t) + \left[\sum_{i=1}^{n} \int_{ih}^{t} \hat{\lambda}_i(t,\tau)u(t-\tau)d\tau \right.$$
$$+ \sum_{i=0}^{m} \int_{ih+h'}^{t} \hat{\lambda}'_i(t,\tau)u(t-\tau)d\tau$$
$$\left. + \hat{c}_0(t)r(t) + \int_{h'}^{t} \hat{c}_1(t,\tau)r(t-\tau)d\tau \right], \qquad (16)$$

with $\hat{\theta}$ and $\omega(t)$ being defined as in (14) – (15) and F and q being defined as in (9) – (11).

REMARK 1. Note from (4), (12), (13) and (16) that Controllers I and III compared, respectively, to Controllers II and IV include finite-time against infinite-time integrals in the generic auxiliary signals refered to in (3). Therefore, the use of Controllers II and IV is only feasible if the plant is stable (Assumption 5).

ASSUMPTIONS

7. The parametrized part of the controller is spectrally controllable and exponentially stable; i.e., F, F_i $(i = 1, 2, \dots, n-1)$ and q are chosen verifying

$$\text{rank}\left[sI - F - \sum_{i=1}^{n-1} e^{-ihs} F_i, q \right] = n-1, \quad \text{all} \quad s \in C, \qquad (17)$$

and

$$D(s) = \text{Det}\left(sI - F - \sum_{i=1}^{n-1} e^{(-ihs)} F_i \right)$$
$$= \sum_{i=0}^{n-1} D_i(s)\mu^i = \sum_{i=0}^{n-1}\sum_{k=0}^{n-1} d_{ki} s^k \mu^i = 0 \qquad (18)$$

has all roots in $Re(s) < 0$, with $F_i = 0$ $(i = 1, 2, \dots, n-1)$ for Controllers III and IV and, furthermore, all $D_{(\cdot)}(\cdot)$-polynomials are monic; i.e., $D_i(s) = s^{n-1} + \bar{D}_i(s)$ with $\deg(\bar{D}_i(s)) = n-2$.

8. The existence of the true parameter vector θ^T, the $\lambda(\cdot)$ -fuctions and the remaining plant parameters and $\lambda(\cdot)$-fuctions is assumed. Two real constants θ'_m and θ'_M, $\theta'_M > \theta'_m \geqslant 0$ are known such that $\theta'_m < \| \left(\theta^T, c_o, k_p k_m^{-1}\right)^T \|$ and an upper-bound $\hat{\eta}'(t)$ of the unmodelled dynamics contribution $|\rho(G_* \Delta_2 + \Delta_1)|$ is known for all $t \geqslant 0$.

Assumption 8 is used in the stability proof and it is coherent with boundedness of the unmodelled dynamics contribution (see also Remark 2 in the next section). Note that $\hat{\eta}'(t)$ could be calculated, for instance, if bounded intervals of variations of the parameters in $\Delta_i(s)$ $(i = 1, 2)$ are known.

4. Adaptive laws. Stability and robustness properties of the adaptive controllers. In this section, an adaptive law based on an extension of a classical σ- modification scheme, (Ioannou and Tsakalis, 1986), in the presence of known delays and unmodelled plant dynamics is given for the case of unknown plant parameters. The tracking error can be calculated from those expressions. In particular, note that

$$e(t) = y(t) - y_m(t) = \psi_0 G_m \Bigg[\tilde{\theta}^T(t)\omega(t) + \tilde{\varphi}(t) + \tilde{c}_0(t) r(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau) r(t+\tau) d\tau\Bigg] + \rho\eta(t), \tag{19}$$

with $\eta(t) = \Delta(D, q^{-1}, q'^{-1})$ and with $\Delta(\cdot)$ defined in Eq. (53), $\psi_0 = c_0^{-1} = k_p k_m^{-1}$, and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$ being the parametrical error which exists since the true parameters and functions exist from Assumption 8 and $\tilde{\varphi}(t) = \hat{\varphi}(t) - \varphi(t)$ being the error function of the updated function

$$\hat{\varphi}(t) = \int_{-h'}^{0} \hat{\lambda}(t,\tau) u(t+\tau) d\tau + \sum_{i=p_1+1}^{2n-1} \int_{-ih}^{0} \hat{\lambda}_i(t,\tau) u(t+\tau) d\tau + \sum_{i=p_2+1}^{n+m-1} \int_{-(ih+h')}^{0} \hat{\lambda}'_i(t,\tau) u(t+\tau) d\tau, \tag{20}$$

related to true unknown function $\varphi(t)$. In (19) – (20) and subsequent equations, the time-domain operators are denoted in the same way as their filter counterparts for the shake of simplicity by omitting the time and delay arguments. The adaptation error is an augmented error given by

$$\epsilon(t) = e(t) + \hat{\psi}_o(t)\zeta(t) = y(t) - y_m(t) + \hat{\psi}_o(t)\zeta(t), \tag{21}$$

for the general case of unknown k_p where $\hat{\psi}_0$ is an extra parameter estimate for $\psi_0 = c_0^{-1}$ whose parametrical error is defined by $\tilde{\psi}_o = c_o\hat{\psi}_0(t) - 1$ and $\zeta(t)$ being the corresponding component in the regressor vector given by

$$\zeta(t) = \hat{\bar{\theta}}^T(t)\bar{\omega}_f(t) + \hat{\varphi}'_f(t) - G_m\left[\hat{\bar{\theta}}^T(t)\bar{\omega}(t) + \hat{\varphi}'(t)\right], \tag{22}$$

where from (5) to (7),

$$\begin{aligned}\hat{\bar{\theta}}(t) &= \left[\hat{\theta}^T(t), \hat{c}_o(t)\right]^T \\ &= \left[\hat{\bar{\theta}}^{(01)T}(t), \hat{\bar{\theta}}^{(02)T}(t), \dots, \hat{\bar{\theta}}^{(n-1,1)T}(t), \hat{\bar{\theta}}^{(n-1,2)T}(t), \hat{c}_0(t)\right]^T,\end{aligned} \tag{23}$$

$$\begin{aligned}\bar{\omega}_f(t) &= \left[\omega_f^T(t), y_m(t)\right]^T = G_m\left[\bar{\omega}_f^T(t), r(t)\right]^T \\ &= G_m\left[\bar{\omega}^{(1)T}(t), \bar{\omega}^{(2)T}(t), \bar{\omega}^{(1)T}(t-h), \bar{\omega}^{(2)T}(t-h), \right. \\ &\quad \left. \dots, \bar{\omega}^{(1)T}(t-(n-1)h), \bar{\omega}^{(2)T}(t-(n-1)h), r(t)\right]^T,\end{aligned} \tag{24}$$

$$\hat{\varphi}'(t) = \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau + \hat{\varphi}(t), \tag{25}$$

$$\hat{\varphi}'_f(t) = \int_{-h'}^{0} \hat{c}_1(t,\tau)y_m(t+\tau)d\tau + \hat{\varphi}_f(t), \tag{26}$$

$$\begin{aligned}\hat{\varphi}_f(t) &= \int_{-h'}^{0} \hat{\lambda}(t,\tau)u_f(t+\tau)d\tau + \sum_{i=p_1+1}^{2n-1} \int_{-ih}^{0} \hat{\lambda}_i(t,\tau)u_f(t+\tau)d\tau \\ &\quad + \sum_{i=p_2+1}^{n+m-1} \int_{-ih+h'}^{0} \hat{\lambda}'_i(t,\tau)u_f(t+\tau)d\tau,\end{aligned} \tag{27}$$

with $u_f(t) = G_m u(t)$. The substitution of (22) and (3) into (21) leads to

$$\epsilon(t) = \bar{\epsilon}(t) + \rho\eta(t) = \psi_0\left[\tilde{\theta}'^T(t)\omega'_f(t) + \tilde{\varphi}'_f(t)\right] + \rho\eta(t), \tag{28}$$

where $\bar{\epsilon}(t)$ is the adaptation error used in the absence of unmodelled dynamics and $\tilde{\theta}'(t) = \left[\tilde{\bar{\bar{\theta}}}^T(t), \tilde{\psi}_0(t)\right]^T$, $\omega'_f(t) = G_m\omega'(t) = G_m\left[\bar{\bar{\omega}}^T(t), y_m(t), \zeta(t)\right]^T$. The parameter adaptive law is

$$\dot{\hat{\theta}}'(t) = -\frac{\Gamma\epsilon(t)\omega'_f(t)}{m^2(t)} - \Gamma\sigma(t)\hat{\theta}'(t), \tag{29}$$

$$\dot{\hat{c}}_1(t,\tau) = -\frac{\gamma\epsilon(t)y_m(t+\tau)}{m^2(t)} - \gamma\sigma_c(t,\tau)\hat{c}_1(t,\tau); \quad \tau \in [-h', 0], \tag{30}$$

$$\dot{\hat{\lambda}}(t,\tau) = -\frac{\gamma\epsilon(t)u_f(t+\tau)}{m^2(t)} - \gamma\sigma_\lambda(t,\tau)\hat{\lambda}(t,\tau); \quad \tau \in [-h', 0], \tag{31}$$

$$\dot{\hat{\lambda}}_i(t,\tau) = -\frac{\gamma\epsilon(t)u_f(t+\tau)}{m^2(t)} - \gamma\sigma_{\lambda_i}(t,\tau)\hat{\lambda}_i(t,\tau); \quad \tau \in [-ih, 0], \quad i = p_1+1, p_1+2, \ldots, 2n-1, \tag{32}$$

$$\dot{\hat{\lambda}}'_i(t,\tau) = -\frac{\gamma\epsilon(t)u_f(t+\tau)}{m^2(t)} - \gamma\sigma_{\lambda'_i}(t,\tau)\hat{\lambda}'_i(t,\tau), \quad \tau \in [-(ih+h'), 0]; \quad i = p_2+1, p_2+2, \ldots, n+m-1, \tag{33}$$

$$\dot{m}(t) = -\delta_0 m(t) + \delta_1(|u(t)| + |u(t-h')| + |y(t)| + 1); \quad m(0) > \delta_1/\delta_0, \tag{34}$$

$$\sigma_\lambda(t,\tau) = \tag{35}$$

$$= \begin{cases} 0 & \text{if } |\hat{\lambda}(t,\tau)| < M_\lambda(t,\tau) \text{ for all } \tau \in [-h', 0], \\ \sigma_{\lambda 0}\left(\dfrac{|\hat{\lambda}(t,\tau)|}{M_\lambda(t,\tau)} - 1\right) & \text{if } M_\lambda(t,\tau) \leqslant |\hat{\lambda}(t,\tau)| \leqslant 2M_\lambda(t,\tau), \\ \sigma_{\lambda 0} & \text{if } |\hat{\lambda}(t,\tau)| > 2M_\lambda(t,\tau) \text{ for all } \tau \in [-h', 0], \end{cases}$$

$$\sigma(t) = \tag{36}$$

$$= \begin{cases} \hat{\eta}^2 \dfrac{3n+m-1-p_1-p_2}{2} \dfrac{\left(\|\hat{\theta}'(t)\| - \theta'_M\right)}{\|\hat{\theta}'(t)\|} \left[\dfrac{\hat{\eta}'^2(t)}{m^2(t)} \right. \\ \left. \quad + \delta_1 m(t) \displaystyle\int_0^t (|u(\tau)| + |u(\tau - h')| + |y(\tau)| + 1) d\tau \right] \text{ if } \|\hat{\theta}'(t)\| > \theta'_M, \\ \hat{\eta}^2 \dfrac{3n+m-1-p_1-p_2}{2} \dfrac{\left(\theta'_m - \|\hat{\theta}'(t)\|\right)}{\|\hat{\theta}'(t)\|} \left[\dfrac{\hat{\eta}'^2(t)}{m^2(t)} + \delta_1 m(t) \right. \\ \left. \quad \times \displaystyle\int_0^t (|u(\tau)| + |u(\tau - h')| + |y(\tau)| + 1) d\tau \right] \text{ if } \epsilon \leqslant \|\hat{\theta}'(t)\| < \theta'_m, \\ 0 \quad \text{if } \|\hat{\theta}'(t)\| < \epsilon \text{ or } \theta'_m \leqslant \hat{\theta}'(t) \leqslant \theta'_M \text{ for some arbitrary} \\ \quad \text{small prefixed positive constant } \epsilon, \end{cases}$$

and similar expressions to $\sigma_\lambda(t,\tau)$ for $\sigma_c(t,\tau)$, $\sigma_{\lambda_{(\cdot)}}(t,\tau)$ and $\sigma_{\lambda'_{(\cdot)}}(t,\tau)$ for $\tau \in [-ih, 0]$ and $\tau \in [-(ih+h'), 0]$, respectively, for a set of functions $M_{\lambda_{(\cdot)}}(t,\tau)$ and $M_{\lambda'_{(\cdot)}}(t,\tau)$, and $\Gamma = \Gamma^T > 0$ is a matrix of order being compatible with that of $\hat{\theta}'(t)$,γ, δ_1 and σ_0 are positive parameters; $M_\lambda(t,\tau) \geqslant |\lambda(t)| \max(\sqrt{c_0}, \sqrt{\psi_0})$, $M_{\lambda_i(t,\tau)} \geqslant |\lambda_i(t)| \max(\sqrt{c_0}, \sqrt{\psi_0})$, $\tau \in [-ih, 0]$, all $i = p_1+1, p_1+2, \ldots, 2n-1$; $M_{\lambda'_i(t,\tau)} \geqslant |\lambda'_i(t)| \max(\sqrt{c_0}, \sqrt{\psi_0})$, $\tau \in [-(ih+h'), 0]$, all $i = p_2+1, p_2+2, \ldots, n+m-1$ for each $t \geqslant 0$. $\hat{\eta}$ is a fixed or time-updated known upper-bound of p and $\hat{\eta}'(t) \geqslant \eta(t)$ is known for all $t \geqslant 0$. The parameter δ_0 is designed according to the constraint $0 < \delta_0 \leqslant \min(p_0, q_0) - \delta_2$ for some $0 < \delta_2 \leqslant \min(p_0, q_0)$ with p_0 satisfying Assumption 6 and $q_0 > 0$is such that the poles of $G_m(s - q_0)$ and the eigenvalues of $F + q_0 I$ are exponentially stable as suggested in Ioannou and Tsakalis (1986) for the delay-free case. The superscripts 'dot' in (29) to (34) stand for time-derivatives with respect to the variable t.

REMARKS

2. The stability and robustness results of Theorem 1 below still hold for the choices $\sigma_\lambda(t,\tau) = \sigma_{\lambda_i}(t,\tau) = \sigma_{\lambda'_{(\cdot)}}(t,\tau) = 0$ for all t, τ provided that $\sigma(t)$ is updated according to (36) since the Lyapunov function candidate proposed for the proofs in Appendix C is proved to be a Lyapunov function for all $t \geqslant 0$ such that $V(t) \geqslant V_0$, V_0 being some positive real constant. However, the proposed updating rule, (35) and their counterparts for the remaining estimates of the functions of the auxiliary set $\bar{\Lambda}$ are more coherent with the fact of applying a σ-modification for all the estimates. Since the σ-modification for such functions can be zeroed for all $t \geqslant 0$, the necessity for a knowledge of the functions $M_\lambda(\cdot)$, $M_{\lambda_{(\cdot)}}(\cdot)$ and $M_{\lambda'_{(\cdot)}}(\cdot)$ has not been introduced in Assumption 8.

3. If the 'prime a priori' knowledge on the controller parameter vector $\hat{\theta}'(t)$ of Assumption 8 is not available but the slighter 'a priori' knowledge of the constant $M_0 \geqslant \|\theta'\| \max(\sqrt{c_0}, \sqrt{\psi_0})$ is available, then Eq. (36) can be substituted by

$$\sigma(t) = \begin{cases} 0 & \text{if } \ \|\hat{\theta}'(t)\| < M_0, \\ \sigma_0 \left(\dfrac{\|\hat{\theta}'(t)\|}{M_0} - 1 \right) & \text{if } \ M_0 \leqslant \|\hat{\theta}'(t)\| \leqslant 2M_0, \\ \sigma_0 & \text{if } \ \|\hat{\theta}'(t)\| > 2M_0, \end{cases} \tag{37}$$

as proposed in Ioannou and Tsakalis (1986). The stability and robustness results still hold in that case but the proofs are more cumbersome since they involve the use of a more general Lyapunov function candidate built with parametrical quadratic errors and a quadratic term of the tracking error. In this case, M_0 can be designed as large as possible by using an 'a priori' known upper-bound of $\|\theta'\|$, but less than $O(1/\rho)$ in order that the fast eigenvalues associated with the parasitic stable modes be of at least $O(1/\rho)$.

4. Note that $u_f(t+\tau)$ is an identical regressor for $\hat{\lambda}(\cdot)$, $\hat{\lambda}_{(\cdot)}(\cdot)$ and $\hat{\lambda}'_{(\cdot)}(\cdot)$ but, however, the identical calculation for the local variable t are performed on different intervals. This makes the various right-hand-side intervals of (4) and (29) – (32) to be different.

Theorem 1. *Under Assumptions* 1–8, *the use of the adaptive Controller structure* I, *Control law* (4) *and Estimation scheme* (29) – (35) *leads to the following properties:*

(i) $\|\hat{\theta}'(t)\|$, *the elements of* $\bar{\Lambda}(h, h', \theta, t)$ *and its associated parametrical errors are uniformly bounded for all* $t \geqslant 0$. *The normalizing signal* $m(t)$ *is also uniformly bounded for all* $t \geqslant 0$.

(ii) *The normalized signals* $|\tilde{\theta}'^T(t)\omega(t)|/m(t)$, $|\tilde{\theta}'^T(t)\omega'_f(t)|/m(t)$, $|\tilde{\eta}'(t)|/m(t)$, $|\tilde{\eta}'_f(t)|/m(t)$, $|u(t)|/m(t)$, $|y(t)|/m(t)$, $|\zeta(t)|/m(t)$, $|\bar{\epsilon}(t)|/m(t)$ *and* $|\epsilon(t)|/m(t)$ *are uniformly bounded for all* $t \geqslant 0$. *Their unnormalized* *counterparts* $|\tilde{\theta}'^T(t)\omega(t)|$, $|\tilde{\theta}'^T(t)\omega'_f(t)|$, *etc., including the input, output, tracking error, adaptation error and* $|\zeta(t)|$ *are uniformly bounded for all* $t \geqslant 0$.

(iii) *For any positive real constant* ρ^*, *these exists residual sets for the adaptation and the tracking errors defined by* $D_\epsilon = \{\lim_{T\to\infty} \sup \frac{1}{T} \int_{t_0}^{t_0+T} |\epsilon(\tau)|^2 d\tau \leqslant \rho\gamma_{1\epsilon} + \gamma_{2\epsilon}, \forall t_0 \geqslant 0, T > 0\}$, *and* $D_e = \{\lim_{T\to\infty} \sup \frac{1}{T} \int_{t_0}^{t_0+T} |e(\tau)|^2 d\tau \leqslant \rho\gamma_{1e} + \gamma_{2e}, \forall t_0 \geqslant 0, T > 0\}$ *with* $\gamma_{1(\cdot)}$ *and* $\gamma_{2(\cdot)}$ *being positive real constants, for all* $\rho \in [0, \rho^*]$.

(iv) *In addition, if* $\rho = 0$ *(namely, in the perfectly modelled situation),* $e(t) \to 0$ *and* $\epsilon(t) = \bar{\epsilon}(t) \to 0$ *as* $t \to \infty$.

The proof is given the Appendix C using partial results of Appendixes A–B. If in the Assumption 1 it is considered a controllable plant, it could be included the model matching like control objective.

Note that, since the Controller structure III is a particularization of the Controller structure I, Theorem 1 also holds for the parametrizations within the Structure III.

5. Further comments and simulated examples. The main purpose of this section is to verify the theory presented in the previous sections by numerical simulations. In the following examples, the performance of the proposed controllers is checked even for unstable plants by using the Controller structure II, in Examples 1 and 2, and the Controller structure I, in Example 3. Example 4 presents a comparative example of the four controllers acting on the same plant. Note that although it is not recommended in the general case the use of Controllers II or IV for unstable plants (see Assumption 1.b), the associate performance is seen to be acceptable in Examples 1 and 2.

The presented scheme is also useful to treat problems with output measurement or computational errors transmitted to the output when implementing the algorithm. For instance, note from (4) – (5), that there exists an output-feedback term in the controller which would act on the output measurement (or its computational error), if any, which could be designed by specifying a prefixed factor in the controller modes to cancel the dominant frequencies of such a noise while solving the diophantine equation for the case of nominal plants of known parameters. In the adaptive case, the same idea could be extended to prefixe a known polynomial factor prior to the parametrical estimation. Similar reasoning can be made for the remaining controllers. Note also that several computational delays appear when the implementing the proposed scheme, namely:

(a) Delays arising from the relative orders of model and plant. The condition on relative orders in Assumption 2 guarantees that the controllers are implementable.

(b) Delays arising from the algorithm implementation. The fourth-order Runge-Kutta method has been used in the numerical calculations. The associated computational delays could be removed in real-time applications since the output is a measurable signal and all the time-integrals in the parameter estimation and controller implementation could be obtained through instrumentation.

If the delays were unknown, they could be estimated by discretization with respect to a small sampling period T by computing from the measured input/state-output delays the number of samples (i.e., an integer d_T such that h (or h') $\simeq d_T T$) required for the corresponding signal to act on the plant output. In the discretization approach to simulate the subsequent examples, the sampling period used was $10^{-3}\min(h, h')$, in order to make its influence irrelevant to the obtained results. If the delays are point delays but they follow an exponential distribution, then the problem can be reduced to one involving point delays only (see De la Sen 1992).

5.1. Example 1. In this example, the perfomance of Controller Structure I is evaluated for the transfer function $G(s) = 1/(s^2+2s-1-\mu)$ with $h = 0.05$, $h' = 0$. The model reference transfer function is $G_m(s) = 1/(s^2+5s+6)$. The proposed Controller structure I by choosing $F = -5$, $q = 1$ and $\hat{\theta}(0) = (-1, 20, -1, -8)$ is applied with $d_0 = 0.001$, $d_1 = 0.002$, $m(0) = 5$, $\Gamma = 125852.5$. There is a unique $\lambda(\cdot)$-function being the remaining $\lambda(\cdot)$- functions in Eqs. (4) zero. In this case, the σ-modification is not used because there is no unmodelled dynamics ($\sigma_0 = 0$). Fig. 1 displays the results of this numerical simulation for a input

$$r(t) = \begin{cases} 5 & \text{if} \quad (t \bmod 10) < 5, \\ 0 & \text{if} \quad (t \bmod 10) \geqslant 5, \end{cases} \tag{38}$$

Now, an external point delay is considered. The plant and the reference model are $G(s) = (2-\mu')/(s^2+2s-1-\mu)$ and $G_m(s) = (2-\mu')/(s^2+5s+6)$ with $h = 0.5$ and $h' = 0.5$ and the other parameters being the same than in

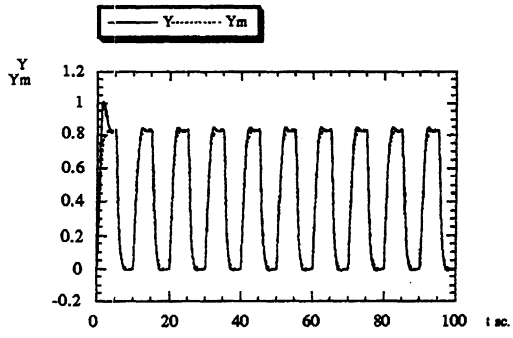

Fig. 1.1

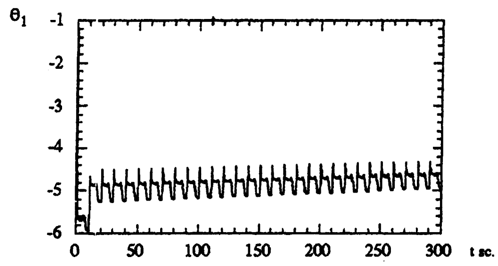

Fig. 1.2

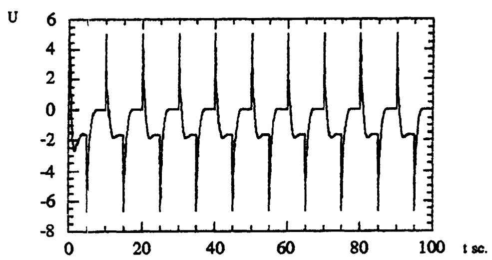

Fig. 1.3

Fig. 1. Example 1. (1) Plant output and model output, (2) Evolution of the $\hat{\theta}_1(t)$ parameter estimate, (3) Control signal.

the previous case. Fig. 2 displays the results of the simulation for a sinusoidal input $r(t) = 10\sin(0.1t)$.

As an evidence of robust perfomance, numerical simulations of the former plant with a plant $G(s) = G_0(s)(1 + \Delta(s))$ are presented with nominal value $G_0(s) = 1/(s^2 + 2s - 1 - \mu)$ and unmodelled dynamics $\Delta(s)$, where $h = 0.1$, $h' = 0$ and the initial estimated parameter vector $\hat{\theta}(0) = (-0.15, -7.5, -4.5, -71)$ and $d_0 = 0.001$, $d_1 = 0.002$, $m(0) = 5$, $\Gamma = 5.5$, $\sigma_0 = 1$, $M_0 = 100$. Fig. 3 displays the results of the robustness performance for a unity step input.

5.2. Example 2. The Mach control for a linearized Wind Tunnel Model has been considered with Controller structure II, (Manitius 1984). In steady-state operating conditions, the dynamic response of the Mach number perturbations δM to small perturbations in the guide vane angle actuator $\delta\theta_A$ is described by the next differential system:

$$\begin{aligned} \tau\delta\dot{M}(t) + \delta M(t) &= k\delta\theta(t-h), \\ \delta\ddot{\theta}(t) + 2\zeta\omega\delta\dot{\theta}(t) + \omega^2\delta\theta(t) &= \omega^2\delta\theta_A(t), \end{aligned} \tag{39}$$

where $\delta\theta(t)$ is the guide vane angle, and τ, k, h, ζ, ω are parameters defining the operating point. These are considered constants with small perturbations. In the state variable form, a term depending on the Mach number perturbation has been added. The underlying reason is the achievement of a closed-loop transfer function for the plant with infinite or finite spectrum to the designer's choice. One of the more important features of the presented controller is its capacity of controlling that class of systems. The system in state variable form is written as

$$\begin{aligned} \dot{x}_1 &= -ax_1 + kax_2(t-h) \\ \dot{x}_2 &= x_3 \ (+\ x_1) \rightarrow \text{added term} \\ \dot{x}_3 &= -\omega^2 x_2 - 2\zeta\omega x_3 + \omega^2 u, \end{aligned} \tag{40}$$

where $a = 1/\tau$, $x_1 = \delta M$, $x_2 = \delta\theta$, $x_3 = \delta\dot{\theta}$, $u = \delta\theta_A$. With $a = 1$, $k = 10$, $\zeta = 3$, $\omega = 4$rad/s and $h = 0.2$, $h' = 0$. The plant transfer and reference model functions are $G(s) = 40/(s^3 + 13s^2 + 16s + 4 - 10s\mu - 120\mu)$ and $G_m(s) = 40/(s^3 + 2s^2 + 12s + 10)$, respectively. Choose $F = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix}$ and $q_1 = 0$, $q_2 = 1$. The initial estimated parameter vector

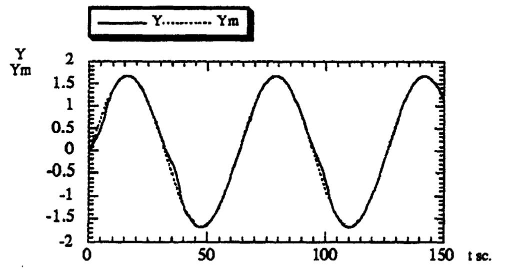

Fig. 2.1

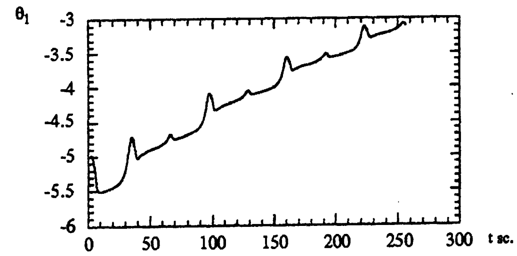

Fig. 2.2

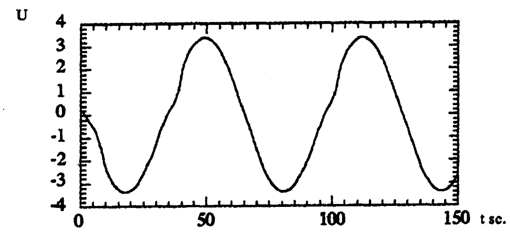

Fig. 2.3

Fig. 2. Example 1. $h' = 0.5$. (1) Planta output and model output, (2) Evolution of the $\hat{\theta}_1(t)$ parameter estimate, (3) Control signal.

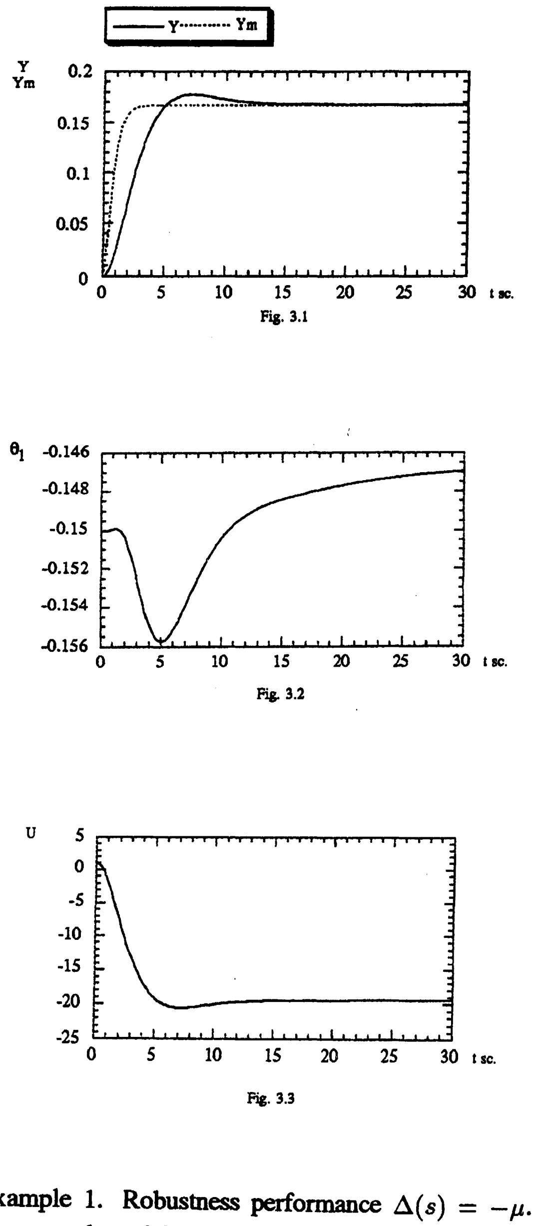

Fig. 3. Example 1. Robustness performance $\Delta(s) = -\mu$. (1) Planta output and model output, (2) Evolution of the $\hat{\theta}_1(t)$ parameter estimate, (3) Control signal.

is $\hat{\theta}(0) = (-73, 7.7, -13.8, -5003, -3000, 10000, -301, -1000, -151, 867.5)$ and $d_0 = 0.0001$, $d_1 = 0.0002$, $m(0) = 5$, $\Gamma = 45100.5$, $\sigma_0 = 0$. The results are shown in Fig. 4 where the initial conditions are zero for the plant and for the reference model. The reference input is a train of pulses

$$r(t) = \begin{cases} 3 & \text{if} \quad (t \bmod 30) < 15, \\ -3 & \text{if} \quad (t \bmod 30) \geqslant 15. \end{cases} \tag{41}$$

5.3. Example 3. Now, the robust performance under Controller Structure I is evaluated by choosing the same plant with $h = 0.2$, $h' = 0$ and reference model as in the previous example. The plant transfer function is $G(s) = G_0(s)(1 + \Delta(s))$ with $G(s) = 40/(s^3 + 13s^2 + 16s + 4 - 10s\mu - 120\mu)$ and $\Delta(s) = \begin{cases} -0.18s & \text{for } t \leqslant 30 \\ 0 & \text{for } t \geqslant 30 \end{cases}$.
Fig. 5 shows the results for this simulation. The initial estimated parameter vector is $\hat{\theta}(0) = (0.5, 1, 3, 144, -0.2, 120, -2.5, -0.5, -1.5, -130)$, and $d_0 = 0.0001$, $d_1 = 0.0002$, $m(0) = 5$,$\Gamma = 993.5$, $\gamma = 30.2$, $M_0 = 215$, $M_{\lambda_0} = 0.4$, $\sigma_0 = 0.2$, $\sigma_{\lambda_0} = 0.2$.

The second evaluation of robustness, summarized in the Fig. 6, is similar to the former one but in this case $\Delta(s) = \begin{cases} 0.15s & \text{for } t > 25 \\ 0 & \text{for } t \leqslant 25 \end{cases}$, $\hat{\theta}(0) = (0.5, 1, 3, 144, -0.2, 120, -2.5, -0.5, -1.5, -130)$ and $d_0 = 0.0001$, $d_1 = 0.0002$, $m(0) = 5\Gamma = 11559.5$, $\gamma = 935.2$, $M_0 = 215$, $M_{\lambda_0} = 0.4$, $\sigma_0 = 0.2$, $\sigma_{\lambda_0} = 0.2$.

5.4. Example 4. All the controllers have been checked on the perfectly modelled, but unknown, plant $G(s) = \dfrac{13}{s^3 + 2s^2 + 3s + 3 + s\mu - \mu}$ which can describe the problem of the transient behaviour of a continuous-current armature-controlled generator motor pair having armature internal resistor, the armature voltage as input and the output being a linear combination of the load position and speed delayed through a delay device such as a D flip-flop. If $h = 0$, then $G(s)$ would have two complex-conjugate poles and one real pole with the output being the load speed. The plant output versus the reference model output for the model $G_m(s) = \dfrac{9}{s^3 + 2s^2 + 4s + 2}$ are shown in Fig. 7 for each controller structure. The used controller data and parameters are $h = 0.1$, $h' = 0$, $F = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix}$ and $q_1 = 0$, $q_2 = 1$, $\hat{\theta}(0) = (-6, -2, 5, 2, 1, -7, 0.3, 3.5, -1.5, -0.5)$ and $d_0 = 0.02$, $d_1 = 0.059$,

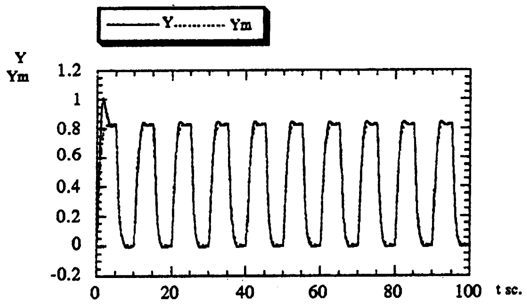

Fig. 4.1

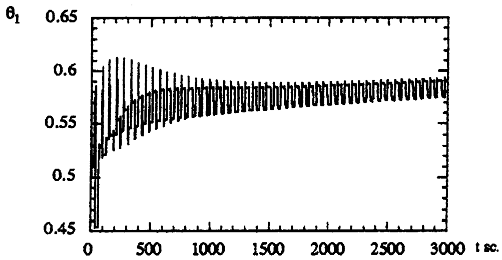

Fig. 4.2

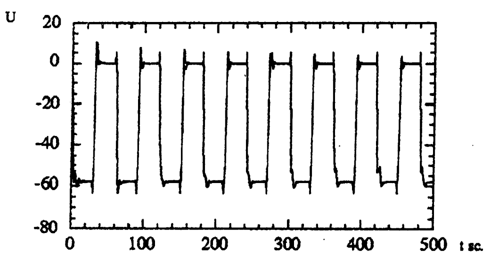

Fig. 4.3

Fig. 4. Example 2. (1) Planta output and model output, (2) Evolution of the $\hat{\theta}_1(t)$ parameter estimate, (3) Control signal.

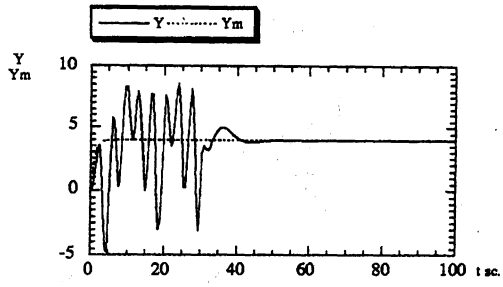

Fig. 5.1

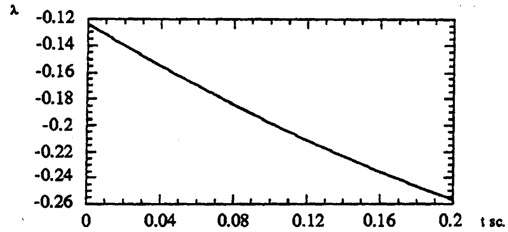

Fig 5.2

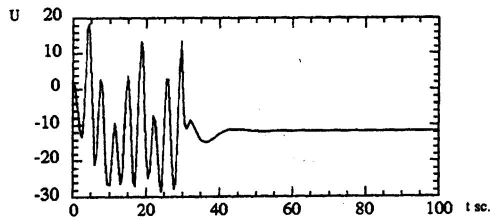

Fig. 5.3

Fig. 5. Example 3. Robustness performance $\Delta(s) = \begin{cases} -0.18s & \text{for } t \leqslant 30 \\ 0 & \text{for } t > 30 \end{cases}$. (1) Planta output and model output, (2) $\hat{\lambda}(\tau, t)$-function in $t = 200$sc. for $\tau \in (0, h)$ with $h = 0.2$, (3) Control signal.

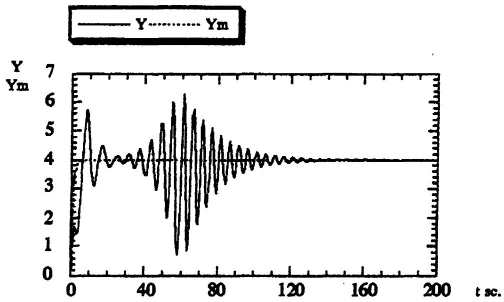

Fig. 6.1

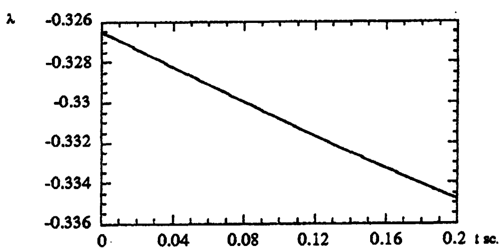

Fig. 6.2

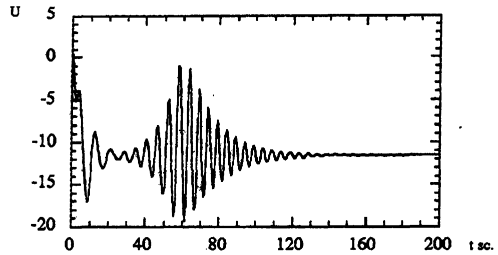

Fig. 6.3

Fig. 6. Example 3. Robustness performance $\Delta(s) = \begin{cases} 0, & \text{for } t \leqslant 25 \\ 0.15s, & \text{for } t > 25 \end{cases}$.
(1) Planta output and model output, (2) $\hat{\lambda}(\tau, t)$-function in $t = 200sc$. for $\tau \in (0, h)$ with $h = 0.2$, (3) Control signal.

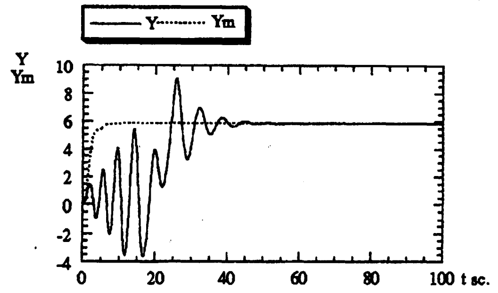

Fig. 7.1

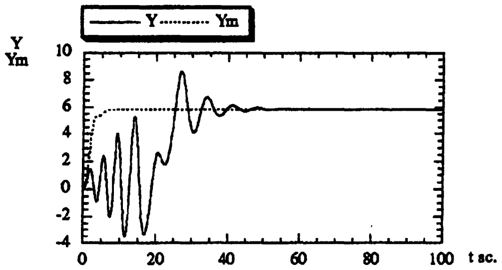

Fig. 7.2

Fig. 7. Example 4. Comparative example of the four control structures with $h = 0.1$, $h' = 0$. (1) Control structure I, (2) Control structure II (to be continued).

$m(0) = 231$, $\Gamma = 31.93$, $\gamma = 931.93$, $\sigma_0 = 0$. Note that Controllers I and II register similar performances while Controllers III and IV also exhibit very close relative performances. On the other hand, the steady-state error is better dumped with Controllers III and IV while the transient overshoot is smaller with Controllers I and II at the expense of greater computational efforts inherent to the more complex nature of such controllers.

6. Conclusions. In this paper, four adaptive controller structures have been proposed for closed-loop plant stabilization of single-input single-output plants involving single internal and external delays when both delays are finite and known. The four controller structures involve a memory effect in the control action to compensate for the presence of delays. Such a memory can act in two ways, namely, the parametrized part of the controller may consist of a linear dynamic system involving (internal) delays and, furthermore, of a set of

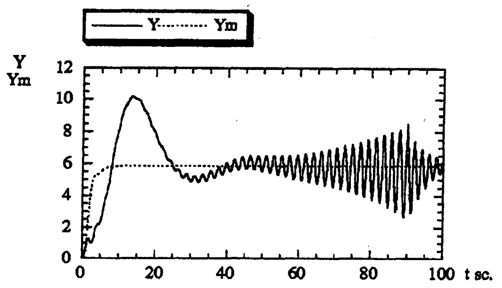

Fig. 7.3

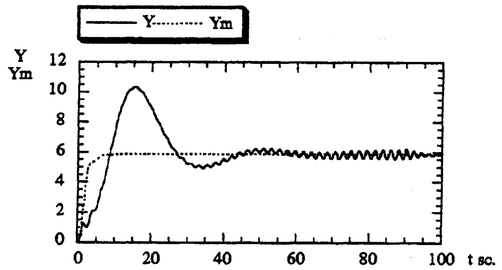

Fig. 7.4

Fig. 7 (continuation). Example 4. Comparative example of the four control structures with $h = 0.1$, $h' = 0$. (3) Control structure III, (4) Control structure IV.

weighting functions which ponderate the input time-integral is additively used to generate the plant input. The first memory effect is used to make possible the achievement of delay-dependent pole-placement control objectives in the non-adaptive version of the controllers synthetized for the nominal case (i.e., the plant is perfectly modelled) of known parameters. It can be omitted and then deleted of the scheme in the case when finite-spectrum assignability is suitable. The second memory effect is used to cancel the unsuitable multiples of the internal delay and their combinations with the external one which are generated through the feedback loop and which are not suitable for the control objective. The plant is allowed to posses unstable zeros but these ones have to be present in the reference model and known in the adaptive case. The adaptive controllers for the case of unknown plants involve the use of a σ-modification scheme together with a signal normalization which ensure signal boundedness and the existence of a residual set for the tracking error even in the presence of stable unmodelled dynamics. The tracking and adaptation error converge to zero asymptotically in the absence of unmodelled dynamics.

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APPENDICES

Proof of Theorem 1 and intermediate results. The next results in Appendices A–B are then used in the proof of the Theorem 1 in Appendix C.

APPENDIX A

Plant output equation from the reference model transfer function. Assume that θ is the constant parameter vector in the nominal situation of known parameters and all parameters without "hat" are the equivalents in the case of known parameters. One gets for Controller I the next identity in the case of known parameters

$$\theta^T \omega(t) = (F_1 + F_2 G_*)u(t) + \rho F_2 (G_* \Delta_2 + \Delta_1)u(t), \tag{42}$$

with $F_1 = \sum_{i=0}^{n-1} \bar{\theta}^{(i1)T}(sI - F - \mu^i F_i)^{-1} q$ and $F_2 = \sum_{i=0}^{n-1} \bar{\theta}^{(i2)T}(sI - F - \mu^i F_i)^{-1} q + \theta_1'$ and with the arguments D, q^{-1} and q'^{-1} in F_1, F_2 and G being omitted for notational simplicity. Define

$$\begin{aligned} F_1'(s) &= F_1(s) + \alpha_1(s), \\ \alpha_1(s) &= (1-\mu')\bar{\lambda}(s) + \sum_{i=p_1+1}^{2n-1} (1-\mu^i)\bar{\lambda}_i(s) + \sum_{i=p_2+1}^{n+m-1} (1-\mu'\mu^i)\bar{\lambda}_i'(s) \end{aligned} \tag{43}$$

and consider also parametrical and auxiliary functions adaptation errors, which exist from Assumption 8,

$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta; \quad \tilde{\lambda}(t,\tau) = \hat{\lambda}(t,\tau) - \lambda(t); \quad \tau \in [-h', 0], \tag{44}$$

$$\tilde{\lambda}_i(t,\tau) = \hat{\lambda}_i(t,\tau) - \lambda_i(t); \quad \tau \in [-ih, 0],$$
$$i = p_1+1, p_1+2, \ldots, 2n-1 \tag{45}$$
$$\tilde{\lambda}'_i(t,\tau) = \hat{\lambda}'_i(t,\tau) - \lambda'_i(t); \quad \tau \in [-(ih+h'), 0],$$
$$i = p_2+1, p_2+2, \ldots, n+m-1, \tag{46}$$
$$\tilde{\varphi}(t) = \hat{\varphi}(t) - \varphi(t) = \int_{-h'}^{0} \tilde{\lambda}(t,\tau)u(t+\tau)d\tau + \sum_{i=p_1+1}^{2n-1} \int_{-ih}^{0} \tilde{\lambda}_i(t,\tau)u(t+\tau)d\tau + \sum_{i=p_2+1}^{n+m-1} \int_{-(ih+h')}^{0} \tilde{\lambda}'_i(t,\tau)u(t+\tau)d\tau. \tag{47}$$

From (4), (43) and (44) – (47), one gets directly

$$u(t) - \theta^T\omega(t) - \varphi(t) = \tilde{\theta}(t)\omega(t) + \tilde{\varphi}(t) + \hat{c}_0(t)r(t) + \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau$$
$$= [1 - F'_1 - F_2G_*]u(t) - \rho F_2(G_*\Delta_2 + \Delta_1)u(t), \tag{48}$$

so that from (1) and (48), the plant output can be expressed as

$$y(t) = \psi_0 G_m \left[\tilde{\theta}^T(t)\omega(t) + \tilde{\varphi}(t) + \hat{c}_0(t)r(t) + \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau\right]$$
$$- \psi_0 G_m \left[\tilde{\theta}^T(t)\omega(t) + \tilde{\varphi}(t) + \hat{c}_0(t)r(t) + \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau\right] G_*u(t) + \rho[G_*\Delta_2 + \Delta_1]u(t)$$
$$= \psi_0 G_m \left[\tilde{\theta}^T(t)\omega(t) + \tilde{\varphi}(t) + \hat{c}_0(t)r(t) + \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau\right]$$
$$- \psi_0 G_m[1 - F'_1 - F_2G_*]u(t) + G_*u(t)$$
$$+ \rho\left\{\psi_0 G_m F_2[G_*\Delta_2 + \Delta_1] + G_*\Delta_2 + \Delta_1\right\}u(t), \tag{49}$$

with $\psi_0 = c_0^{-1}$. Theorem 1 for the nominal system leads to

$$G_m = \frac{c_0[1+(1-\mu')\bar{c}(s)]G_*}{1-F_1'-F_2G_*} \Rightarrow G_* = \frac{(1-F_1')G_m}{F_2G_m + c_0[1+(1-\mu')\bar{c}(s)]}. \quad (50)$$

Obtaining $u(t)$ from (48) and using of (50) into (49) leads to the next two equivalent expressions which generalize those obtained in Ioannou and Tsakalis (1986) for the delay-free case.

$$y(t) = \psi_0 G_m \left[\tilde{\theta}^T(t)\omega(t) + \tilde{\varphi}(t) + \hat{c}_0(t)r(t) + \int_{-h'}^{0} \hat{c}_1(t,\tau)r(t+\tau)d\tau\right] + \rho\Delta u(t) \quad (51)$$

$$= \left\{\psi_0 G_m + \rho\frac{\Delta}{1-F_1'-F_2G_* - \rho F_2[\Delta_1 + G_*\Delta_2]}\right\} \times \left(\tilde{\theta}^T(t)\omega(t) + \tilde{\varphi}(t) + \hat{c}_0(t)r(t) + \int_{-h'}^{0} c_1(t,\tau)r(t+\tau)d\tau\right), \quad (52)$$

where the filter Δ is defined by

$$\Delta = \Delta_1 + G_*\Delta_2 + \psi_0 G_m F_2(G_*\Delta_2 + \Delta_1) = (1+\psi_0 G_m F_2)\Delta_1 + (1+\psi_0 G_m F_2)\Delta_2 \frac{(1-F_1')G_m}{F_2G_m + c_0[1+(1-\mu')\bar{c}(s)]}. \quad (53)$$

Since G_m, Δ_1 and Δ_2 are exponentially stable as well as $\bar{c}(s)$, the transfer function Δ is exponentially stable.

Appendix B

Mathematical Developments for Stability and Robustness Analysis (Controllers I and III). The following technical result generalizes one proved for the delay-free case and is used in the proof of Theorem 1.

Lemma B.1. *Consider the system* $\eta(t) = \Delta(t, q^{-1}, q'^{-1})U(t)$ *where* $\Delta(s)$ *is proper and exponentially stable. Assume also that* $0 < \delta_1 \leqslant \delta_1(t) \leqslant \bar{\delta} < \infty$ *and* $U(t) \leqslant |u(t)| + |u(t-h')| + |y| + m(t)$ *all* $t \geqslant 0$. *Then,*

(i) $|\eta(t)|/m(t) \leqslant \delta + \bar{\delta}(t)$ *for some positive real constant* δ *and a function* $\bar{\delta}(t)$ *which depend on initial conditions and decays exponentially to zero with a rate at least as fast as* $e^{(-\delta_0 t)}$.

(ii) *The normalized signals* $|\eta(t)/m(t)|$, $|\omega(t)/m(t)|$ *and* $|\omega_f(t)/m(t)|$ *are uniformly bounded for all* $t \geqslant 0$.

Proof. (Outline) (i) First, consider the case when $\Delta(s)$ is strictly proper. Thus, it exists a fundamental matrix $\psi(t)$ (De la Sen, 1988a; De la Sen, 1988b, De la Sen, 1994) such that the state-space solution of $\Delta(s)$ satisfies

$$x(t) = \psi(t,0)x_0 + \sum_{i=1}^{\rho_1} \int_0^{h_i} \psi(t-\tau)A_i\varphi(\tau - h_i)d\tau + \int_0^t \psi(t-\tau)bu(\tau)d\tau + \int_0^t \psi(t-\tau)b_1u(\tau - h')d\tau \tag{54}$$

if there are ρ_1 points internal (uncommensurate or commensurate; i.e., $h_i = ih$) delays h_i. Note that (54) is the state-trajectory of a general system with point delays subject to any piecewise continuous function of initial conditions $\varphi : \left[\max_{1\leqslant i\leqslant\rho_1}(h_i), 0\right] \to R$. Taking norms in (54) and using the triangle inequality, one gets

$$\|x(t)\| \leqslant \delta_3(x_0)\|x_0\|e^{(-\delta_0 t)} + \delta_4 \int_0^t e^{(-\delta_0(t-\tau))}v(\tau)d\tau + \int_0^t e^{(-\delta_0+\delta_2/2)(t-\tau)}m(\tau)d\tau, \tag{55}$$

where $v(t) = |u(t)| + |u(t-h')| + |y(t)| + 1$, some positive real constants δ_4 and

$$\delta_3(x_0) = \begin{cases} a\left[1 + \max_{1\leqslant i\leqslant\rho_1}\left(\dfrac{\|A_i\|}{\|x_0\|}\right) \sup_{\tau\in[-\bar{h},0]}(\|\varphi(\tau)\|)\left(\sum_{i=1}^{\rho_1}\dfrac{1-e^{(-\delta_0 h_i)}}{\delta_0}\right)\right] & \text{if } x_0 \neq 0, \\ a \max_{1\leqslant i\leqslant\rho_1}(\|A_i\|) \sup_{\tau\in[-\bar{h},0]}(\|\varphi(\tau)\|)\left(\sum_{i=1}^{\rho_1}\dfrac{1-e^{(-\delta_0 h_i)}}{\delta_0}\right) & \text{if } x_0 = 0, \end{cases} \tag{56}$$

with $\bar{h} = \max_{1\leqslant i\leqslant\rho_1}(h_i)$ (in the case of commensurate delays $\bar{h}(\rho_1 h)$) being such that $\|\psi(t,0)\| \leqslant ae^{(-\delta_0+\delta_2/2)t}$ for all $t \geqslant 0$ which is fulfilled since the poles of $\Delta(s)$ have real parts non less than $\delta_0 + \delta_1$.

From (34) one gets

$$m(\tau) = m(0)e^{-\delta_0 \tau} + \int_0^\tau \delta_1 e^{-\delta_0(\tau-\tau')}(1+v(\tau'))\,d\tau'. \tag{57}$$

Substituting (56) into (55) and reversing the order of integration, one obtains for some positive real constants δ_5 and δ_6,

$$\begin{aligned}\|x(t)\| \leqslant & \left\{\delta_3(x_0)\|x_0\| + 2\delta_5 m(0)[1 - e^{(-\delta_2 t/2)}]/\delta_2\right\} e^{-\delta_0 t} \\ & + (\delta_4 + \delta_6)\int_0^t e^{-\delta_0(t-\tau)}v(\tau)d\tau \\ & + \delta_6 \int_0^t e^{-\delta_0(t-\tau)}m(\tau)d\tau. \end{aligned} \tag{58}$$

and (i) follows by comparing (58) to (57) and noting that $m(t) \geqslant \delta_1/\delta_0$ and that, for some nonzero vector f, $\eta(t) = f^T x(t)$. In the case when $\Delta(s)$ is nonstrictly proper, $\eta(t) = f^T x(t) + f_1 U(t)$ for some nonzero constant f_1 with $|U(t)| \leqslant \delta_7 m(t)$ for $\delta_7 > 0$ so that (58) still holds with the change $\delta_6 \to \delta_6 + \delta_7$ and the proof of (i) is complete.

(ii) One gets from (42).

$$\begin{aligned}\omega(s) = & \operatorname{Diag}\left(\sum_{i=0}^{n-1}(sI - F - \mu^i F_i)^{-1}q,\right. \\ & \left.\sum_{i=0}^{n-1}(sI - F - \mu^i F_i)^{-1}q + \theta_1'\right)(u(t), y(t))^T, \\ \omega_f(s) = & G_m \omega(s), \end{aligned} \tag{59}$$

where $\sum_{i=0}^{n-1} sI - F - \mu^i F_i)$ has all its eigenvalues in $\mathrm{Re}(s) < 0$ from Assumption 7. Since $\|u(t), y(t)\|_2 \leqslant |u(t)| + |y(t)| + m(t)$ and the transfer function associate with the mapping $[u(t), y(t)]^T \to \omega(t)$ is exponentially stable from (59), then $|\omega(t)/m(t)|$ and $|\omega_f(t)/m(t)|$ are uniformly bounded for all $t \geqslant 0$ following the same steps as in (i). From (23) – (29), the adaptive algorithm can be also written in terms of the parameter-adaptive errors as

$$\dot{\tilde{\theta}}'(t) = \frac{\psi_0 \Gamma}{m^2(t)} \left[\tilde{\theta'}^T(t)\omega_f'(t) + \tilde{\varphi}_f'(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau')y_m(t+\tau')d\tau' \right] \omega_f'(t)$$

$$- \Gamma\sigma(t)\hat{\theta}'(t) - \rho\frac{\eta(t)}{m^2(t)}\Gamma\omega_f'(t), \qquad (60)$$

$$\dot{\tilde{c}}_1'(t) = \frac{\gamma\psi_0}{m^2(t)} \left[\tilde{\theta'}^T(t)\omega_f'(t) + \tilde{\varphi}_f'(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau')y_m(t+\tau')d\tau' \right] y_m(t+\tau)$$

$$- \gamma\sigma_c(t,\gamma)\hat{c}_1'(t,\tau) - \rho\frac{\gamma\eta(t)}{m^2(t)}y_m(t+\tau) \quad \tau \in [-h',0], \qquad (61)$$

$$\dot{\tilde{\lambda}}(t) = \frac{\gamma\psi_0}{m^2(t)} \left[\tilde{\theta'}^T(t)\omega_f'(t) + \tilde{\varphi}_f'(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau')u(t+\tau')d\tau' \right] u(t+\tau)$$

$$- \gamma\sigma_\lambda(t,\gamma)\hat{\lambda}(t,\tau) - \rho\frac{\gamma\eta(t)}{m^2(t)}u(t+\tau) \quad \tau \in [-h',0], \qquad (62)$$

$$\dot{\tilde{\lambda}}_i(t) = \frac{\gamma\psi_0}{m^2(t)} \left[\tilde{\theta'}^T(t)\omega_f'(t) + \tilde{\varphi}_f'(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau')u(t+\tau')d\tau' \right] u(t+\tau)$$

$$- \gamma\sigma_{\lambda_i}(t,\gamma)\hat{\lambda}_i(t,\tau) - \rho\frac{\gamma\eta(t)}{m^2(t)}u(t+\tau) \quad \tau \in [-ih,0];$$

$$i = p_1+1, p_1+2, \ldots, 2n-1, \qquad (63)$$

$$\dot{\tilde{\lambda}}_i'(t) = \frac{\gamma\psi_0}{m^2(t)} \left[\tilde{\theta'}^T(t)\omega_f'(t) + \tilde{\varphi}_f'(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau')u(t+\tau')d\tau' \right] u(t+\tau)$$

$$- \gamma\sigma_{\lambda_i'}(t,\gamma)\hat{\lambda}_i'(t,\tau) - \rho\frac{\gamma\eta(t)}{m^2(t)}u(t+\tau) \quad \tau \in [-(ih+h'),0];$$

$$i = p_2+1, p_2+2, \ldots, n+m-1, \qquad (64)$$

where, from (24) – (26) and (47),

$$\tilde{\varphi}_f'(t) = \hat{\varphi}_f'(t) - \varphi_f'(t) = G_m \left[\tilde{\varphi}(t) + \int_{-h'}^{0} \tilde{c}_1(t,\tau)y_m(t+\tau)d\tau \right]. \qquad (65)$$

APPENDIX C

Proof of Theorem 1

(i) Take the Lyapunov functional candidate

$$V(t) = \frac{1}{2}\tilde{\theta}'^T(t)\Gamma^{-1}\tilde{\theta}'(t) + \frac{1}{2}\gamma^{-1}\Big\{ \int_{-h'}^{0} \Big(\tilde{c}_1^2(t,\tau) + \tilde{\lambda}^2(t,\tau)\Big)\, d\tau + \sum_{i=p_1+1}^{2n-1} \int_{-ih}^{0} \tilde{\lambda}_i^2(t,\tau)d\tau + \sum_{i=p_2+1}^{n+m-1} \int_{-(ih+h')}^{0} \tilde{\lambda}'^2_i(t,\tau)d\tau \Big\} + \frac{1}{2}m^2(t). \tag{66}$$

The time-derivative of (66) along (60) – (65) is given by

$$\begin{aligned}\dot{V}(t) =& \tilde{\theta'}^T(t)\Gamma^{-1}\dot{\tilde{\theta}}'(t) + \gamma^{-1}\Big\{ \int_{-h'}^{0} \Big(\tilde{c}_1(t,\tau)\dot{\tilde{c}}_1(t,\tau) + \tilde{\lambda}(t,\tau)\dot{\tilde{\lambda}}(t,\tau)\Big)\, d\tau \\ &+ \sum_{i=p_1+1}^{2n-1} \int_{-ih}^{0} \tilde{\lambda}_i(t,\tau)\dot{\tilde{\lambda}}_i(t,\tau)d\tau \\ &+ \sum_{i=p_2+1}^{n+m-1} \int_{-(ih+h')}^{0} \tilde{\lambda}'_i(t,\tau)\dot{\tilde{\lambda}}'_i(t,\tau)d\tau \Big\} + m(t)\dot{m}(t) \\ =& -2\dot{V}_1(t) - \dot{V}_2(t) - \rho\frac{\eta(t)}{m^2(t)}\bar{\epsilon}(t) + m(t)\dot{m}(t) = -\sum_{i=1}^{3}\dot{V}_i(t) \\ &- \delta_0 m^2(t) + \delta_1 m(t)\int_0^t (|u(\tau)| + |u(\tau-h')| + |y(\tau)| + 1)d\tau, \end{aligned} \tag{67}$$

where

$$\dot{V}_1(t) = \frac{\psi_0}{2m^2(t)}\Bigg[\Big(\tilde{\theta'}^T(t)\omega'_f(t)\Big)^2 + \Big(\int_{-h'}^{0} \tilde{c}_1(t,\tau)y_m(t+\tau)d\tau\Big)^2 + \Big(\int_{-h'}^{0} \tilde{\lambda}(t,\tau)u_f(t+\tau)d\tau\Big)^2 + \sum_{i=p_1+1}^{2n-1}\Big(\int_{-ih}^{0} \tilde{\lambda}_i(t,\tau)u_f(t+\tau)d\tau\Big)^2$$

$$+\sum_{i=p_2+1}^{n+m-1}\left(\int_{-(ih+h')}^{0}\tilde{\lambda}'_i(t,\tau)u_f(t+\tau)d\tau\right)^2\Bigg], \tag{68}$$

$$\begin{aligned}\dot{V}_2(t)=&\Bigg[\sigma(t)\hat{\theta'}^T(t)\tilde{\theta}'(t)+\int_{-h'}^{0}\sigma_c(t,\tau)\hat{c}_1(t,\tau)\tilde{c}_1(t,\tau)d\tau\\&+\int_{-h'}^{0}\sigma_\lambda(t,\tau)\hat{\lambda}(t,\tau)\tilde{\lambda}(t,\tau)d\tau\\&+\sum_{i=p_1+1}^{2n-1}\int_{-ih}^{0}\sigma_{\lambda_i}(t,\tau)\hat{\lambda}_i(t,\tau)\tilde{\lambda}_i(t,\tau)d\tau\\&+\sum_{i=p_2+1}^{n+m-1}\int_{-(ih+h')}^{0}\sigma_{\lambda'_i}(t,\tau)\hat{\lambda}'_i(t,\tau)\tilde{\lambda}'_i(t,\tau)d\tau\Bigg],\end{aligned} \tag{69}$$

$$\begin{aligned}\dot{V}_3(t)=&\frac{1}{2}\left[\frac{\psi_0\tilde{\theta'}^T(t)\omega'_f(t)}{m(t)}+\rho\frac{\eta(t)}{m(t)}\right]^2-\frac{\rho^2}{2}\frac{\eta^2(t)}{m^2(t)}\\&+\frac{1}{2}\left[\int_{-h'}^{0}\frac{\psi_0\tilde{c}_1(t,\tau)y_m(t+\tau)}{m(t)}+\rho\frac{\eta(t)}{m(t)}\right]^2-\frac{\rho^2}{2}\frac{\eta^2(t)}{m^2(t)}\\&+\frac{1}{2}\left[\int_{-h'}^{0}\frac{\psi_0\tilde{\lambda}(t,\tau)u_f(t+\tau)}{m(t)}+\rho\frac{\eta(t)}{m(t)}\right]^2-\frac{\rho^2}{2}\frac{\eta^2(t)}{m^2(t)}\\&+\frac{1}{2}\sum_{i=p_1+1}^{2n-1}\left[\int_{-ih}^{0}\frac{\psi_0\tilde{\lambda}_i(t,\tau)u_f(t+\tau)}{m(t)}+\rho\frac{\eta(t)}{m(t)}\right]^2\\&\qquad-\left(\frac{2n-p_1-1}{2}\right)\rho^2\frac{\eta^2(t)}{m^2(t)}\\&+\frac{1}{2}\sum_{i=p_2+1}^{n+m-1}\left[\int_{-(ih+h')}^{0}\frac{\psi_0\tilde{\lambda}'_i(t,\tau)u_f(t+\tau)}{m(t)}+\rho\frac{\eta(t)}{m(t)}\right]^2\\&\qquad-\left(\frac{n+m-p_2-1}{2}\right)\rho^2\frac{\eta^2(t)}{m^2(t)}.\end{aligned} \tag{70}$$

Note that the first term of (69) fulfills $\sigma(t)\hat{\theta}'^T(t)\tilde{\theta}'(t)$ for $\|\hat{\theta}'(t)\| > \theta'_M$ or $\epsilon \leqslant \|\hat{\theta}'(t)\| < \theta'_m$ and $\sigma(t)\hat{\theta}'^T(t)\tilde{\theta}'(t) = 0$, since $\sigma(t) = 0$, for $\theta'_m \leqslant \|\hat{\theta}'(t)\| \leqslant \theta'_M$ or for $\|\hat{\theta}'(t)\| < \epsilon$ (see (36)). Similar inequalities occur for all the remaining terms involving their respective $\sigma_{(\cdot)}(\cdot)$ and $M_{(\cdot)}(\cdot)$. Thus, $\dot{V}_2(t) \geqslant 0$ for all $t \geqslant 0$ so that $\dot{V}(t) < 0$, whenever $V(t) > V_0$ for same fixed constant, and $V(t)$ and all the parameters and auxiliary function estimates are uniformly bounded for all $t \in P_1 = \{t \in R_0^+ : V(t) > 0\}$. (These properties also hold for the modification of the adaptive algorithm (29) indicated in Remarks 2–3). Now, define $P_2 = \{t \in R_0^+ : V(t) \leqslant V_0\} = P_{21} \cup P_{22}$ where $P_{21} = \left\{t \in P_2 : \dot{V}(t) \leqslant 0\right\}$ and $P_{22} = \left\{t \in P_2 : \dot{V}(t) > 0\right\}$ are disjoint sets which can be empty depending on the initial conditions of the algorithm. Note that for $\sigma(t) \neq 0$, $t \in P_1 \cup P_{21}$ and $V(t) > 0$, $\dot{V}(t) \leqslant 0$ while for $\sigma(t) = 0$, $t \in P_1 \cup P_2$ but if $t \in P_{22}$ then $V(t)$ increases up till a time t such that $t \in P_1 \cup P_{21}$. Thus, the above reasonings and results about boundedness still apply for all $t \in P_{21}$. Now, assume that $t \in P_{22}$. Thus, for all $t \in P_{22}$ with $t \geqslant t_1$, $V(t)$ increases positively up till a time $t_2 \in P_1 \cup P_{21}$ such that $|t_2 - t_1|$ is finite and $V(t_2) = V_0$. As a result, there is a denumerable or finite set $P_{22}^{(i)}$ of open time subintervals which are disjoint, of finite length such that in their boundaries (i.e., points of $P_{11} \cup P_{21}$), $\tilde{\theta}'(\cdot)$, $m(t)$ and the auxiliary function error estimates, with respect to the elements of the auxiliary $\bar{\Lambda}$-set, are uniformly bounded at a sequence of time instants separated by finite intervals. Since $\tilde{\theta}'(\cdot)$, $m(t)$ and the estimation error functions related to the elements of $\bar{\Lambda}(\theta, h, h', t)$ are continuously differentiable on $[0, \infty)$ from (29), they have to be uniformly bounded on R_0^+. Note that boundedness on $P_1 \cup P_{21}$ and continous differentiability on R_0^+ implies boundedness on each (finite-length) $P_{21}^{(\cdot)}$ interval by using simple contradiction arguments. Thus, property (i) is proved.

(ii) The integration of (67) by using (28) and (68) leads to

$$\frac{1}{T}\int_{t_0}^{t_0+T}\left(\dot{V}_1(\tau)+\dot{V}_2(\tau)\right)d\tau = -\frac{1}{T}\int_{t_0}^{t_0+T}\left(\dot{\tilde{V}}(\tau)+\dot{V}_1(\tau)+\frac{\rho\eta(\tau)}{m^2(\tau)}\bar{\epsilon}\right)d\tau$$

$$= -\frac{1}{T}\int_{t_0}^{t_0+T}\left(\dot{V}(\tau)+\dot{V}_3(\tau)\right)d\tau \leqslant \frac{g_1}{T}+\rho^2\frac{\gamma}{2},\ t \in (P_1 \cup P_{21}) \qquad (71)$$

where $\dot{\tilde{V}}(\tau) = d/d\tau\left(V(\tau) - m^2(\tau)/2\right)$, since $\tilde{V}(\tau)$ is also bounded because

$m(t)$ and $V(t)$ are bounded from (i) and since $\left(-\dot{V}_3(t)\right) \leqslant \rho^2(3n+m+1-p_1-p_2)\eta^2(t)/\left(2m^2(t)\right) \leqslant \rho^2\gamma/2$, for all $t \geqslant 0$ from (70) and Lemma B.1(i) where $g_1 = \sup_{t_0 \in R_0^+}\left(|\tilde{V}(t_0)| - |\tilde{V}(t_0+T)|\right)$. Also $\dot{V}_1(t)$ is uniformly bounded from (71) since $\dot{V}_2(t) \geqslant 0$, all $t \geqslant 0$. Thus, from (68) and (71), $|\tilde{\theta}'^T(t)\omega'_f(t)|/m(t)$, $|\tilde{\varphi}'_f(t)|/m(t)$ are uniformly bounded for all $t \in (P_1 \cup P_{21})$ since $|\eta(t)|/m(t)$ is uniformly bounded for all $t \geqslant 0$ from Lemma B.1(i). If remains to be proved that these signals are also bounded for all $t \in P_{22}$. First, note that $|\tilde{\theta}'^T(t)\omega'_f(t)|/m(t) \leqslant \|\tilde{\theta}'(t)\|\|\omega'_f(t)\|/m(t)$ is bounded for all $t \geqslant 0$ since $\|\tilde{\theta}'(t)\|$ and $\|\omega'_f(t)\|/m(t)$ are bounded from Property (i) above and Lemma B.1(ii). Also, direct calculus with (7) – (8) leads to

$$\frac{|\tilde{\varphi}'_f(t)|}{m(t)} \leqslant g_2\frac{u_f(t+\tau)|}{m(t)}, \quad \tau \in [-h', 0], \tag{72}$$

$$\begin{aligned} g_2 = \sup_{t\geqslant 0}\Bigg\{ & \sup_{\tau\in[-h',0]}\left(|\tilde{\lambda}(t,\tau)|\right) \\ & + (2n-p_1-1)\max_{p_1+1\leqslant i\leqslant 2n-1}\left(\sup_{\tau\in[-ih,0]}\left(|\tilde{\lambda}_i(t,\tau)|\right)\right) \\ & \times (n+m-p_2-1)\max_{p_2+1\leqslant i\leqslant n+m-1}\left(\sup_{\tau\in[-(ih+h'),0]}\left(|\tilde{\lambda}'_i(t,\tau)|\right)\right) \\ & + \sup_{\tau\in[-h',0]}\left(|\tilde{c}_1(t,\tau)|\right)\Bigg\}. \end{aligned}$$

Now, assume that $|\tilde{\varphi}'_f(t)|/m(t)$ diverges so that $|u_f(t+\tau)|/m(t)$ diverges for all $\tau \in [-h', 0]$ from (72). Thus, since G_m is strictly Hurwitz, $|\tilde{\varphi}'(t+\tau)|/m(t)$ also diverges and from (7) – (8), (4) and (19), one gets

$$\begin{aligned} & |u(t+\tau)/m(t) \leqslant \\ \leqslant & \frac{1}{m(t)e^{-\delta_0\tau} + \int_t^{t+\tau}\delta_1(\tau)\left[|u(\tau')| + |u(\tau'-h')| + |y(\tau')| + 1\right]d\tau} \\ & \times \Bigg(\|\hat{\theta}'(t)\|g_3(x_0)e^{-\delta_0\tau}|\omega'(t)| \\ & + (g'_2g_2 + g_4\delta_0^{-1})\int_{t-(2n-1)h-h'}^{t}\left(|u(\tau)| + |y(\tau)|\right)d\tau\Bigg), \end{aligned} \tag{73}$$

where $\omega'_f(t)$ has been calculated and upper-bounded, in a similar way as in (54), by using a exponentially stable state transition matrix and $g_3(x_0)$ is defined for (8) similar as $\delta_3(x_0)$ in (56). Also, g'_2 is a constant relating upper-bounds of parametrical error estimates to updated parameter estimates which are bounded from Property (i). Since the right-hand-side of (73) is bounded, $|u(t+\tau)|/m(t)$, $\tau \in [-h', 0]$ and then $|\tilde{\varphi}'_f(t)|/m(t)$ and $|\tilde{\varphi}'(t)|/m(t)$, can not diverge on R_0^+. Thus, $|\tilde{\theta}'^T(t)\omega'(t)|/m(t)$, $|\tilde{\theta}'^T(t)\omega'_f(t)|/m(t)$, $|\tilde{\varphi}'(t)|/m(t)$ and $|\tilde{\varphi}'_f(t)|/m(t)$ are uniformly bounded for all $t \geqslant 0$. Thus, $|\bar{\epsilon}(t)|/m(t)$, $|\epsilon(t)|/m(t)$ and $|\eta(t)|/m(t)$ are uniformly bounded from Lemma B.1(i) and the fact that $m(t)$ is uniformly bounded from (i) implies also that $\bar{\epsilon}(t)$,$\epsilon(t)$, $e(t)$, $y(t)$ and $u(t)$ are uniformly bounded.

(iii) Note from (68) and (71) since $\dot{V}_2(t) \geqslant 0$ that

$$2\Big(\frac{\bar{g}_1}{T} + \rho^{*2}\frac{\gamma}{2}\Big) \geqslant \frac{2}{T}\int_{t_0}^{t_0+T} \dot{V}_1(\tau)d\tau$$

$$= \frac{2\psi_0}{T}\int_{t_0}^{t_0+T} \frac{\left(\tilde{\theta}'^T(\tau)\omega'_f(\tau)\right)^2 + \tilde{\varphi}'^2_f(\tau)}{m^2(\tau)}d\tau \geqslant \frac{2}{T}\int_{t_0}^{t_0+T}\frac{\epsilon^2(\tau)}{m^2(\tau)}d\tau$$

$$\geqslant \frac{2}{T\sup\limits_{0\leqslant\rho\leqslant\rho^*}\left(\sup\limits_{t_0\leqslant\tau\leqslant t_0+\tau} m(\tau)\right)}\int_{t_0}^{t_0+T}\epsilon^2(\tau)d\tau, \tag{74}$$

and the result follows for the adaption error. Note also from (48) that $e^2(t) = \big(y(t) - y_m(t)\big)^2 = \big(G_m\big(\bar{\bar{\tilde{\theta}}}^T(t)\bar{\omega}(t) + \tilde{\varphi}'(t) + \rho\eta(t)\big)\big)^2 \leqslant 2\big[\big(G_m\big(\bar{\bar{\tilde{\theta}}}^T(t)\bar{\omega}(t) + \tilde{\varphi}'(t)\big)\big)^2 + \rho^*\gamma^2\big]$ and the result for $e^2(t)$ follows since $|\bar{\bar{\tilde{\theta}}}^T(t)\bar{\omega}(t) + \tilde{\varphi}'(t)|$ is uniformly bounded and $\frac{1}{T}\int_{t_0}^{t_0+T} e^2(\tau)d\tau \leqslant \sup\limits_{t_0\leqslant\tau\leqslant t_0+T}|e^2(\tau)|$.

(iv) If $\rho = 0$ then $\epsilon(t) = \bar{\epsilon}(t)$ from (28) and $\dot{V}(t) < 0$ for all finite $t > 0$ and $\lim\limits_{t\to\infty}\dot{V}(t) = 0$. From (67) and (68),. $\dot{V}_2(t) + \dot{V}_3(t) + m\dot{m}(t) > 0$ from the choice of the $\sigma_{(\cdot)}(\cdot)$ -funtions in (29) and $\dot{V}_1(t) > 0$. Thus, since the signs of both additive terms are identical, $\lim\limits_{t\to\infty}\dot{V}_1(t) = \lim\limits_{t\to\infty}\dot{V}_2(t) = 0$. Now, note from (28) that $\epsilon^2(t) = \left(\tilde{\theta}'^T(t)\omega'_f(t) + \tilde{\varphi}'_f(t)\right)^2 \leqslant \frac{4\dot{V}_1(t)}{\psi_0}m^2(t)$ since

$\rho = 0$ and $\left(\tilde{\theta'}^T(t)\omega'_f(t) + \tilde{\varphi}'_f(t)\right)^2 \leqslant 2\left[\left(\tilde{\theta'}^T(t)\omega'_f(t)\right)^2 + \tilde{\varphi'}^2_f(t)\right)\right]$. Now, $\epsilon^2(t) \to 0$ as $t \to \infty$ since $\dot{V}_1(t) \to 0$ as $t \to \infty$ and $m(t)$ is uniformly bounded. Assume $k_\rho = 1$, then $\eta(t) = 0$ and, since $\epsilon(t) = \tilde{\theta'}^T(t)\omega'_f(t) + \tilde{\varphi}'_f(t) \to 0$ as $t \to \infty$, $u_f(t) = \hat{\theta}'^T(t)\omega'_f(t) + \hat{\varphi}'_f(t)$ converges to $u^*_f(t) = G_m u^*(t) = G_m\left(\theta'^T(t)\omega'_f(t) + \varphi'_f(t)\right)$ as $t \to \infty$ where $u^*(t)$ and $u^*_f(t)$ are the nominal and filtered nominal plant inputs for $\rho = 0$, the current regressor and auxiliary functions and the nominal parameters. Since $u^*_f(t) \to u_f(t)$ as $t \to \infty$, then $u^*(t) \to u(t)$ as $t \to \infty$ since G_m is exponentially stable. Now, (A.7) implies that $e(t) = G_m\left(u(t) - u^*(t)\right)$ for $\rho = 0$ so that $e(t) \to 0$ as $t \to \infty$ since G_m is exponentially stable and $\left(u(t) - u^*(t)\right)$ converges to zero asymptotically. If $k_p \neq 1$, then an extra controller parameter has to be used. The facts that $V(t)$ is bounded and $\dot{V}(t) < 0$, $\dot{V}_1(t) < 0$ for all $t \geqslant 0$ imply directly from (28) that $\lim_{t\to\infty} \int_0^t \psi_0\left(\tilde{\theta'}^T(\tau)\omega'_f(\tau) + \tilde{\varphi}'_f(\tau)\right)/m^2(\tau)d\tau = \lim_{t\to\infty} \int_0^t \frac{\epsilon(\tau)}{m^2(\tau)}d\tau < \infty$, since $\frac{\epsilon^2(\tau)}{m^2(\tau)}$ is uniformly bounded and integrable on $[0, \infty)$, with $\tilde{\theta}'(t)$ being bounded and $\frac{d}{dt}\left(\frac{\tilde{\theta}'^T(t)\omega'_f(t) + \tilde{\varphi}'_f(t)}{m^2(t)}\right)$ is bounded so that $\left(\tilde{\theta}'^T\omega'_f(t)/m^2(t)\right)$ is uniformly continuous. Barbalat's lemma, (Ioannou and Tsakalis, 1986), implies that $\left(\left(\tilde{\theta}'^T\omega'_f(t)\right)^2 + \varphi'_f(t)\right)/m^2(t) \to 0$ as $t \to \infty$, so that $\left(\bar{\tilde{\theta}}'^T\bar{\omega}(t) + \tilde{\varphi}(t)\right)/m^2(t) \to 0$ as $t \to \infty$ and $e(t) \to 0$ as $t \to \infty$, and the proof is complete.

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SISTEMŲ SU VĖLINIMU ADAPTYVUS ROBASTINIS VALDYMAS

Manuel De la SEN ir Josu JUGO

Straipsnyje nagrinėjamas adaptyvus valdymo algoritmas, kuris yra robastinis adaptyvios ir multiplikatyvios objekto dinamikos atžvilgiu. Analizuojamos kelios adaptyvios valdymo struktūros, kai objekto eilė žinoma, o parametrai nežinomi. Du valdymo įrenginiai vėlinimą turi, o kiti du – vėlinimo neturi. Valdymo įrenginiuose naudojami papildomi signalai, kurie stabilizuoja valdymo procesus. Pateikiami valdymo uždavinių matematiniai sprendimai bei modeliavimo pavyzdžiai.