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ON ADAPTIVE THRESHOLD INTERVALS FOR STOPPING RECURSIVE LEAST SQUARES IN THE SPACE OF SIGNALS

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Abstract. In the papers (Kaminskas, 1973; Kaminskas and Nemura, 1975) the stopping rule of recursive least squares (RLS) is worked out using the length of the confidence interval for the respective current meaning of the true output signal of a linear dynamic system. The aim of the given paper is the development of techniques for calculating threshold intervals of respective criteria, used in such a stopping rule. In this connection adaptive threshold intervals based on the Cramer–Rao lower bound according to Pupeikis (1995) are proposed here, too. The results of numerical simulation by IBM PC/AT are given.

Key words: dynamic system, recursive algorithm, threshold, least squares.

1. Stopping rule of RLS. Consider a single input x_k and output U_k of a linear discrete-time system, described by the difference equation

$$u_k = y_k + N_k = -a_1 u_{k-1} - \ldots - a_p u_{k-p} + b_o x_k + \ldots + b_q x_{k-q} + N_k, \quad (1)$$

where y_k is the current meaning of the true output signal of the initial linear dynamic system; a_i, $i = \overline{1,p}$ and b_j, $j = \overline{0,q}$ are unknown parameters to be estimated by processing some input-output observations x_k and u_k; p, q are known positive integers; N_k is a sequence of independent Gaussian variables with zero mean and σ_N^2.

To calculate the estimate $\widehat{\mathbf{c}}_{s+1}$ of the parameter vector $\mathbf{c}^T = (a_1, \ldots, a_p, b_0, \ldots, b_q)$ we use ordinary RLS of the form

$$\widehat{\mathbf{c}}_{s+1} = \widehat{\mathbf{c}}_s + \mathbf{\Gamma}_{s+1} \nabla_c e_{s+1} e_{s+1}, \quad (2)$$

$$\mathbf{\Gamma}_{s+1} = \mathbf{\Gamma}_s - \frac{\mathbf{\Gamma}_s \nabla_c e_{s+1} \nabla_c^T e_{s+1} \mathbf{\Gamma}_s}{1 + \nabla_c^T e_{s+1} \mathbf{\Gamma}_s \nabla_c e_{s+1}}, \quad (3)$$

$$e_{s+1} = u_{s+1} - \nabla_c^T e_{s+1} \hat{\mathbf{c}}_s, \tag{4}$$

$$\boldsymbol{\Gamma}_0 = \gamma \boldsymbol{I}, \quad \gamma >> 1. \tag{5}$$

Here

$$\hat{\mathbf{c}}_{s+1}^T = \left(\hat{\mathbf{a}}^T, \hat{\mathbf{b}}^T\right)_{s+1} = \left(\hat{a}_1, \ldots, \hat{a}_p, \hat{b}_0, \ldots, \hat{b}_q\right)_{s+1}, \tag{6}$$

$$\hat{\mathbf{c}}_s^T = \left(\hat{\mathbf{a}}^T, \hat{\mathbf{b}}^T\right)_s = \left(\hat{a}_1, \ldots, \hat{a}_p, \hat{b}_0, \ldots, \hat{b}_q\right)_s \tag{7}$$

are the vectors of unknown parameter estimates obtained by recursive processing of $s+1$ samples and s samples of x_k and U_k, $k = 1, 2, \ldots, s, s+1$, respectively;

$$\nabla_c e_{s+1} = (-u_s, \ldots, -u_{s+1-p}, x_{s+1}, \ldots, x_{s+1-q})^T \tag{8}$$

is a vector of p and $q+1$ most recent observations of input x_k and output U_k; $\boldsymbol{\Gamma}_s$ is an $m \times m$ positive definite matrix; $\boldsymbol{I}$ is a $m \times m$ unit matrix; $m = p+q+1$.

According to Kaminskas and Nemura (1975) the stopping rule for RLS in the space of signals is based on the statistic

$$t = \frac{\hat{y}_s - y_s}{\hat{\sigma}_{N_s}^2 \sqrt{\nabla_c^T e_s \boldsymbol{\Gamma}_s \nabla_c e_s}}, \tag{9}$$

where t denotes Student's statistic with t-distribution and $s - m$ degrees of freedom;

$$\hat{y}_s = \nabla_c e_s \hat{\mathbf{c}}_s \tag{10}$$

is the value of an output signal of the statical mathematical model of system (1) according to Cypkin (1984);

$$\nabla_c e_s = (-u_{s-1}, \ldots, -u_{s-p}, x_s, \ldots, x_{s-q})^T \tag{11}$$

is a vector of p and $q+1$ most recent observations of input x_k and output U_k; $\hat{\sigma}_{N_s}^2$ is the estimate of variance σ_N^2, that can be calculated recursively (Kaminskas, 1973).

Then, two criteria

$$\varrho_s^{(1)} = 2\hat{\sigma}_{N_s}^2 t_\alpha \sqrt{\nabla_c^T e_s \boldsymbol{\Gamma}_s \nabla_c e_s}, \tag{12}$$

$$\varrho_s^{(2)} = \max_i \left\{ 2\hat{\sigma}_{N_s}^2 t_\alpha \sqrt{\nabla_c^T e_s \boldsymbol{\Gamma}_s \nabla_c e_s} \right\}, \quad i = \overline{n, s} \tag{13}$$

are elaborated in Kaminskas and Nemura (1975).

The first criterion defines the length of the confidence interval of such a form

$$\hat{y}_s - \hat{\sigma}^2_{N_s} t_\alpha \sqrt{\nabla_c^T e_s \Gamma_s \nabla_c e_s} \leqslant y_s \leqslant \hat{y}_s + \hat{\sigma}^2_{N_s} t_\alpha \sqrt{\nabla_c^T e_s \Gamma_s \nabla_c e_s}, \quad (14)$$

where t_α is such that

$$P\{|t| \leqslant t_\alpha\} = 1 - \alpha, \quad (15)$$

and it is tabulated, dependent on the significance level α and $s - m$ degrees of freedom.

The second criterion defines the maximal length of confidence regions obtained for the true meaning of system output y_s.

Recursive calculations by RLS are stopped when

$$\varrho_s^{(i)} \leqslant \varrho_0, \quad i = \overline{1,2}, \quad (16)$$

where ϱ_0 is the threshold to be chosen beforehand.

2. Calculation of adaptive thresholds. Just like in Kaminskas (1972) there exist *five* main uncertainties while using the stopping rule proposed in Kaminskas (1973), Kaminskas and Nemura (1975). *First*, for both criteria (12) and (13) the same threshold ϱ_0 is chosen; *second*, there are no suggestions whatsoever as to the choice of ϱ_0; *third*, the efficiency of different $\varrho_s^{(i)}$, $i = \overline{1,2}$ is shown not so clearly; *fourth*, the stopping criteria are worked out only for the case of additive Gaussian noise; *fifth*, the ability of stopping criteria (12), (13) to terminate the recursive computation for more complex noise model structures (for example, the ARMAX model) is not investigated there. Therefore, we try to obtain here the threshold values using the Cramer–Rao lower bound (Rao, 1968; Pupeikis, 1988; Pupeikis, 1995). Then according to Cypkin (1984), for the asymptotically optimal algorithm (2) – (5) the asymptotic covariance matrix of errors (ACME) is

$$V = \lim_{k \to \infty} kV(\hat{\mathbf{c}}) = I^{-1}(p_0) A^{-1}\left(\mathbf{c}, \sigma^2(p_0)\right), \quad (17)$$

because in such a case the Cramer–Rao inequality

$$V \geqslant I^{-1}(p_0) A^{-1}\left(\mathbf{c}, \sigma^2(p_0)\right) \quad (18)$$

turns into an equality.

Here

$$\boldsymbol{V}(\hat{\mathbf{c}}) = M\{(\hat{\mathbf{c}}_s - \mathbf{c})(\hat{\mathbf{c}}_s - \mathbf{c})^T\} \tag{19}$$

is the covariance matrix of errors (CME);

$$\boldsymbol{I}(p_0) = M\left\{\frac{p_0'(\boldsymbol{N})}{p_0(\boldsymbol{N})}\right\} \tag{20}$$

is Fisher's information; $p_0(\boldsymbol{N})$ and $p_0'(\boldsymbol{N})$ are the probability density function and its first derivative, respectively; $\boldsymbol{A}(\mathbf{c}, \sigma^2(p_0))$ is the normed information matrix (NIM); $\sigma^2(p_0)$ is the variance of noise $\boldsymbol{N}_k$;

$$\tilde{\boldsymbol{I}}(\mathbf{c}) = \boldsymbol{I}(p_0)\boldsymbol{A}(\mathbf{c}, \sigma^2(p_0)); \tag{21}$$

$M\{\cdot\}$ is the average value.

The estimate of ACME can be calculated by

$$\hat{\boldsymbol{V}}_s = s\hat{\sigma}_{e_s}^2 \left(\boldsymbol{\Phi}_s^T \boldsymbol{\Phi}_s\right)^{-1} \tag{22}$$

in an off-line operation and

$$\hat{\boldsymbol{V}}_k = k\hat{\sigma}_{e_k}^2 \boldsymbol{\Gamma}_k, \quad k = 1, 2, \ldots, s, s+1, \ldots, \tag{23}$$

in the on-line one.

Here $\hat{\sigma}_{e_s}^2$ is the estimate of variance of σ_e^2 after processing s pairs of input–output observations; $\mathbf{e} = (e_1, \ldots, e_s)^T$ is the vector of residuals (4);

$$\boldsymbol{\Phi}_s = \begin{bmatrix} -u_i & \ldots & -u_{i-p+1} & x_{i+1} & \ldots & x_{i-q+1} \\ -u_{i+1} & \ldots & -u_{i-p+2} & x_{i+2} & \ldots & x_{i-q+2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -u_{s-2} & \ldots & -u_{s-p-1} & x_{s-1} & \ldots & x_{s-1} \\ -u_{s-1} & \ldots & -u_{s-p} & x_s & \ldots & x_s \end{bmatrix} \tag{24}$$

is the matrix of input–output observations; $i = \max(p, q)$.

Hence, taking into account (17), (23) it follows for minimal values of thresholds that

$$\varrho_{k\,\min}^{(1)} = 2\hat{\sigma}_{N_k}^2 t_\alpha \sqrt{\left(\nabla_c^T e_k \tilde{\boldsymbol{I}}^{-1}(\mathbf{c}) \nabla_c e_k\right) k^{-1} \sigma_{e_k}^{-2}}, \tag{25}$$

$$\varrho_{k\,\min}^{(2)} = \max_i \left\{2\hat{\sigma}_{N_k}^2 t_\alpha \sqrt{\left(\nabla_c^T e_k \tilde{\boldsymbol{I}}^{-1}(\mathbf{c}) \nabla_c e_k\right) k^{-1} \sigma_{e_k}^{-2}}\right\}, \tag{26}$$

$$k = s+1, s+2, \ldots, s+l, \ldots, \quad i = \overline{n, k}.$$

The maximal values of thresholds could be calculated by the same formulas supposing $k = s$ in (25), (26). Then

$$\varrho^{(1)}_{s\,\max} = 2\widehat{\sigma}^2_{N_s} t_\alpha \sqrt{\left(\nabla_c^T e_s \widetilde{\boldsymbol{I}}^{-1}(\mathbf{c}) \nabla_c e_s\right) s^{-1} \sigma_{e_s}^{-2}}, \tag{27}$$

$$\varrho^{(2)}_{s\,\max} = \max_i \left\{ 2\widehat{\sigma}^2_{N_s} t_\alpha \sqrt{\left(\nabla_c^T e_s \widetilde{\boldsymbol{I}}^{-1}(\mathbf{c}) \nabla_c e_s\right) s^{-1} \sigma_{e_s}^{-2}} \right\}, \tag{28}$$

$$i = \overline{n, s},$$

respectively.

It might be mentioned that the respective minimal and the maximal value of thresholds are time varying not only because of current s, k, $\widehat{\sigma}^2_{N,k}$, $\widehat{\sigma}^2_{N,s}$ and $\nabla_c e_k$, $\nabla_c e_s$, but also because of the values of some parameters whose current estimates ought to be substituted into $\widetilde{\boldsymbol{I}}(c)$ of the form (21).

Then, recursive calculations by RLS are stopped if one of the conditions

$$\varrho^{(1)}_{s\,\max} \geqslant \varrho^{(1)}_k \geqslant \varrho^{(1)}_{k\,\min}, \tag{29}$$

$$\varrho^{(2)}_{s\,\max} \geqslant \varrho^{(2)}_k \geqslant \varrho^{(2)}_{k\,\min}, \tag{30}$$

$$k = s+1, s+2, \ldots, s+l, \ldots$$

is satisfied or even both the conditions are satisfied at the same time.

Here

$$\varrho^{(1)}_k = 2\widehat{\sigma}^2_{N_k} t_\alpha \sqrt{\nabla_c^T e_k \boldsymbol{\Gamma}_k \nabla_c e_k}, \tag{31}$$

$$\varrho^{(2)}_k = \max_i \left\{ 2\widehat{\sigma}^2_{N_k} t_\alpha \sqrt{\nabla_c^T e_k \boldsymbol{\Gamma}_k \nabla_c e_k} \right\}, \quad i = \overline{n, k}, \tag{32}$$

according to Kaminskas and Nemura (1975).

3. Time varying threshold intervals for the first order object. Anologously as in Pupeikis (1995), we consider here a discrete-time object of the form

$$u_k + a u_{k-1} = b_0 x_k + N_k, \tag{33}$$

as an example, where a and b_0 are the coefficients of difference equation.

In such a case ACME and NIM are

$$V = \sigma_e^2 \frac{\begin{bmatrix} K_x(0) & K_{ux}(1) \\ K_{ux}(1) & K_u(0) \end{bmatrix}}{K_x(0)\, K_u(0) - K_{ux}^2(1)}, \tag{34}$$

and

$$A^{-1}(\mathbf{c}, \sigma_N^2) = \begin{bmatrix} \frac{1-a^2}{\sigma_N^2 + b_0\sigma_x^2} & 0 \\ 0 & 1/\sigma_x^2 \end{bmatrix}, \tag{35}$$

respectively, where

$$e_k = u_k - b_0 x_k + a u_{k-1} \tag{36}$$

is the residual; $K_x(\cdot)$, $K_u(\cdot)$, $K_{ux}(\cdot)$ are the estimates of input–output autocovariance and crosscovariance function values, respectively; $\sigma_{e_k}^2$ and $\sigma_{x_k}^2$ are variances of residuals and input signal, respectively.

Then inequality (18) can be rewritten in such a way

$$\sigma_e^2 \frac{\begin{bmatrix} K_x(0) & K_{ux}(1) \\ K_{ux}(1) & K_u(0) \end{bmatrix}}{K_x(0)\, K_u(0) - K_{ux}^2(1)} \geqslant \sigma_{N_k}^2 \begin{bmatrix} \frac{1-a^2}{\sigma_{N_k}^2 + b_0\sigma_{x_k}^2} & 0 \\ 0 & \frac{1}{\sigma_{x_k}^2} \end{bmatrix} \tag{37}$$

since Fisher's information

$$I(p_0) = 1/\sigma_N^2. \tag{38}$$

In view of the mentioned expressions time varying thresholds (25)–(28) can be obtained for object (33) by the formulas:

$$\varrho_{k\,\min}^{(1)} = 2\hat{\sigma}_{N_k}^2 t_\alpha \sqrt{k^{-1} f_k \left(v_{1_k} u_{k-1}^2 + v_{2_k} x_k^2\right)}, \tag{39}$$

$$\varrho_{k\,\max}^{(1)} = 2\hat{\sigma}_{N_s}^2 t_\alpha \sqrt{s^{-1} f_s \left(v_{1_s} u_{s-1}^2 + v_{2_s} x_s^2\right)}, \tag{40}$$

$$\varrho_{s\,\min}^{(2)} = \max_i \left\{ 2\hat{\sigma}_{N_k}^2 t_\alpha \sqrt{k^{-1} f_k \left(\nabla_c e_i \begin{bmatrix} v_{1_k} & 0 \\ 0 & v_{2_k} \end{bmatrix} \nabla_c e_i \right)} \right\}, \tag{41}$$

$$i = \overline{n, s},$$

$$\varrho_{s\,\max}^{(2)} = \max_i \left\{ 2\hat{\sigma}_{N_s}^2 t_\alpha \sqrt{s^{-1} f_s \left(\nabla_c e_i \begin{bmatrix} v_{1_s} & 0 \\ 0 & v_{2_s} \end{bmatrix} \nabla_c e_i \right)} \right\}, \tag{42}$$

$$i = \overline{n, s},$$

$$v_{1_k} = \frac{1-\hat{a}_k^2}{\hat{\sigma}_{N_k}^2 + \hat{b}_{0_k}\sigma_{x_k}^2}, \quad v_{2_k} = \frac{1}{\hat{\sigma}_{x_k}^2}, \tag{43}$$

$$v_{2_s} = \frac{1-\hat{a}_s^2}{\hat{\sigma}_{N_s}^2 + \hat{b}_{0_s}\hat{\sigma}_{x_s}^2}, \quad v_{2_s} = \frac{1}{\hat{\sigma}_{x_s}^2}, \tag{44}$$

$$f_k = \frac{\hat{\sigma}_{N_k}^2}{\hat{\sigma}_{e_k}^2}, \qquad f_s = \frac{\hat{\sigma}_{N_s}^2}{\hat{\sigma}_{e_s}^2}. \tag{45}$$

Equations (56)–(67) could be realized in an on-line operation, if the estimates $\hat{a}_k$, $\hat{b}_{0_k}$, $\hat{K}_x(0,k)$, $\hat{K}_u(\tau,k)$, $\hat{K}_{ux}(\tau,k)$, $\tau = \overline{0,1}$, $\hat{\sigma}_{x_k}^2$, $\hat{\sigma}_{e_k}^2$, $\hat{\sigma}_{N_k}^2$ are substituted into the above mentioned expressions instead of their unknown values, respectively. They may be calculated in such a way:

$$\begin{bmatrix} \hat{a}_{1_k} \\ \hat{b}_{0_k} \end{bmatrix} = \frac{\begin{bmatrix} -\hat{K}_x(0,k) & \hat{K}_u(1,k)+\hat{K}_{ux}(1,k) & \hat{K}_{xu}(0,k) \\ -\hat{K}_{ux}(1,k) & \hat{K}_u(1,k)+\hat{K}_u(0,k) & \hat{K}_{ux}(0,k) \end{bmatrix}}{\hat{K}_x(0,k)\,\hat{K}_u(0,k) - \hat{K}_{ux}^2(1,k)}, \tag{46}$$

$$\hat{K}_x(0,k) = \hat{K}_x(0,k-1) + \frac{1}{1+k}\left[x_k x_k - \hat{K}_x(0,k-1)\right], \quad \tau = \overline{0,1},$$

$$\hat{K}_u(\tau,k) = \hat{K}_u(\tau,k-1) + \frac{1}{1+k}\left[u_{k-\tau}u_k - \hat{K}_u(\tau,k-1)\right],$$

$$\hat{K}_{ux}(\tau,k) = \hat{K}_{ux}(\tau,k-1) + \frac{1}{1+k}\left[u_{k-\tau}x_k - \hat{K}_{ux}(\tau,k-1)\right],$$

$$\hat{\sigma}_{x_k}^2 = \hat{\sigma}_{x_{k-1}}^2 + \frac{1}{k-1}\left[(x_k - \overline{x}_k)^2 - \hat{\sigma}_{x_{k-1}}^2\right],$$

$$\overline{x}_k = x_{k-1} + \frac{1}{k}(x_k - \overline{x}_{k-1}),$$

$$\hat{\sigma}_{e_k}^2 = \hat{\sigma}_{e_{k-1}}^2 + \frac{1}{k-1}\frac{\left(u_k - \hat{b}_{0_k}x_k + \hat{a}_k u_{k-1}\right)^2}{1+\eta_k},$$

$$\eta_k = \frac{\hat{K}_x(0,k-1)u_{k-1}^2 - \hat{K}_{ux}(1,k-1)x_k u_{k-1}}{(k-1)\left[\hat{K}_x(0,k)\hat{K}_u(0,k) - \hat{K}_{ux}^2(1,k)\right]}$$

$$+ \frac{\hat{K}_{ux}(1,k-1)u_{k-1}^2 + \hat{K}_u(0,k-1)x_k^2}{(k-1)\left[\hat{K}_x(0,k)\hat{K}_u(0,k) - \hat{K}_{ux}^2(1,k)\right]},$$

$$\widehat{\sigma}^2_{N_k} = \widehat{\sigma}^2_{N_{k-1}} + \frac{1}{k-1}\left[\left(\widehat{N}_k - \overline{N}_k\right)^2 - \widehat{\sigma}^2_{N_{k-1}}\right],$$

$$\overline{N}_k = \overline{N}_{k-1} + \frac{1}{k}\left(\widehat{N}_k - \overline{N}_{k-1}\right),$$

$$\widehat{N}_k = \widehat{v}_k + \widehat{a}_k \widehat{v}_{k-1},$$

$$\widehat{v}_k = u_k - \widehat{y}_k = u_k - \widehat{b}_{0_k} x_k + \widehat{a}_k \widehat{y}_{k-1},$$

where G_k is the estimate of y_k.

For higher order objects the stopping conditions are considerably more complicated, but not so much that their determination were impossible. Recommendations referring to the information matrix can be found in (Cypkin, 1984; Klein and Melard, 1994).

5. Simulation results. The stopping rule (16) with adaptive thresholds (25) – (28) for discrete-time objects with $a = 0.7$, $b_0 = 1$ and $a = 0.985$, $b_0 = 1$ in (33) was investigated by numerical simulation using IBM PC/AT. Realizations of independent Gaussian variables ξ_k with zero mean and unitary variance and a sequence of the second order AR model of the form

$$x_k = x_{k-1} - 0.5x_{k-2} + \xi_k, \quad k = \overline{1,500}, \tag{47}$$

were used as an input sequence x_k. Ten experiments with different realizations of noise N_k at the noise level $\sigma_N^2/\sigma_y^2 = 0.5$ were carried out. In each i-th experiment, first, the estimates of parameters of equation (33), the criterion $\varrho_k^{(1)}$, of the shape (31), and its maximal threshold values (40) were obtained. Afterwards, the same values and the minimal threshold value (39) were calculated recursively, using the above mentioned on-line procedure.

In Table 1 we present the estimates, averaged by 10 experiments,

$$\overline{\widehat{a}} = \frac{1}{10}\sum_{i=1}^{10} \widehat{a}^{(i)}, \tag{48}$$

$$\overline{\widehat{b}} = \frac{1}{10}\sum_{i=1}^{10} \widehat{b}^{(i)}, \tag{49}$$

the criterion

$$\overline{\varrho}_k^{(1)} = \frac{1}{10}\sum_{i=1}^{10} \varrho_{k,i}^{(1)}, \tag{50}$$

and its maximal threshold value

$$\varrho^{(1)}_{s\ \max} = \frac{1}{10}\sum_{i=1}^{10} \varrho^{(1)}_{s,i\ \max}, \tag{51}$$

with their confidence intervals Δ, calculated in each i-th experiment after processing $s = 15$ values of observations (x_k, u_k).

Table 1. Estimates (48), (49), criterion (50) and its maximal threshold values (51), averaged by 10 experiments, with confidence intervals Δ after processing 15 values of observations

$\overline{\hat{a}}$	$\overline{\hat{b}}$	$\overline{\varrho}^{(1)}_{k\ \max}$	$\overline{\varrho}^{(1)}_{k}$
Parameters $a = 0.7$, $b_0 = 1$ in (33)			
Input – Gaussian process			
0.39 ± 0.03	0.84 ± 0.07	0.64 ± 0.19	2.14 ± 0.37
Input – AR process			
0.24 ± 0.04	1.17 ± 0.12	0.513 ± 0.21	3.73 ± 0.72
Parameters $a = 0.985$, $b_0 = 1$ in (33)			
Input – Gaussian process			
0.92 ± 0.01	0.81 ± 0.02	4.41 ± 1.08	9.87 ± 1.73
Input – AR process			
0.95 ± 0.01	0.89 ± 0.04	10.15 ± 3.45	16.1 ± 2.79

Table 2 illustrate the same estimates, the respective criterion and its minimal threshold value

$$\varrho^{(1)}_{k\ \min} = \frac{1}{10}\sum_{i=1}^{10} \varrho^{(1)}_{k,i\ \min}, \tag{52}$$

averaged by 10 experiments and calculated in each experiment after processing different number of observations (x_k, u_k). The first line of each k corresponds to the meanings calculated using a Gaussian process as input, and the second line shows the meanings obtained by applying a sequence of the form (47) as input. It follows from the simulation and estimation results, presented here, that

Table 2. Estimates (48), (49), criterion (50) and its minimal threshold values (52), averaged by 10 experiments, with confidence intervals depending on k

k	$\widehat{\widehat{a}}$	$\widehat{\widehat{b}}$	$\overline{\varrho}^{(1)}_{k\,\min}$	$\overline{\varrho}^{(1)}_{k}$
	Parameters $a = 0.7$, $b_0 = 1$ in (33)			
100	0.69 ± 0.01	0.99 ± 0.01	0.27 ± 0.06	4.03 ± 0.93
	0.70 ± 0.01	1.00 ± 0.01	0.31 ± 0.06	5.19 ± 1.05
200	0.70 ± 0.01	0.99 ± 0.01	0.20 ± 0.03	4.63 ± 0.62
	0.69 ± 0.01	1.00 ± 0.01	0.23 ± 0.03	5.84 ± 0.97
300	0.70 ± 0.01	0.99 ± 0.01	0.24 ± 0.02	7.03 ± 0.56
	0.70 ± 0.01	1.01 ± 0.02	0.17 ± 0.02	6.32 ± 0.54
400	0.70 ± 0.01	1.00 ± 0.01	0.11 ± 0.02	3.68 ± 0.76
	0.70 ± 0.01	0.99 ± 0.01	0.14 ± 0.02	6.17 ± 0.94
500	0.70 ± 0.01	0.99 ± 0.01	0.14 ± 0.01	5.08 ± 0.72
	0.70 ± 0.01	0.98 ± 0.01	0.14 ± 0.01	7.41 ± 0.97
	Parameters $a = 0.985$, $b_0 = 1$ in (33)			
100	0.95 ± 0.01	0.98 ± 0.01	11.77 ± 2.89	64.73 ± 11.70
	0.95 ± 0.01	0.97 ± 0.01	21.26 ± 5.43	119.30 ± 18.67
200	0.98 ± 0.01	0.99 ± 0.01	13.60 ± 3.15	91.41 ± 20.30
	0.98 ± 0.01	1.00 ± 0.01	19.03 ± 4.21	138.81 ± 30.71
300.	0.97 ± 0.01	0.97 ± 0.01	44.87 ± 17.30	223.39 ± 51.84
	0.97 ± 0.01	1.00 ± 0.01	58.30 ± 24.07	283.44 ± 63.91
400	0.98 ± 0.01	1.01 ± 0.01	21.82 ± 4.79	184.32 ± 35.07
	0.98 ± 0.01	1.00 ± 0.01	28.30 ± 6.95	278.68 ± 51.03
500	0.98 ± 0.01	0.99 ± 0.01	49.37 ± 6.71	423.11 ± 36.00
	0.98 ± 0.01	1.00 ± 0.01	51.12 ± 8.72	542.28 ± 46.15

for both inputs there is no decrease in criterion (31) for an increased number of observations k even *in a mean sense*. Therefore, condition (29) for averaged measures (51), (52) will not be satisfied even for $k = 500$. In that situation we have no opportunity to use the adaptive threshold intervals proposed above.

6. Conclusions. The results of numerical simulations carried out by computer even for the first order system (33) prove the inapplicability of criterion (12) to RLS stopping. On the other hand, the applicability of criterion (13) is restricted beforehand, because it is necessary to store all the vectors $\nabla_c e_i$, $i = \overline{n, s}$, in computer memory. Therefore, there still remains a problem to work out effective criteria, that could be successfully used for RLS stopping in the space of signals.

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APIE ADAPTYVIUS SLENKSČIŲ INTERVALUS, STABDANT REKURENTINĮ MAŽIAUSIŲJŲ KVADRATŲ ALGORITMĄ SIGNALŲ ERDVĖJE

Rimantas PUPEIKIS

Analitinio tyrimo būdu, taikant Kramerio-Rao nelygybę, sudaryti adaptyvūs slenksčių intervalai prof. V. Kaminsko ir prof. A. Nemuros kriterijams, kuriuos jie pasiūlė parametrų įverčių skaičiavimams stabdyti, pasiekus pageidaujamą šių įverčių tikslumą. Net ir riboti modeliavimo rezultatai, gauti PC/AT pagalba (Lentelės 1, 2), parodė, kad šių autorių kriterijai aplamai negali būti taikomi rekurentiniam mažiausiųjų kvadratų algoritmui stabdyti.