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Endogenous Reliability Calculus of Objective-Weighted MCDM: Differential Sensitivity and Certified Rank Robustness

Ibrahim M. HEZAM
Department of Statistics and Operations Research, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia
e-mail: ialmishnanah@ksu.edu.sa

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Abstract. Objective weighting in multi-criteria decision-making (MCDM) makes criterion weights functions of the same performance data used to rank alternatives. Data uncertainty therefore propagates through both normalized performances and weight re-estimation. This paper develops an endogenous reliability calculus for the complete raw-data-to-ranking pipeline. The composite Jacobian is formulated for general differentiable aggregation, while closed-form pairwise and certification results are developed under additive aggregation. Pairwise sensitivity is decomposed exactly into a direct performance component and an endogenous reweighting component, yielding local conditions for amplification, attenuation, neutrality, and cancellation. A Conditioning Transfer Principle further shows that ill-conditioned objective weights need not imply an ill-conditioned decision because weight and ranking sensitivities are different projections of the same information Jacobian. The analysis then moves from local diagnostics to finite guarantees. A deterministic raw-data certificate is derived for rank preservation; information-separable weighting permits exact criterion-wise decomposition, whereas coupled rules with a suitable factorization admit a monotone factor-clustering hierarchy. Standard deviation, Entropy, and CRITIC instantiate local-dispersion, information-divergence, and network-coupled structures, respectively. Numerical experiments verify the analytical Jacobians, distinguish weight conditioning from decision conditioning, and show amplification, near-neutrality, and local attenuation. The worked SD example additionally certifies the unique winner for every componentwise relative perturbation up to 7.210%, while an explicit adverse perturbation produces a reversal at 7.211%. The framework therefore shifts robustness analysis from independent parameter perturbation toward reliability analysis of the complete endogenous decision mechanism.

Key words: multi-criteria decision-making, objective weighting, endogenous weights, sensitivity analysis, robustness certification, rank stability, dependency topology, Entropy weighting, CRITIC.

1. Introduction

Multi-criteria decision-making (MCDM) ranks a finite set of alternatives against several, often conflicting, criteria. The final recommendation depends on both observed performances and criterion weights, so sensitivity analysis is central to a credible decision model. A large literature has therefore examined how perturbations in weights,

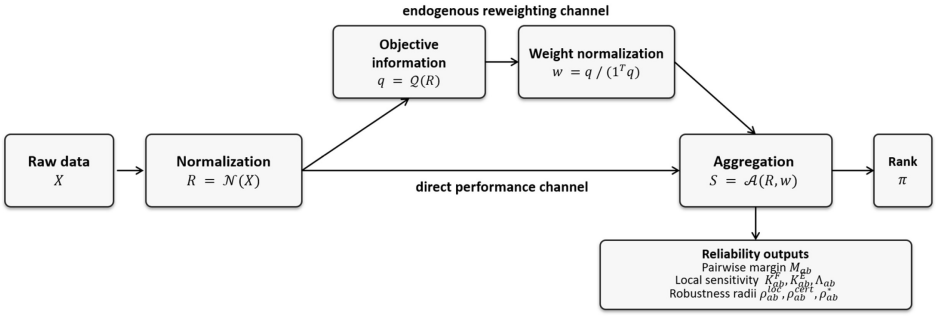


Fig. 1. Endogenous raw-data-to-ranking pipeline and the two perturbation channels.

performances, normalization, and related model components affect rankings (Barron and Schmidt, 1988; Mareschal, 1988; Triantaphyllou and Sánchez, 1997; Więckowski and Sałabun, 2023).

Objective weighting changes the structure of that sensitivity problem. Methods such as standard deviation (SD), Entropy, CRITIC, and MEREC infer criterion importance from the decision matrix itself rather than from an external preference vector (Diakoulaki *et al.*, 1995; Keshavarz-Ghorabaee *et al.*, 2021; Mukhametzyanov, 2021). Because the weights are functions of the performance data, any measurement, estimation, forecasting, or rounding error in the raw matrix propagates into weight re-estimation as well as into the normalized performances. Treating those weights as fixed after their initial calculation therefore analyses a different perturbation mechanism from the one used to construct the original model.

The relevant object is a composite data-driven map. A perturbation of the raw matrix first changes normalized performances and simultaneously changes the information scores from which the objective weights are recalculated. Both effects then enter the aggregation stage before a ranking is produced. Figure 1 makes this dual pathway explicit.

Existing MCDM research already provides strong foundations for weight sensitivity, performance perturbation, stochastic uncertainty, rank-reversal analysis, and robustness measures. Recent work also studies the stability of objective weighting under perturbed data, stochastic objective weights, and robust redesign of weighting schemes. These contributions establish that both performance and data-derived weights can be uncertain. The unresolved question considered here is more specific: how does raw-data uncertainty propagate analytically through an *existing* objective-weighted pipeline when the weights are recalculated from the same perturbed matrix, and how can the resulting ranking be certified over a finite uncertainty region?

This paper answers that question by treating pairwise score margins as the primitive reliability quantities. The composite differential first separates direct performance propagation from the additional effect of endogenous weight re-estimation. That decomposition reveals whether reweighting reinforces a perturbation, opposes it, or is locally neutral. The analysis then distinguishes instability of the weight vector from instability of the final decision, since both arise from the information Jacobian but through different projections.

The framework also moves beyond local sensitivity. A finite raw-data certificate is derived for preservation of pairwise dominance, and the structure of the objective-weighting rule is used to determine how that certification problem decomposes. Information-separable rules such as SD and Entropy admit exact criterion-wise decomposition under Cartesian uncertainty. Correlation-coupled rules such as CRITIC instead produce interacting factors, motivating a hierarchy that trades retained dependency against computational effort.

Accordingly, the aim is not to introduce a new objective-weighting rule or to argue that one weighting principle is intrinsically preferable to another. The paper instead asks a reliability question that arises after a data-derived weighting rule has been selected: when the underlying performance data are perturbed and the weights are recomputed consistently, how does uncertainty propagate through the complete decision mechanism, and what can be guaranteed about the resulting ranking?

The contribution lies in the reliability analysis of the complete data-driven decision pipeline rather than in the design of another weighting method. First, the framework derives the composite response of a decision model in which the same data affect both normalized performances and objective weights, and it separates this response exactly into direct and endogenous components. Second, it distinguishes sensitivity of the estimated weights from sensitivity of the decision itself and connects local perturbation analysis to finite rank-stability guarantees in raw-data space. Third, the information-dependency structure of the weighting rule is shown to govern not only how uncertainty propagates, but also how the certification problem decomposes computationally. SD, Entropy, and CRITIC therefore serve as representative instances of separable and coupled information structures rather than as competing weighting methods; the framework is intended to sit downstream of the weighting-rule selection problem and assess the reliability of the resulting data-driven decision mechanism.

The remainder of the paper is organized as follows. Section 2 positions the contribution against the closest sensitivity, uncertainty, robustness, and objective-weighting literature. Section 3 defines the endogenous framework, Section 4 develops the differential calculus, and Section 5 establishes finite robustness results. Section 6 specializes the theory and provides a fully worked numerical illustration. Section 7 reports computational validation and an external published case. Section 8 discusses the implications and limitations, and Section 9 concludes.

2. Related Work and Research Gap

Sensitivity analysis in MCDM was initially developed around criterion weights and performance evaluations. Barron and Schmidt (1988) studied the smallest changes in additive multiattribute value models that produce indifference or preference reversal, while Mareschal (1988) derived stability intervals for individual criteria and groups of criteria. Triantaphyllou and Sánchez (1997) subsequently examined the sensitivity of WSM, WPM, and AHP to changes in both criterion weights and performance values, and Wolters and

Mareschal (1995) extended additive-model sensitivity analysis to performance changes and minimum weight modifications. More recently, Więckowski and Sałabun (2023) reviewed the broader MCDA sensitivity-analysis literature and confirmed that perturbation analysis remains a central means of assessing the credibility and robustness of decision conclusions. These contributions established that ranking boundaries can be studied systematically, but they generally treat criterion weights and performance evaluations as separately specified model inputs.

Objective weighting changes that structure because the criterion weights are themselves functions of the decision data. Classical examples include CRITIC, which combines dispersion with inter-criterion conflict (Diakoulaki *et al.*, 1995), and information- or dispersion-based approaches such as Entropy and standard deviation (Mukhametzyanov, 2021). MEREC later introduced a removal-effect principle for determining objective criterion importance (Keshavarz-Ghorabaee *et al.*, 2021). The weighting literature has continued to expand considerably. Ayan *et al.* (2023) review several more recent methods, including CILOS, IDOCRIW, FUCOM, LBWA, SAPEVO-M, and MEREC, together with their hybrid and fuzzy extensions. Of particular relevance to data-derived weighting, Zavadskas and Podvezko (2016) introduced CILOS and integrated its criterion-impact-loss logic with Entropy in the IDOCRIW method, whereas Ecer and Pamucar (2022) proposed LOPCOW, which derives objective weights from logarithmic percentage changes in the decision data. These methods broaden the mechanisms through which criterion importance can be inferred from observed data or combined with preference information. Their main methodological question, however, is how criterion weights should be constructed, integrated, or refined; the present study addresses what happens to the decision once a data-dependent weighting mechanism has already been specified.

A second research stream examines uncertainty in MCDM through probabilistic analysis or repeated perturbation. SMAA represents uncertain measurements and preferences by probability distributions and explores the resulting weight and ranking spaces (Lahdelma *et al.*, 1998; Tervonen and Lahdelma, 2007). Mukhametzyanov and Pamucar (2018) perturb decision-matrix entries within prescribed error ranges and use repeated simulation to obtain statistical information about the ranks of alternatives. Więckowski and Sałabun (2024) extend this perspective through COMSAM, which permits simultaneous modification of multiple decision parameters and examines the resulting preference changes over a broader perturbation space. These approaches provide valuable empirical information about robustness, but their principal outputs are distributions, scenarios, or observed ranking responses rather than an analytical decomposition of the complete data-to-weight-to-ranking mechanism.

Recent work has moved closer to uncertainty in data-derived weights. Anagnostopoulou *et al.* (2025) examine the stability of several objective weighting methods, including SD, coefficient of variation, Entropy, CRITIC, and MEREC, under perturbations of normalized decision data. Their results show that the stability of the calculated weights depends on both the weighting rule and the normalization procedure. Roszkowska (2026) investigates the relationship between normalization, entropy as a descriptor of criterion-data structure, and TOPSIS ranking stability, while explicitly distinguishing this use of

entropy from Entropy weighting itself. Guliyev and Deveci (2026) introduce a stochastic Entropy-based weighting model in which Monte Carlo simulation generates distributions rather than single deterministic weights. Li (2026) addresses uncertain decision matrices from a different direction by developing a robust two-stage weight-assignment model based on uncertainty sets and data-driven robust optimization. Pala (2025), in turn, constructs criterion weights with rank stability as an explicit design objective. Collectively, these studies establish that both objective weights and final rankings can be sensitive to uncertainty in the underlying information. Their emphasis, however, is on empirical weight stability, stochastic representation, normalization effects, or the redesign of weights for robustness rather than on the analytical propagation of raw-data perturbations through an already specified objective-weighting rule.

The diversity of contemporary weighting research is also evident in recent hybrid and fuzzy developments. Stevic *et al.* (2026) combine MEREC and FUCOM within the STENOZA decision framework, thereby integrating objective and subjective weighting information. Mehvari and Esmaeili (2026) combine CRITIC with trapezoidal fuzzy TOPSIS and use sensitivity analysis to examine the robustness of a railway-track selection model. Stanujkic *et al.* (2026) develop a bounded fuzzy extension of SWARA aimed at improving the interpretability of subjective weight elicitation, whereas Fan *et al.* (2026) embed CRITIC in a Pythagorean fuzzy group-decision framework that also incorporates pairwise-preference graph neural similarity and TOPSIS. These studies illustrate how weighting, uncertainty representation, and ranking procedures are increasingly combined within richer decision architectures. They are methodologically relevant to the present study because they reinforce the need to examine reliability after a weighting mechanism has been selected, but their primary contribution lies in the construction or integration of decision models rather than in the endogenous propagation of data uncertainty through the weighting mechanism itself.

A further neighbouring literature focuses directly on rank-reversal robustness. Faramondi *et al.* (2023) formulate an optimization problem that identifies the minimum perturbation of pairwise-comparison information capable of reversing an ordinal ranking and derive conditions under which reversal cannot occur within prescribed uncertainty bounds. Górecki *et al.* (2024) complement this deterministic perspective with a probabilistic analysis of rank-reversal robustness under random perturbations of pairwise-comparison matrices. Paradowski *et al.* (2025) introduce explicit ranking-stability and conditioning measures for MCDA, emphasizing that decision robustness and the conditioning of the underlying problem provide information beyond a nominal ranking alone. These studies are closely related to the distinction between local sensitivity and finite robustness adopted here, but they operate on different uncertainty structures and do not consider criterion weights that are recomputed endogenously from the same perturbed performance matrix used for ranking.

Taken together, the literature covers criterion-weight sensitivity, performance perturbation, stochastic uncertainty, modern objective and hybrid weighting, empirical stability of data-derived weights, robust weight design, and minimum-perturbation analysis of rank reversal. The analytical object considered here is different. We take the objective-weighting rule as given and treat the raw decision matrix as the common uncertain input

to both normalized performance evaluation and weight estimation. The resulting data-to-weight-to-ranking pipeline is therefore analysed as a single endogenous map. This makes it possible to separate the direct performance effect from the additional effect generated by weight recomputation, to distinguish sensitivity of the estimated weight vector from sensitivity of the resulting decision, and to connect local differential analysis with finite rank certification through the dependency structure of the information map. Our review did not identify a direct precedent that combines these elements within one framework under systematic weight recomputation. The novelty claimed here is consequently not a new weighting formula, ranking algorithm, or general robustness measure, but a reliability calculus for an existing objective-weighted decision pipeline in which data uncertainty and weight estimation are treated consistently as parts of the same decision mechanism.

3. General Endogenous Decision Framework

3.1. *Decision Setting and Composite Map*

Consider $m \geqslant 2$ alternatives and $n \geqslant 2$ criteria. Let $X = [x_{ij}] \in \mathcal{D}_X \subseteq \mathbb{R}^{m\times n}$ be the raw decision matrix. A normalization operator $\mathcal{N}$ maps X to a normalized matrix $R = [r_1, \ldots, r_n]$; an information operator $\mathcal{Q}$ maps R to nonnegative raw information scores $q = (q_1, \ldots, q_n)^T$; a normalization operator $\mathcal{W}$ maps q to a unit-sum weight vector; and an aggregation operator $\mathcal{A}$ maps (R, w) to alternative scores S.

$$R = \mathcal{N}(X), \qquad q = \mathcal{Q}(R), \qquad Q = \mathbf{1}^T q > 0, \qquad w = \mathcal{W}(q) = \frac{q}{Q}. \tag{1}$$

The complete raw-data-to-score map is

$$F(X) = \mathcal{A}\big(\mathcal{N}(X), \mathcal{W}\big(\mathcal{Q}\big(\mathcal{N}(X)\big)\big)\big). \tag{2}$$

The framework is deliberately modular. The normalization rule, information functional, and aggregation rule may be changed provided their regularity assumptions are respected. The principal pairwise results below use the weighted-sum model (WSM), $S = Rw$, because rank differences then admit a transparent analytical form.

For alternatives a and b, define $v_{ab} = e_a - e_b$ and $d_{ab} = R^T v_{ab}$, whose jth component is $d_{ab,j} = r_{aj} - r_{bj}$. Under WSM,

$$M_{ab} = S_a - S_b = d_{ab}^T w. \tag{3}$$

The sign of M_{ab} determines pairwise preference. Consequently, a rank reversal is a boundary-crossing event $M_{ab} = 0$ (see Table 1).

3.2. *Regularity Domain*

Let X_0 denote the observed matrix. The differential results are stated on an open neighbourhood $\mathcal{O}_X$ in which $\mathcal{N}$, $\mathcal{Q}$, and $\mathcal{A}$ are continuously differentiable and $Q(R) > 0$.

Table 1
Main notation.

Symbol	Meaning
$X \in \mathbb{R}^{m\times n}$	raw decision matrix
$R = \mathcal{N}(X)$	normalized matrix
$q = \mathcal{Q}(R)$	unnormalized objective-information scores
$Q = \mathbf{1}^T q$	total information
$w = q/Q$	normalized objective weights
$S = \mathcal{A}(R, w)$	alternative scores
$v_{ab} = e_a - e_b$	pairwise contrast vector
$d_{ab} = R^T v_{ab}$	criterion-wise performance differences
$M_{ab} = S_a - S_b$	pairwise score margin
$D_R q$	information Jacobian
$\mathbf{1}$	All-ones vector of dimension determined by context

Method-specific restrictions are added when needed. In particular, SD is not classically differentiable at zero dispersion; Entropy derivatives require strictly positive normalized entries in the interior formulation; and CRITIC correlations require positive dispersion for every criterion. Finite robustness regions must remain inside the corresponding regular domain.

3.3. *Composite Differentiability*

Proposition 1 (Composite differentiability). *Suppose that $\mathcal{N}$, $\mathcal{Q}$, and $\mathcal{A}$ are continuously differentiable on the regular domain and $Q > 0$. Then F is continuously differentiable, and*

$$D_X F = [D_R \mathcal{A} + D_w \mathcal{A} D_R w] D_X \mathcal{N}, \tag{4}$$

$$D_R w = \frac{1}{Q}\left(I - w\mathbf{1}^T\right) D_R q. \tag{5}$$

Hence

$$D_X F = \left[D_R \mathcal{A} + \frac{1}{Q} D_w \mathcal{A}\left(I - w\mathbf{1}^T\right) D_R q \right] D_X \mathcal{N}. \tag{6}$$

Proof. Since $w = q/Q$ and $dQ = \mathbf{1}^T dq$, differentiation gives $dw = Q^{-1}(I - w\mathbf{1}^T)dq$. The multivariate chain rule then yields (4)–(6). □

The two terms in (6) have different interpretations. $D_R \mathcal{A} D_X \mathcal{N}$ is the direct performance response obtained when the current weights are locally frozen. The remaining term is the endogenous response generated by re-estimating the objective weights from the perturbed data.

3.4. *Information-Dependency Topology*

Definition 1 (*Information-dependency graph*). Partition R by criterion as $R = [r_1, \ldots, r_n]$. The structural information-dependency graph $\mathcal{G}_q = (V, E)$ has vertices

$V = \{1, \ldots, n\}$ and contains a directed cross-edge $k \to j$ whenever $D_{r_k} q_j$ is not identically zero on the regular domain. At a particular R, the edge is active when $D_{r_k} q_j(R) \neq 0$.

The graph describes dependence of the unnormalized information scores, not independence of the normalized weights. Even when $q_j = q_j(r_j)$, all $w_j = q_j/Q$ remain coupled through the common denominator Q.

Proposition 2 (Graph-Jacobian equivalence). *At any regular R, the active cross-edges of $\mathcal{G}_q(R)$ coincide exactly with the nonzero off-diagonal criterion blocks of $D_R q$.*

Proof. The (j, k) criterion block of $D_R q$ is $D_{r_k} q_j$. The result follows directly from Definition 1. □

A rule with $q_j = q_j(r_j)$ is termed information-separable. SD and Entropy belong to this class. CRITIC is information-coupled because q_j depends on correlations between criterion j and the remaining criteria. This distinction becomes central both for differential propagation and for finite certification.

4. Differential Reliability Calculus

4.1. *Pairwise direct/endogenous decomposition*

The pairwise margin is the primitive reliability quantity because rankings change at its zero level set.

Theorem 1 (Pairwise sensitivity decomposition). *Under WSM and $Q > 0$,*

$$dM_{ab} = w^T d d_{ab} + \frac{1}{Q}(d_{ab} - M_{ab}\mathbf{1})^T dq. \tag{7}$$

Proof. Differentiate $M_{ab} = d_{ab}^T q/Q$, use $dQ = \mathbf{1}^T dq$, $q/Q = w$, and $d_{ab}^T q/Q = M_{ab}$. □

The first term is the direct performance effect. The second is the endogenous reweighting effect. A frozen-weight analysis retains only the first term.

The coefficient attached to dq_j is $(d_{ab,j} - M_{ab})/Q$. Thus, if $d_{ab,j} = M_{ab}$, a perturbation acting only through the information score of criterion j has no first-order endogenous effect on the pairwise margin. This neutrality condition is independent of the particular objective-weighting rule.

For a perturbation of normalized criterion k,

$$\nabla_{r_k} M_{ab} = w_k v_{ab} + \frac{1}{Q}\sum_{j=1}^{n}(d_{ab,j} - M_{ab})\nabla_{r_k} q_j. \tag{8}$$

If the information map is separable, only the $j = k$ term remains. If it is coupled, the sum contains cross-criterion channels. This is the differential meaning of the information-dependency graph.

4.2. *Frozen and Endogenous Local Sensitivity*

Let $\xi \in \mathbb{R}^p$ denote primitive perturbation coordinates, for example, vectorized relative errors in the raw matrix. Let f_{ab} be the gradient of the frozen-weight pairwise margin at $\xi = 0$, and let h_{ab} be the additional gradient generated by endogenous weight recomputation. Then

$$\nabla_\xi M_{ab}(0) = f_{ab} + h_{ab}. \tag{9}$$

For a perturbation norm $\|\cdot\|$, let $\|\cdot\|_{\mathrm{d}}$ denote its dual norm. Define

$$\kappa_{ab}^F = \|f_{ab}\|_{\mathrm{d}}, \qquad \kappa_{ab}^E = \|f_{ab} + h_{ab}\|_{\mathrm{d}}, \qquad \Lambda_{ab} = \frac{\kappa_{ab}^E}{\kappa_{ab}^F}\big(\kappa_{ab}^F > 0\big). \tag{10}$$

The ratio Λ_{ab} is a descriptive consequence of the decomposition, not a new ranking index.

4.3. *Amplification and Attenuation Geometry*

Under Euclidean perturbations, the interaction between the direct and endogenous channels has an exact geometry.

Theorem 2 (Amplification-attenuation geometry). *Assume* $\|f_{ab}\|_2 > 0$ *and* $h_{ab} \neq 0$. *Define* $\eta_{ab} = \|h_{ab}\|_2/\|f_{ab}\|_2$ *and* $\gamma_{ab} = \langle f_{ab}, h_{ab}\rangle/(\|f_{ab}\|_2\|h_{ab}\|_2)$. *Then*

$$\Lambda_{ab,2}^2 = 1 + 2\eta_{ab}\gamma_{ab} + \eta_{ab}^2. \tag{11}$$

Therefore amplification occurs if $\gamma_{ab} > -\eta_{ab}/2$, *attenuation if* $\gamma_{ab} < -\eta_{ab}/2$, *and norm-neutrality if equality holds. Complete first-order cancellation occurs exactly when* $h_{ab} = -f_{ab}$.

Proof. Expand $\|f_{ab} + h_{ab}\|_2^2$ and divide by $\|f_{ab}\|_2^2$. □

For componentwise relative perturbations, $\|\xi\|_\infty \leqslant \rho$, the dual norm is l_1 and amplification is assessed through $\|f + h\|_1 - \|f\|_1$. No universal angle representation exists for l_1.

4.4. *Local Reversal Approximation*

If $M_{ab}(0) > 0$, first-order expansion gives $M_{ab}(\xi) = M_{ab}(0) + \nabla_\xi M_{ab}(0)^T\xi + o(\|\xi\|)$. The corresponding local reversal approximation is

$$\rho_{ab}^{\mathrm{loc},E} = \frac{M_{ab}(0)}{\kappa_{ab}^E}, \qquad \rho_{ab}^{\mathrm{loc},F} = \frac{M_{ab}(0)}{\kappa_{ab}^F}. \tag{12}$$

These are tangent approximations, not finite guarantees. Winner-level local robustness requires the minimum over all competitors, so attenuation against the nominal runner-up does not imply improved winner robustness if another competitor becomes critical.

4.5. *Conditioning Transfer*

The same information Jacobian governs both weight sensitivity and the endogenous part of pairwise decision sensitivity, but through different projections.

Proposition 3 (Conditioning Transfer Principle). *On a regular domain,*

$$D_R w = \frac{1}{Q} P_w D_R q, \qquad P_w = I - w\mathbf{1}^T, \tag{13}$$

$$D_R M_{ab}^{\text{endo}} = \frac{1}{Q}(d_{ab} - M_{ab}\mathbf{1})^T D_R q. \tag{14}$$

Hence a large or diverging $\|D_R w\|$ does not by itself imply a comparable divergence of the pairwise decision derivative.

To make the asymptotic statement precise, suppose along a path $\varepsilon \downarrow 0$ that $Q \sim c_Q \varepsilon^{\alpha}$, $\|P_w D_R q\| \sim c_W \varepsilon^{\beta}$, and $\|(d_{ab} - M_{ab}\mathbf{1})^T D_R q\| \sim c_M \varepsilon^{\gamma}$, with nonzero leading constants. Then

$$\|D_R w\| \sim \frac{c_W}{c_Q}\varepsilon^{\beta-\alpha}, \qquad \left\|D_R M_{ab}^{\text{endo}}\right\| \sim \frac{c_M}{c_Q}\varepsilon^{\gamma-\alpha}. \tag{15}$$

If a leading projected coefficient cancels, only the corresponding $O(\cdot)$ statement is justified until the next nonzero term is identified. This qualification prevents weight-conditioning results from being transferred mechanically to decision conditioning.

5. Certified Decision Robustness

5.1. *Raw-Data Uncertainty Model*

Let X_0 be the observed matrix and write

$$X(E) = X_0 + H \odot E, \qquad \|E\| \leqslant \rho, \tag{16}$$

where $H \geqslant 0$ is a scaling matrix. Setting $H = |X_0|$ gives componentwise relative perturbations. Let $\mathcal{U}_\rho(X_0)$ denote the resulting uncertainty region. The entire region is assumed to remain within the regular domain of the selected normalization and weighting rules.

For pair (a, b) define the finite-region derivative bound

$$L_{ab}(\rho) = \max_{\|E\| \leqslant \rho} \left\|\nabla_E M_{ab}\big(X(E)\big)\right\|_{\mathrm{d}}. \tag{17}$$

5.2. *Finite Deterministic Certificate*

Theorem 3 (Finite rank-stability certificate). *Suppose the perturbation ball is convex, the pairwise margin is continuously differentiable throughout the corresponding regular*

uncertainty region, and $M_{ab}(X_0) > 0$. *Then*

$$\left|M_{ab}\big(X(E)\big) - M_{ab}(X_0)\right| \leqslant \rho L_{ab}(\rho), \quad \forall \|E\| \leqslant \rho. \tag{18}$$

Consequently, if

$$M_{ab}(X_0) > \rho L_{ab}(\rho), \tag{19}$$

then $M_{ab}(X(E)) > 0$ *throughout the uncertainty region. If* (19) *holds for every* $b \neq a$, *alternative* a *remains the unique winner.*

Proof. For fixed E, apply the fundamental theorem of calculus to $\phi(t) = M_{ab}(X(tE))$, $t \in [0, 1]$, and use Hölder's inequality together with the definition of $L_{ab}(\rho)$. □

The derivative-based certificate and the true minimum reversal radius are

$$\begin{aligned} \rho_a^L &= \sup\big\{\rho : M_{ab}(X_0) > \rho L_{ab}(\rho)\, \forall b \neq a\big\}, \\ \rho_a^* &= \min_{b \neq a} \inf\big\{\|E\| : M_{ab}\big(X(E)\big) \leqslant 0\big\}. \end{aligned} \tag{20}$$

Whenever the conditions of Theorem 3 hold on the relevant uncertainty region, $\rho_a^L \leqslant \rho_a^*$. More generally, any radius established as safe by Theorem 3, Theorem 4, or a valid factor bound from Theorem 5 is a certified lower bound on ρ_a^*. The local radius in (12) has no universal ordering relative to ρ_a^*.

5.3. *Exact Certification Under Information Separability*

The Lipschitz certificate is general but can be conservative. Under additive aggregation and information separability, the sign problem simplifies substantially. If $q_j = q_j(r_j)$, define

$$\phi_{ab,j}(r_j) = q_j(r_j) d_{ab,j}(r_j), \qquad N_{ab}(R) = \sum_{j=1}^{n} \phi_{ab,j}(r_j). \tag{21}$$

Because $M_{ab} = N_{ab}/Q$ and $Q > 0$, the signs of M_{ab} and N_{ab} coincide.

Theorem 4 (Exact separable certification). *Assume criterion-wise normalization,* $q_j = q_j(r_j)$, *a Cartesian uncertainty set* $\mathcal{U} = \mathcal{U}_1 \times \cdots \times \mathcal{U}_n$, *continuity of each* $\phi_{ab,j}$ *on compact* $\mathcal{U}_j$, *and* $Q > 0$ *throughout* $\mathcal{U}$. *Then*

$$\min_{R \in \mathcal{U}} N_{ab}(R) = \sum_{j=1}^{n} \min_{r_j \in \mathcal{U}_j} \phi_{ab,j}(r_j). \tag{22}$$

Thus a *dominates* b *throughout* $\mathcal{U}$ *if and only if the right-hand side of* (22) *is strictly positive.*

Proof. Each factor is bounded below by its own minimum, giving one inequality. Compactness and continuity ensure that each minimum is attained; Cartesian independence makes the tuple of individual minimizers jointly feasible, giving equality. □

The theorem converts an mn-variable worst-case problem into n independent subproblems of at most m variables each.

5.4. *Coupled Factor Certification*

Suppose instead that the pairwise numerator admits a factorization

$$N_{ab}(R) = \sum_{e\in\mathcal{E}} \psi_e(R_{V_e}), \tag{23}$$

where factor e depends only on criterion variables indexed by V_e. Let $\mathcal{P} = \{B_1, \ldots, B_L\}$ be a partition of the factors and define

$$L_{\mathcal{P}} = \sum_{l=1}^{L} \min_{R_{V(B_l)}\in\mathcal{U}_{V(B_l)}} \left\{ \sum_{e\in B_l} \psi_e(R_{V_e}) \right\}, \qquad V(B_l) = \bigcup_{e\in B_l} V_e. \tag{24}$$

Theorem 5 (Factor-certification hierarchy). *For any factor partition,*

$$L_{\mathcal{P}} \leqslant \min_{\mathcal{U}} N_{ab}. \tag{25}$$

If $\mathcal{P}_2$ is obtained by merging blocks of a finer partition $\mathcal{P}_1$, then

$$L_{\mathcal{P}_1} \leqslant L_{\mathcal{P}_2} \leqslant \min_{\mathcal{U}} N_{ab}. \tag{26}$$

A single full block gives equality.

Proof. For every feasible point, each block sum is at least its blockwise minimum. Summing yields (25). Joint minimization of two merged blocks is at least the sum of their separate minima, giving monotonicity under merging.

The result provides a certification-complexity hierarchy: more retained dependency produces a tighter bound at a higher computational cost.

For larger decision matrices, the certification stage can be organized hierarchically rather than solved as one full-dimensional global problem. A practical implementation first evaluates the nominal winner, all winner–competitor margins, and the corresponding local sensitivity measures. Pairs that can already be certified by a coarse derivative or factor bound require no further refinement; validated global optimization is reserved for the unresolved or near-critical comparisons. For information-separable rules, Theorem 4 reduces each remaining pairwise problem to n independent subproblems of at most m variables, which can be solved in parallel. For coupled rules, Theorem 5 suggests beginning with a fine factor partition and merging only those blocks for which the lower bound

is insufficient. The monotonicity of the hierarchy ensures that each merge can only tighten the bound. This staged procedure does not remove the intrinsic growth of validated global optimization with subproblem dimension, but it confines the most expensive calculations to the comparisons and dependency blocks that actually determine the certificate. □

6. Model-Class Instantiations

All three instantiations use positive column-sum normalization. For criterion j, let $c_j = \mathbf{1}^T x_j$ and

$$r_j = \frac{x_j}{c_j}, \qquad D_{x_j} r_j = \frac{1}{c_j}\big(I - r_j \mathbf{1}^T\big). \tag{27}$$

This normalization is invariant to positive rescaling of an entire criterion column.

6.1. *Standard-Deviation weighting*

Let $u = m^{-1}\mathbf{1}$ and

$$q_j^{SD} = \sigma_j = \frac{\|r_j - u\|_2}{\sqrt{m}}. \tag{28}$$

On the smooth domain $\sigma_j > 0$,

$$\nabla_{r_j}\sigma_j = \frac{r_j - u}{m\sigma_j}, \qquad \nabla_{x_j}\sigma_j = \frac{1}{c_j}\left[\frac{r_j - u}{m\sigma_j} - \sigma_j \mathbf{1}\right]. \tag{29}$$

SD is information-separable. Substituting (29) into (8) gives

$$\nabla_{r_j} M_{ab} = w_j v_{ab} + \frac{d_{ab,j} - M_{ab}}{Q_{SD}} \frac{r_j - u}{m\sigma_j}. \tag{30}$$

At exact uniformity, SD is continuous but directionally nondifferentiable. The apparent $1/\sigma_j$ term does not imply a divergent gradient norm along a fixed approach direction because $\|(r_j - u)/(m\sigma_j)\|_2 = 1/\sqrt{m}$.

For $r_j = u + \varepsilon\delta_j$, $\mathbf{1}^T\delta_j = 0$,

$$q_j^{SD} = \frac{\varepsilon}{\sqrt{m}}\|\delta_j\|_2, \qquad w_j^{SD} \to \frac{\|\delta_j\|_2}{\sum_k \|\delta_k\|_2}. \tag{31}$$

The rank numerator is separable with $\phi_{ab,j}^{SD} = \sigma_j d_{ab,j}$, so Theorem 4 applies directly.

6.2. *Entropy weighting*

For strictly positive normalized entries, define normalized Shannon entropy and information divergence by

$$H_j = -\frac{1}{\ln m}\sum_{i=1}^{m} r_{ij}\ln r_{ij}, \qquad q_j^{Ent} = 1 - H_j = \frac{D_{KL}(r_j\|u)}{\ln m}. \tag{32}$$

Let $k_m = 1/\ln m$ and $L_j = \sum_i r_{ij}\ln r_{ij}$. Then

$$\nabla_{r_j} q_j^{Ent} = k_m(\ln r_j + \mathbf{1}), \qquad \nabla_{x_j} q_j^{Ent} = \frac{k_m}{c_j}(\ln r_j - L_j\mathbf{1}). \tag{33}$$

Entropy is information-separable and smooth at the uniform point when restricted to the simplex interior. The simplex-tangent derivative vanishes at $r_j = u$. Near uniformity,

$$q_j^{Ent} = \frac{m}{2\ln m}\varepsilon^2\|\delta_j\|_2^2 - \frac{m^2}{6\ln m}\varepsilon^3\sum_i \delta_{ij}^3 + O\big(\varepsilon^4\big). \tag{34}$$

Consequently,

$$w_j^{Ent} \to \frac{\|\delta_j\|_2^2}{\sum_k \|\delta_k\|_2^2}. \tag{35}$$

Thus, SD and Entropy have different information geometry near complete nondiscrimination: SD information is first order in departure magnitude, whereas Entropy information is second order. Nevertheless, both normalized weight maps are generically $O(\varepsilon^{-1})$-sensitive near global uniformity, while the final pairwise decision derivative may remain bounded by Proposition 3.

The closed-simplex Entropy score is continuous under the convention $0\ln 0 = 0$, but the classical logarithmic derivative is singular at zero components. A strictly positive baseline combined with relative perturbations $\rho < 1$ preserves positivity.

6.3. *CRITIC weighting*

Following Diakoulaki *et al.* (1995), CRITIC combines within-criterion dispersion and cross-criterion conflict.

Let $P = I - m^{-1}\mathbf{1}\mathbf{1}^T$, $z_j = Pr_j$, $s_j = \|z_j\|_2$, $\sigma_j = s_j/\sqrt{m}$, and $a_j = z_j/s_j$. For $\sigma_j > 0$,

$$\varrho_{jk} = a_j^T a_k, \qquad q_j^C = \sigma_j\sum_{k\neq j}(1-\varrho_{jk}). \tag{36}$$

The correlation derivatives are

$$\nabla_{r_j}\varrho_{jk} = \frac{a_k - \varrho_{jk}a_j}{s_j}, \qquad \nabla_{r_k}\varrho_{jk} = \frac{a_j - \varrho_{jk}a_k}{s_k}. \tag{37}$$

Writing $g_{jk} = \nabla_{r_k} q_j^C$, the information-Jacobian blocks simplify to

$$g_{jj} = \frac{1}{\sqrt{m}}\left[(n-1)a_j - \sum_{l\neq j} a_l\right], \tag{38}$$

$$g_{jk} = -\frac{1}{\sqrt{m}}\frac{\sigma_j}{\sigma_k}(a_j - \varrho_{jk}a_k), \quad j \neq k. \tag{39}$$

Hence,

$$\|g_{jk}\|_2 = \frac{1}{\sqrt{m}}\frac{\sigma_j}{\sigma_k}\sqrt{1-\varrho_{jk}^2}. \tag{40}$$

The cross block can diverge when $\sigma_k \to 0$ while criterion j remains dispersed and the two standardized directions do not become nearly collinear. The instability is angular: $g_{jk}^T a_k = 0$, so radial changes in criterion k do not generate the singular first-order correlation response.

At the decision level,

$$\nabla_{r_k} M_{ab} = w_k v_{ab} + \frac{1}{Q_C}\sum_{j=1}^{n}(d_{ab,j} - M_{ab})g_{jk}. \tag{41}$$

The potentially singular cross-information terms may cancel after projection through the pairwise score geometry, illustrating the Conditioning Transfer Principle.

CRITIC is not criterion-separable, but its pairwise numerator has a natural edge factorization:

$$N_{ab}^C = \sum_{j<k}(1-\varrho_{jk})(\sigma_j d_{ab,j} + \sigma_k d_{ab,k}). \tag{42}$$

Thus, Theorem 5 applies with criterion-pair factors. CRITIC also has a global degeneracy absent from SD and Entropy: if all pairwise correlations equal $+1$, then every conflict term vanishes and $Q_C = 0$ even when all criteria have positive dispersion.

6.4. *Worked Numerical Illustration: from Raw Data to Robustness*

The preceding sections are compact by design, but the composition of normalization, information extraction, weight recomputation, and pairwise sensitivity can be difficult to visualize from formulas alone. This subsection therefore follows one small problem from the raw matrix to the robustness interpretation. SD weighting is used because its information map is separable and its derivatives are explicit. The purpose is pedagogical rather than evidential; the same matrix is reused in Section 7 as the amplification benchmark, where the other weighting structures are compared on equal terms. Each step below identifies the equation from the general theory that performs the calculation.

Consider three alternatives evaluated on three benefit criteria:

$$X_0 = \begin{bmatrix} 56.3197 & 21.5800 & 38.3211 \\ 23.3562 & 30.7321 & 26.4973 \\ 20.3241 & 47.6879 & 35.1817 \end{bmatrix}.$$

Step 1. Normalize the raw data
Column-sum normalization gives $r_j = x_j/c_j$, where $c_j = \mathbf{1}^T x_j$. The column sums are 100, 100, and 100.0001, respectively, and Eq. (27) gives

$$R = \begin{bmatrix} 0.563197 & 0.215800 & 0.383211 \\ 0.233562 & 0.307321 & 0.264973 \\ 0.203241 & 0.476879 & 0.351817 \end{bmatrix}.$$

The normalization is criterion-wise, so a common positive rescaling of an entire raw column does not change the corresponding normalized profile.

Step 2. Compute the SD information scores
With $m = 3$ and $u = (1/3, 1/3, 1/3)^T$, Eq. (28) gives

$$q^{SD} = (\sigma_1, \sigma_2, \sigma_3)^T = (0.163009, 0.108161, 0.050008)^T,$$

and therefore

$$Q = \mathbf{1}^T q = 0.321178.$$

Criterion 1 has the largest normalized dispersion and consequently receives the largest raw information score.

Step 3. Normalize the information scores into weights
Using $w = q/Q$ from Eq. (1),

$$w = (0.507535, 0.336762, 0.155703)^T.$$

At this point the key modelling issue becomes visible: these weights are not external constants. They were generated from X_0 through R and q, so a perturbation of the raw matrix generally changes both the normalized performances and the weights.

Step 4. Calculate scores and identify the closest score competitor
Under WSM, $S = Rw$, which gives

$$S = (0.418182, 0.263292, 0.318526)^T.$$

Hence,

$$A_1 > A_3 > A_2.$$

The closest competitor to the winner is A_3. For this pair,

$$d_{13} = R^T(e_1 - e_3) = (0.359956, -0.261079, 0.031394)^T,$$

and Eq. (3) gives

$$M_{13} = d_{13}^T w = S_1 - S_3 = 0.099657 > 0.$$

The sign of this margin, rather than either score in isolation, is the quantity that must be protected against perturbation.

Step 5. Inspect the endogenous leverage of each criterion
Theorem 1 shows that the weight-reestimation channel is scaled by $d_{13} - M_{13}\mathbf{1}$. Numerically,

$$d_{13} - M_{13}\mathbf{1} = (0.260299, -0.360736, -0.068263)^T.$$

This vector has an immediate interpretation. Increasing the information importance of criterion 1 tends to reinforce A_1 because $d_{13,1} > M_{13}$. Increasing the relative importance of criteria 2 or 3 tends to reduce the margin because their pairwise advantages lie below the current aggregate margin. Equation (7) therefore explains *why* weight recomputation can either reinforce or oppose the direct performance effect.

Step 6. Differentiate with respect to relative raw-data errors
Use the raw-data uncertainty model in Eq. (16) with $H = X_0$, so that $X(E) = X_0 \odot (1 + E)$. Combining the normalization derivative in Eq. (27), the SD derivative in Eq. (29), and the pairwise decomposition in Eq. (7) gives the following gradients with respect to the nine relative-error coordinates.

The frozen-weight component is

$$G^F = \nabla_E M_{13}^F = \begin{bmatrix} 0.182951 & 0.091647 & 0.057794 \\ -0.042669 & 0.027020 & -0.001295 \\ -0.140282 & -0.118667 & -0.056499 \end{bmatrix}.$$

The endogenous reweighting correction is

$$G^{endo} = \begin{bmatrix} 0.140144 & 0.114010 & -0.023005 \\ -0.069475 & 0.065005 & 0.028478 \\ -0.070669 & -0.179015 & -0.005473 \end{bmatrix},$$

and the full endogenous gradient is

$$G^E = G^F + G^{endo} = \begin{bmatrix} 0.323096 & 0.205657 & 0.034789 \\ -0.112145 & 0.092025 & 0.027183 \\ -0.210951 & -0.297682 & -0.061972 \end{bmatrix}.$$

For example, the negative entry associated with the third alternative under criterion 2 means that increasing that raw observation decreases M_{13} and therefore moves the decision toward A_3. The gradient matrix makes the adverse data directions explicit rather than treating uncertainty as an abstract scalar percentage.

Step 7. Compare frozen and endogenous local sensitivity
For componentwise relative uncertainty, $\|E\|_\infty \leqslant \rho$, the dual norm in Eq. (10) is l_1. Because the gradients are displayed as 3×3 arrays only for readability, their l_1 norms below are the entrywise norms of the corresponding vectorized gradients. Thus,

$$\kappa_{13}^{F} = \left\|\mathrm{vec}\left(G^{F}\right)\right\|_1 = 0.718825, \qquad \kappa_{13}^{E} = \left\|\mathrm{vec}\left(G^{E}\right)\right\|_1 = 1.365499.$$

Using the ratio defined in Eq. (10),

$$\Lambda_{13} = \frac{\kappa_{13}^{E}}{\kappa_{13}^{F}} = 1.8996.$$

The complete model is therefore about 90% more locally sensitive than the frozen-weight model for this pair. The difference is created entirely by the weight-reestimation channel; the raw matrix and the aggregation rule are otherwise unchanged.

Step 8. Translate sensitivity into a local reversal radius
Equation (12) converts the baseline margin and local sensitivity into a first-order distance to the pairwise boundary:

$$\rho_{13}^{loc,F} = \frac{0.099657}{0.718825} = 0.13864,$$

whereas

$$\rho_{13}^{loc,E} = \frac{0.099657}{1.365499} = 0.07298.$$

Thus, a frozen-weight analysis suggests a local tolerance of about 13.9%, whereas complete endogenous recomputation reduces the corresponding local estimate to about 7.3%. The local radius is still a tangent approximation; it is not yet a finite guarantee.

Step 9. Recompute the complete model under a finite 5% perturbation
To see the mechanism directly, take a 5% perturbation in the locally most adverse sign direction,

$$E^{\mathrm{adv}} = -0.05\mathrm{sign}\left(G^{E}\right) = 0.05\begin{bmatrix} -1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Equation (16) gives

$$X\left(E^{\mathrm{adv}}\right) = \begin{bmatrix} 53.503715 & 20.501000 & 36.405045 \\ 24.524010 & 29.195495 & 25.172435 \\ 21.340305 & 50.072295 & 36.940785 \end{bmatrix}.$$

Under the frozen-weight approximation, the recomputed normalized performances give

$$M_{13}^{F}\left(E^{\mathrm{adv}}\right) = 0.063616.$$

If the SD weights are recomputed consistently from the perturbed matrix, they change to

$$w\left(E^{\mathrm{adv}}\right) = (0.447962, 0.382619, 0.169419)^{T},$$

and the actual endogenous margin falls to

$$M_{13}\left(E^{\mathrm{adv}}\right) = 0.030667.$$

The mechanism is transparent. Criterion 1, which strongly favours A_1, loses weight from 0.5075 to 0.4480, while criterion 2, which favours A_3, gains weight from 0.3368 to 0.3826. Reweighting therefore reinforces the adverse direct perturbation.

The local calculus also predicts this finite calculation closely. The first-order endogenous estimate is

$$M_{13}(0) - 0.05\kappa_{13}^{E} = 0.031382,$$

compared with the fully recomputed value 0.030667. The frozen approximation is 0.063716, compared with the exact frozen-path value 0.063616. This agreement explains why the local calculus is diagnostically useful while also motivating the finite certification step.

Step 10. Separate a numerical boundary from a certificate

Continuing full recomputation along the adverse direction in Step 9 places the zero crossing at approximately $\rho = 0.072102$. This construction supplies an upper bound on the true reversal radius, but by itself does not exclude a smaller perturbation elsewhere in the uncertainty box.

A finite guarantee requires a lower bound valid over the entire box. Because SD is information-separable, Eq. (22) permits criterion-wise certification. We evaluated each three-dimensional criterion-error box by adaptive branch-and-bound with outward-rounded interval arithmetic. At $\rho = 0.07210$, rigorous lower enclosures for the three factors of N_{13} are

$$\underline{\phi}_{13,1} = 0.04239645, \qquad \underline{\phi}_{13,2} = -0.04109876, \qquad \underline{\phi}_{13,3} = -0.00129673,$$

which give

$$\underline{N}_{13} = 9.6 \times 10^{-7} > 0.$$

The corresponding validated lower bound for the second competitor is $\underline{N}_{12} = 0.0249066 > 0$. Hence, A_1 remains the unique winner for every componentwise relative perturbation satisfying $\|E\|_\infty \leqslant 0.07210$. Conversely, the explicit adverse direction from Step 9 produces $M_{13} < 0$ at $\rho = 0.07211$. The winner-level reversal radius is therefore tightly bracketed by

$$0.07210 < \rho_1^* \leqslant 0.07211.$$

This calculation illustrates the distinction between a local prediction and a guarantee. Equation (12) predicts the boundary from one tangent calculation; Eqs. (21)–(22), combined with validated interval bounds, exclude every possible reversal below the certified level.

The decision interpretation is immediate. The frozen-weight model suggests a local tolerance of about 13.9%, whereas consistent weight recomputation lowers the local estimate to about 7.3%. The validated calculation sharpens this diagnosis: A_1 is guaranteed to remain the unique winner for every admissible componentwise relative perturbation up to 7.210%, while the explicit adverse perturbation reverses the $A_1 - A_3$ ordering at 7.211%. Thus, the framework answers two distinct practical questions: when should instability be expected, and how much data error can be guaranteed not to change the decision?

7. Numerical and Computational Validation

The experiments are designed to test specific claims from the theory rather than to compare weighting methods by nominal ranking performance.

7.1. *Experimental Protocol*

Unless noted otherwise, all experiments use positive matrices, column-sum normalization, and componentwise relative perturbations

$$X(E) = X_0 \odot (1 + E), \qquad \|E\|_\infty \leqslant \rho < 1. \tag{43}$$

The dual norm is therefore l_1. Analytical derivatives are evaluated with respect to the relative-error coordinates. Computations were implemented in Python using NumPy and SciPy. Numerical boundary searches combine vertex screening, differential evolution, and local polishing with fixed random seeds; such results are reported as numerical estimates or explicit feasible reversal paths, not as formal certificates. The finite guarantee in Section 6.4 was checked separately by adaptive branch-and-bound with outward-rounded interval arithmetic over the complete relative-error box. For a larger instance, applying validated branch-and-bound directly to every winner–competitor pair over the full mn-dimensional uncertainty box would generally be unnecessary and may become computationally prohibitive. We therefore distinguish between a low-cost diagnostic stage and a

selective certification stage. First, the nominal scores, pairwise margins, analytical derivatives, and local reversal radii are computed for all competitors of the nominal winner. Second, inexpensive sufficient bounds are used to identify comparisons that are already safely separated from zero. Only unresolved comparisons are passed to validated finite-region optimization. For SD and Entropy, criterion separability permits those calculations to be divided into independent criterion-level subproblems. For CRITIC and other coupled rules with a sparse factor representation, certification can begin from small factor blocks and increase the retained dependency only where required by the bound. The expensive part of the computation is therefore the validated global minimization used for finite certification, rather than the local differential analysis itself. The approach remains exact whenever the reported lower bounds are globally valid; local or heuristic searches are retained only as diagnostics or feasible upper-bound constructions.

7.2. *Analytical Derivative Verification*

The complete analytical score Jacobian from Eqs. (4)–(6) was compared with centred finite differences using $h = 10^{-6}$. For each dimension, 20 positive square matrices were sampled from $U(0.5, 5)$ using random seed 20260917. Table 2 reports the maximum absolute discrepancy over all score and perturbation coordinates.

Table 2
Maximum absolute analytical-finite-difference Jacobian discrepancy over 20 random matrices per size.

Matrix size	SD	Entropy	CRITIC
3×3	1.11×10^{-10}	4.49×10^{-10}	1.02×10^{-10}
5×5	5.22×10^{-11}	1.15×10^{-10}	6.32×10^{-11}
10×10	2.97×10^{-11}	4.73×10^{-11}	3.70×10^{-11}

All discrepancies remain below 5×10^{-10}. The CRITIC agreement is particularly informative because it simultaneously checks the own-criterion and cross-criterion derivative blocks.

7.3. *Matched Amplification, Near-Neutrality, and Local Attenuation*

Let $X^{(A)}$ denote the worked matrix X_0 from Section 6.4. To contrast it with a near-neutral configuration, consider

$$X^{(N)} = \begin{bmatrix} 47.9119 & 45.4892 & 48.1508 \\ 33.7097 & 31.5905 & 33.3368 \\ 18.3783 & 22.9202 & 18.5125 \end{bmatrix}. \tag{44}$$

For each method, the local quantities in Table 3 follow directly from Eqs. (9)–(12).

Table 3
Matched local amplification and near-neutrality.

Matrix	Method	Critical local pair	Λ	$\rho^{\mathrm{loc},F}$	$\rho^{\mathrm{loc},E}$
$X^{(A)}$	SD	$A_1 - A_3$	1.900	0.13864	0.07298
	Entropy	$A_1 - A_3$	2.779	0.21476	0.07729
	CRITIC	$A_1 - A_3$	2.556	0.08946	0.03500
$X^{(N)}$	SD	$A_1 - A_2$	0.9984	0.17687	0.17715
	Entropy	$A_1 - A_2$	0.9973	0.17682	0.17731
	CRITIC	$A_1 - A_2$	0.9986	0.17665	0.17689

The following constructive matrices are included to establish the existence of local attenuation under each weighting structure; they are illustrative counterexamples rather than estimates of how frequently attenuation occurs.

$$X^{(SD)} = \begin{bmatrix} 0.29677 & 59.98221 & 22.36097 \\ 49.16214 & 10.04134 & 35.51563 \\ 50.54109 & 29.97645 & 42.12340 \end{bmatrix},$$

$$X^{(EN)} = \begin{bmatrix} 0.256029 & 67.089837 & 22.919704 \\ 53.122806 & 6.540279 & 36.950763 \\ 46.621165 & 26.369883 & 40.129533 \end{bmatrix},$$

$$X^{(CR)} = \begin{bmatrix} 41.003307 & 46.517209 & 7.680462 \\ 12.341729 & 46.281712 & 44.237561 \\ 46.654964 & 7.201079 & 48.081977 \end{bmatrix}.$$

Table 3 shows not only whether endogenous weight recomputation matters, but also the magnitude of its effect on local decision reliability. For the amplification matrix $X^{(A)}$, the values $\Lambda = 1.900$, 2.779, and 2.556 for SD, Entropy, and CRITIC indicate that recomputing the weights increases the local pairwise sensitivity by approximately 90.0%, 177.9%, and 155.6%, respectively, relative to the frozen-weight approximation. The associated local reversal radii fall from 0.13864 to 0.07298 for SD, from 0.21476 to 0.07729 for Entropy, and from 0.08946 to 0.03500 for CRITIC. Thus, ignoring weight recomputation would overstate the local tolerance to data error in this example. By contrast, for the near-neutral matrix $X^{(N)}$, all three values of Λ lie within 0.3% of unity, so recomputing the weights has almost no first-order effect on the inferred robustness. The explicit adverse perturbation paths reach the $A_1 - A_3$ boundary at approximately 0.072102 for SD, 0.07751 for Entropy, and 0.03532 for CRITIC. These paths establish feasible reversals and hence upper bounds on the corresponding true reversal radii; they are not finite robustness certificates.

The attenuation examples in Table 4 illustrate the opposite mechanism. Here, $\Lambda < 1$, meaning that the endogenous reweighting response partially offsets the direct performance response and therefore increases the corresponding local reversal radius. This effect should be interpreted as pair-specific and local: attenuation for one comparison does not imply that the nominal winner is globally more robust unless all competing alternatives are also checked.

Table 4
Local attenuation examples.

Method	Matrix	Winner-critical competitor	Λ	$\rho^{\mathrm{loc},F}$	$\rho^{\mathrm{loc},E}$
SD	$X^{(SD)}$	$A_3 - A_2$	0.9516	0.12514	0.13151
Entropy	$X^{(EN)}$	$A_3 - A_2$	0.9392	0.06532	0.06955
CRITIC	$X^{(CR)}$	$A_2 - A_3$	0.8398	0.05687	0.06771

Table 5
Near-uniform weight and decision conditioning.

Method	ε	$\|D_E w\|_2$	$\|\nabla_E M_{12}\|_2$
SD	10^{-1}	1.0980	0.4068
SD	10^{-2}	11.2270	0.3912
SD	5×10^{-4}	225.1331	0.3890
SD	10^{-4}	1125.7958	0.3889
Entropy	10^{-1}	2.2210	0.5565
Entropy	10^{-2}	23.6546	0.5504
Entropy	5×10^{-4}	474.6852	0.5486
Entropy	10^{-4}	2373.7311	0.5485

7.4. *Weight Conditioning Versus Decision Conditioning*

To test Proposition 3, define $R(\varepsilon) = u\mathbf{1}^T + \varepsilon D$ with $u = (1/3, 1/3, 1/3)^T$ and $D = [\delta_1, \delta_2, \delta_3]$, where

$$\delta_1 = \frac{(1, -1, 0)^T}{\sqrt{2}}, \qquad \delta_2 = 0.8\frac{(1, 0, -1)^T}{\sqrt{2}}, \qquad \delta_3 = 1.2\frac{(0.2, 0.8, -1)^T}{\sqrt{1.68}}. \tag{45}$$

The two reported conditioning quantities correspond directly to the projections in Eqs. (13)–(15). As $\varepsilon \downarrow 0$, the matrix approaches complete nondiscrimination.

The weight-map sensitivity grows at approximately the predicted $O(\varepsilon^{-1})$ rate, while the pairwise decision sensitivity remains bounded. Figure 2 makes the separation visible.

Large percentage changes in objective weights should therefore not be treated as a failure signal by themselves; the relevant reliability quantity is the decision margin onto which those changes project.

7.5. *Certified Safety and Finite Boundary Behaviour Under Separable weighting*

The SD worked example provides both a finite guarantee and a numerical failure boundary. The interval calculation in Section 6.4 proves that A_1 is the unique winner throughout the complete box $\|E\|_\infty \leqslant 0.07210$, while an explicit feasible perturbation reverses the $A_1 - A_3$ order at $\rho = 0.07211$. Thus,

$$0.07210 < \rho_1^* \leqslant 0.07211.$$

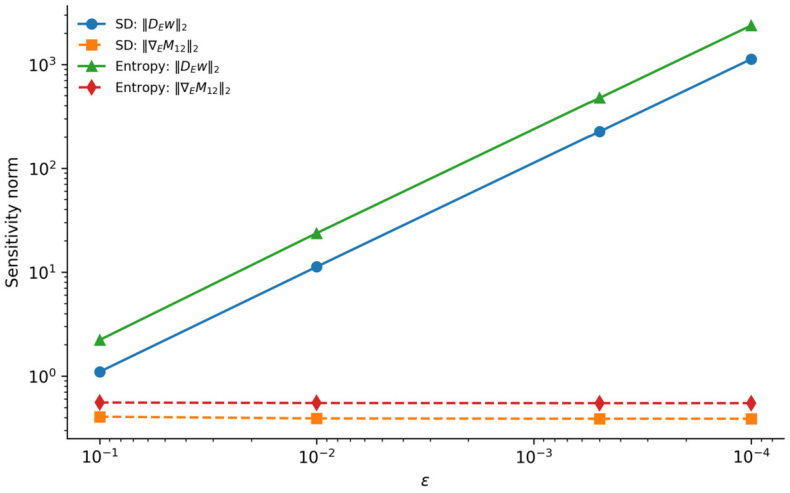


Fig. 2. Near-uniform conditioning. Weight sensitivity diverges while pairwise decision sensitivity remains bounded.

Table 6
Local approximation versus numerical finite boundary for the amplification matrix.

Method	Weight treatment	ρ^{loc}	Numerical boundary $\hat{\rho}^*$	Relative local error
SD	frozen	0.13864	0.13671	1.41%
SD	endogenous	0.07298	0.07210	1.22%
Entropy	frozen	0.21476	0.20989	2.32%
Entropy	endogenous	0.07729	0.07422	4.13%

For comparison across weighting rules, Theorem 4 reduces the SD and Entropy boundary searches to three independent criterion-level problems. Numerical global-search minimization of the factors in Eqs. (21)–(22), using vertex screening, differential evolution, and local polishing, gives the estimates in Table 6.

For the endogenous SD model, the local radius of 0.07298 is close to the finite boundary, while interval validation sharpens the result to the rigorous bracket $0.07210 < \rho_1^* \leqslant 0.07211$. The distinction is important operationally. The local radius is a first-order estimate of when a change in ranking should be expected, whereas the validated lower bound states that no admissible perturbation anywhere in the complete uncertainty box can change the winner below that level. Thus, for example, a 5% componentwise relative-error region lies inside the certified safe region for this case, while at 7.211% an explicit adverse perturbation already produces a reversal. For Entropy, the local radius 0.07729 is also close to the numerical boundary 0.07422, but this boundary remains a numerical estimate because no validated global lower bound was established. The local quantities are therefore useful for screening and diagnosis, whereas only a certified finite bound supports a formal decision guarantee.

Table 7
Numerical illustration of the CRITIC factor hierarchy.

Numerical level	Estimated minimum
Independent edges, $\hat{L}_{\text{edge}}$	0.00637112
Two-edge clusters, $\hat{L}_{\text{cluster}}$	0.00654796
Joint numerator, $\hat{L}_{\text{joint}}$	0.00655289

Table 8
Published supplier-selection case-study matrix from Chen (2020).

Alternative	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.95	36	0.19	53	0.73	11	0.93
A_2	0.98	39	0.17	52	0.75	13	0.89
A_3	0.93	33	0.21	57	0.69	11	0.92
A_4	0.91	37	0.23	56	0.77	12	0.87
A_5	0.92	35	0.16	51	0.76	10	0.86

7.6. *CRITIC Factor Hierarchy*

A positive 4×4 matrix was generated from

$$\tilde{x}_{ij} = 2 + 0.85\sin(0.9ij + 0.4j) + 0.55\cos(1.3i + 0.7j) + 0.12i + 0.08j, \quad (46)$$

followed by column scaling to 100. Under CRITIC, A_4 is the baseline winner and A_3 the closest score competitor. At $\rho = 0.0075$, the baseline pairwise rank numerator is 0.0123997. The factor representation in Eqs. (23)–(26) was evaluated numerically at three dependency levels: independent edges, three two-edge clusters, and joint optimization of the complete numerator (see Table 7).

The ordering of the estimates is consistent with Theorem 5: retaining more shared-variable dependence moves the numerical estimate upward toward the joint minimum. Because these values come from differential-evolution searches rather than validated global lower bounds, they are not themselves certificates. Their role is computational: the edge calculation is inexpensive, while small clusters recover almost all of the joint numerical value.

7.7. *External Published Case: Building-Material Supplier Selection*

Chen (2020) reports a five-alternative, seven-criterion supplier-selection case-study matrix whose numerical values were quoted or revised with reference to building-material suppliers in Taiwan. The original study combines Entropy, AHP, and TOPSIS; the matrix is reused here only as an external published data set. Let X^{raw} denote the published matrix in Table 8. Criteria C_2 (price) and C_6 (delivery time) are costs, whereas the remaining criteria are benefits.

For the present analysis, the two cost criteria C_2 and C_6 were converted to benefit orientation by reciprocal transformation. The resulting benefit-oriented matrix is denoted

Table 9
Endogenous reliability results for the external supplier-selection case.

Method	M_{43}	Λ_{43}	$\rho_{43}^{\mathrm{loc},F}$	$\rho_{43}^{\mathrm{loc},E}$	Adversarial-line boundary
SD	0.000399	1.440	0.000930	0.000646	0.000647
Entropy	0.005170	1.746	0.011713	0.006710	0.006769
CRITIC	0.000703	1.436	0.001647	0.001147	0.001148

by X, and all subsequent normalization, objective weighting, and perturbation calculations are performed on this matrix. Thus, the analysis follows

$$X^{raw} \to X \underbrace{\mathcal{N}} R \to q \to w \to S.$$

All three objective-weighted WSM models select A_4, with A_3 as the locally critical competitor. The pairwise scores are calculated from Eq. (3), the direct and endogenous sensitivity components from Eqs. (7)–(10), and the local reversal measure from Eq. (12). The corresponding objective-weight vectors are reported in the reproducibility archive (see Table 9).

The published supplier-selection matrix provides a more practical illustration of the same mechanism. Although SD, Entropy, and CRITIC all select A_4, agreement on the nominal winner does not imply comparable reliability. Under SD, the local tolerance decreases from 0.000930 under frozen weights to 0.000646 when the weights are recomputed, a reduction of approximately 30.5%. The corresponding reduction is about 42.7% for Entropy, from 0.011713 to 0.006710, and about 30.4% for CRITIC, from 0.001647 to 0.001147. In relative-error terms, these endogenous local radii correspond to approximately 0.0646%, 0.6710%, and 0.1147%, respectively. The practical implication is that a ranking can appear stable when judged only from the nominal scores while remaining sensitive to small data errors once the induced change in objective weights is taken into account. The adversarial-line boundaries in the last column are very close to the corresponding local estimates, but they remain explicit feasible reversal paths and therefore provide upper bounds on the true reversal radii rather than certified minima.

7.8. *Practical Implementation and Decision-Support Workflow*

The framework can be implemented as a reliability layer attached to an existing objective-weighted decision model; a decision maker does not need to evaluate the derivatives manually. The required inputs are the raw decision matrix, criterion orientations and preprocessing rules, the selected normalization and objective-weighting method, the aggregation rule, and a specification of plausible data error. Starting from these inputs, the software first reproduces the nominal decision pipeline by computing the normalized matrix, information scores, objective weights, alternative scores, and nominal winner. It then evaluates the composite derivatives in Eqs. (4)–(6) and, for each winner–competitor comparison, separates the direct and endogenous components using Eqs. (7)–(10).

The resulting diagnostic layer can report, for every relevant pair, the nominal score margin, the frozen and endogenous sensitivities κ_{ab}^{F} and κ_{ab}^{E}, the amplification ratio Λ_{ab},

and the corresponding local reversal radii $\rho_{ab}^{loc,F}$ and $\rho_{ab}^{loc,F}$. These quantities answer different practical questions. The score margin records nominal separation; Λ indicates whether weight recomputation amplifies or attenuates the effect of data error; and the corresponding local reversal radius provides a first-order estimate of how close the comparison is to reversal. The software can therefore identify the robustness-critical competitor even when that alternative is not the closest competitor in nominal score.

When a finite guarantee is required, a second computational layer evaluates whether a user-specified error range can be certified as decision-preserving. The general bound in Theorem 3 can be applied first. For information-separable rules, Theorem 4 permits the certification calculation to be divided into independent criterion-level problems; for coupled rules, Theorem 5 allows factor blocks to be refined adaptively until the available lower bound is sufficient or further joint optimization is required. At the winner level, the reported safe radius is determined by the most restrictive winner–competitor comparison. A validated lower bound can therefore be translated directly into a statement of the form: the nominal winner is guaranteed to remain unchanged for every admissible data perturbation within the specified uncertainty region.

A practical decision-support interface should also distinguish the status of the evidence. A locally estimated radius is a diagnostic, a feasible adversarial perturbation establishes that reversal is possible at or below a given level, and a validated finite bound certifies that reversal is impossible below that level. This distinction prevents numerical search results from being presented as guarantees. In operational use, a decision maker could compare an anticipated measurement or data-quality tolerance with the certified safe radius. If the expected error lies within the certified region, the ranking is supported against that uncertainty model; if it approaches or exceeds the local reversal scale, the system can flag the critical comparison for data verification, improved measurement, or additional robustness analysis.

The Python implementation used in this study provides a prototype of this computational workflow. NumPy and SciPy are used for the nominal calculations, analytical sensitivities, and numerical boundary searches, while validated interval calculations are invoked separately when a formal certificate is required. A production decision-support implementation could encapsulate these steps behind a conventional user interface and return the nominal winner together with the critical competitor, endogenous amplification or attenuation, local tolerance, certification status, and any identified adverse reversal path. In this way, the mathematical framework functions as an audit layer for an existing MCDM procedure rather than as an additional weighting method that the decision maker must operate separately.

8. Discussion

8.1. *Direction and Conditioning of Endogenous Reweighting*

The central result is not that objective weighting is intrinsically unstable. It is that data-derived reweighting creates a second perturbation channel whose direction depends on

the current decision geometry. Equation (7) shows that the endogenous contribution is scaled by $d_{ab,j} - M_{ab}$, so the same information-score response may reinforce one pairwise comparison and oppose another. Theorem 2 formalizes that interaction locally, and the numerical examples exhibit all three practically relevant regimes: strong amplification, near-neutrality, and attenuation. A frozen-weight study can therefore be accurate in one problem and materially optimistic in another; its adequacy should be assessed rather than assumed.

Weight instability must also be separated from decision instability. Proposition 3 shows that the weight map and the pairwise decision map apply different projections to $D_R q$. Near complete nondiscrimination, both SD and Entropy produce increasingly sensitive normalized weights while the alternative contrasts shrink simultaneously. Table 5 confirms the consequence: $\|D_E w\|$ grows by orders of magnitude, yet the pairwise derivative remains bounded. For reliability analysis, variation in the weight vector is therefore an intermediate diagnostic, not the final object of interest.

8.2. *Dependency Topology and Certification Complexity*

The information-dependency graph exposes a structural distinction that nominal weight vectors do not show. SD and Entropy are criterion-local at the information-score level: changing criterion k changes only q_k before normalization couples the final weights. CRITIC is network-coupled because a perturbation of criterion k can alter several q_j through correlation. Its cross derivative in Eq. (39) also identifies a specific low-dispersion mechanism: when a weakly dispersed criterion changes direction under a small perturbation, its correlations with other criteria can change sharply. This is an angular correlation instability rather than a generic low-variance effect.

The same topology controls finite certification. Information-separable rules allow exact criterion-wise decomposition of the pairwise numerator. Coupled rules retain shared variables across factors, so even exact independent-factor minimization is generally conservative. Numerical factor calculations become certifying only when valid global lower bounds for the relevant subproblems are established. Theorem 5 provides a principled continuum between inexpensive factor-level calculations and full joint optimization. The CRITIC experiment illustrates this trade-off numerically: small clusters retain most of the joint value while requiring substantially less coupled optimization. More generally, for weighting rules with sparse factorization, certification difficulty is governed by dependency width as well as by the raw number of criteria.

8.3. *Interpreting Robustness Quantities and Method Choice*

Local sensitivity, certified safety, and the true reversal boundary answer different questions. The local radius in Eq. (12) is a tangent diagnostic. In the worked example, it predicts the finite adverse path closely, but agreement in one case does not turn a local approximation into a guarantee. A certified safe radius instead excludes *all* reversals inside a specified region. The interval calculation in Section 6.4 demonstrates this distinction directly: all perturbations through 7.210% are safe, while an explicit reversal occurs at 7.211%.

Operationally, the reported quantities should be read at different levels. The amplification ratio Λ_{ab} diagnoses whether endogenous weight recomputation strengthens the local effect of data perturbation ($\Lambda_{ab} > 1$), weakens ($\Lambda_{ab} < 1$), or leaves essentially unchanged ($\Lambda_{ab} \approx 1$). The local reversal radii $\rho_{ab}^{loc,\,F}$ and $\rho_{ab}^{loc,\,E}$ translate the corresponding sensitivities into first-order distances to the pairwise reversal boundary and are therefore useful for screening potentially fragile comparisons. A certified finite radius answers a stronger question: it defines a region in which reversal is mathematically excluded. The true reversal radius ρ_a^*, when known or tightly bracketed, marks the actual failure threshold. These quantities are complementary rather than interchangeable: Λ_{ab} explains the effect of endogenous reweighting, the local radii provide first-order warning measures, and the finite certificate provides the decision guarantee.

The identity of the closest score competitor does not need to coincide with the robustness-critical competitor because score proximity and perturbation distance measure different geometry. Winner-level robustness should therefore be checked against every competitor. Likewise, the choice of weighting rule changes more than the nominal weights: SD, Entropy, and CRITIC transmit data uncertainty through different information geometries. This does not imply that the most robust weighting method should automatically be selected. Robustness complements, rather than replaces, the substantive justification for the weighting principle.

From a decision-support perspective, the framework is most useful when the uncertainty analysis is related to the expected quality of the underlying data. If the certified safe radius exceeds a defensible bound on measurement, reporting, or estimation error, the nominal recommendation has an explicit robustness justification under that uncertainty model. Conversely, a small local radius or a large amplification ratio identifies a comparison for which additional data verification may be more valuable than simply reporting the nominal ranking. The framework can therefore support not only post hoc robustness assessment but also data-quality management by indicating where greater measurement precision is most consequential for the decision. Failure to obtain a certificate should not, however, be interpreted as evidence that the decision is unstable; it means only that stability has not been established over the specified uncertainty region by the available bound.

8.4. *Scope and Limitations*

The composite Jacobian in Proposition 1 applies to general differentiable aggregation, but the closed-form pairwise decomposition and finite certification results are developed for additive aggregation. TOPSIS, VIKOR, multiplicative models, and other nonlinear ranking operators have different decision-boundary geometry and require separate certification analysis. The explicit instantiations also use column-sum normalization; min-max normalization is only piecewise smooth because active extrema can change.

The certificate of Theorem 3 can be conservative, and numerical global-search procedures are not formal certificates unless valid lower bounds establish globality. The interval branch-and-bound calculation in the worked SD example provides a tightly bracketed certified boundary, but it should not be interpreted as evidence that full-dimensional validated

optimization will remain inexpensive as the decision matrix grows. The relevant scaling mechanism depends on the dependency structure of the weighting rule. Under information separability, increasing the number of criteria increases the number of independent certification subproblems, whereas increasing the number of alternatives increases the dimension of each criterion-level problem. Under coupled weighting, the effective certification dimension is additionally controlled by the width of the dependency blocks retained in the factorization. Parallel criterion-wise computation, early certification of clearly safe winner–competitor pairs, and adaptive factor clustering can therefore reduce the practical burden substantially, but large tightly coupled instances may still require more scalable validated optimization, stronger analytical bounds, or adaptive decomposition methods. The present uncertainty model addresses performance-data error; missing data, uncertain criterion orientation, linguistic assessments, model-form uncertainty, and uncertainty in the normalization rule remain outside scope. The external supplier-selection case demonstrates the mechanism on an independently published decision matrix, but it does not by itself establish empirical generality across application domains. Broader validation on independently sourced problems in areas such as engineering selection, logistics, energy, healthcare, and sustainability would be valuable for assessing how often endogenous amplification, attenuation, and narrow certified tolerances arise in practice. Finally, SD, Entropy, and CRITIC were selected to represent distinct dependency classes, not to exhaust objective weighting. Extending the reliability calculus to MEREC, CILOS, IDOCRIW, LOPCOW, and related data-derived weighting rules is a natural direction for future work, but each extension requires method-specific analysis of its data-to-weight map, regularity conditions, and dependency structure.

9. Conclusion

This paper develops an endogenous reliability calculus for objective-weighted MCDM. When criterion weights are estimated from the same performance matrix used to rank alternatives, raw-data uncertainty propagates through two channels: normalized performances change directly, and the objective weights are re-estimated from the perturbed data. The composite formulation makes both channels explicit.

Under additive aggregation, the pairwise differential separates exactly into direct performance and endogenous reweighting components. This decomposition yields local amplification, attenuation, neutrality, and cancellation conditions, while the Conditioning Transfer Principle shows why unstable weights do not need to imply an unstable ranking. The framework then moves from local sensitivity to finite robustness. Information-separable weighting permits exact criterion-wise decomposition of the pairwise worst-case problem; factorizable coupled weighting leads instead to a factor hierarchy whose tightness depends on how much interaction structure is retained.

The numerical study supports these claims. Analytical Jacobians agree with finite differences to numerical precision, endogenous reweighting can be amplifying or locally stabilizing depending on the data geometry, and weight sensitivity can diverge while decision

sensitivity remains bounded. The worked SD example also provides a concrete decision guarantee: A_1 remains the unique winner for every componentwise relative perturbation through 7.210%, while an explicit adverse perturbation reverses the order at 7.211%. The published supplier-selection case-study matrix shows that the same endogenous mechanism can be material outside the constructed examples.

The methodological implication is simple: when objective weights are data-derived, reliability analysis should propagate uncertainty through the full data-driven pipeline rather than perturb performances while holding those weights fixed. Future work can extend the calculus to nonlinear ranking operators, nonsmooth normalization, tighter interval-validated certification, and sparse dependency structures in larger decision systems.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Declaration of Competing Interest

The author declares that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement.

The author confirms responsibility for the following: Conceptualization; Methodology; Formal analysis; Investigation; Data curation; Software; Visualization; Validation; Writing – original draft; Writing – review and editing; Project administration.

Data availability

The numerical data and matrices analysed in this study are reported in the manuscript or derived from the published source cited in Section 7.7. The Python code, numerical inputs, and computational routines used to reproduce the numerical experiments and validation results are available from the corresponding author upon reasonable request.

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I.M. Hezam is a professor in the Department of Statistics and Operations Research, College of Science, King Saud University, Riyadh, Saudi Arabia. His research interests include operations research, mathematical optimization, multi-criteria decision-making, robust and stochastic optimization, and decision analysis. His work focuses on developing analytical and computational methods for optimization and decision-making under uncertainty, with applications in engineering, healthcare, transportation, and related decision-support problems.