

Regret Theory-Driven Method for Addressing Multi-Attribute Decision-Making under Probabilistic Double Hierarchy Linguistic Term Set and Application to Information System Investment Project Selection

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Abstract. Taking into account the irrational elements and regret aversion of decision makers (DMs) during the decision-making process, regret theory (RT) and the TODIM methods have been integrated into a decision-making framework to develop an enhanced multi-attribute decision-making (MADM) method (PDHL-RT-TODIM) within probabilistic double hierarchy linguistic (PDHL) environment. Specifically, extending the perceived utility function in RT to determine the regret and joy values of the overall advantage flow of alternatives calculated by TODIM method in PDHL environment. Then, a correlation coefficient (CC) and standard deviation (SD) integral (CCSD) method was created using the probabilistic double hierarchy linguistic set (PDHLTS) distance metric and PDHL weight arithmetic operator to establish the objective weights of attributes. Additionally, the effectiveness of this proposed method was illustrated through numerical examples for information system investment project selection, and its stability, efficiency, and benefits were further confirmed through sensitivity analysis and comparisons with existing methods.

Key words: multi-attribute decision-making (MADM), probabilistic double hierarchy linguistic term set (PDHLTS), regret theory (RT), CCSD method, information system investment project selection.

1. Introduction

In reality, because of the intricate nature of the socio-economic landscape and the uncertainty of human thought processes, DMs frequently utilize linguistic expressions to convey the assessment information of various attributes (Diao *et al.*, 2022; Tang *et al.*, 2022; Wang *et al.*, 2024; Zhang, S.Q. *et al.*, 2021; Zhao *et al.*, 2021; Zuo *et al.*, 2020). Therefore, Zadeh

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(1974) proposed fuzzy linguistic sets to describe qualitative evaluation information, but more often than not, DMs also use several linguistic terms to express the evaluation information of attributes (for example, when evaluating an artwork, an appreciator may use both “good” and “very good” to evaluate it). Inspired by the hesitant fuzzy set (Torra, 2010) and the linguistic term set (LTS) (Xu, 2004), Rodriguez *et al.* (2012) introduced the hesitant fuzzy linguistic term set (HFLTS). While the HFLTS is capable of conveying much of the evaluation information related to DM, it has two main drawbacks: firstly, the evaluation linguistic used by HFLTS is overly simplistic and the evaluation information is too generalized. Secondly, HFLTS fails to capture the significance of each hesitant fuzzy linguistic component. To compensate for its shortcomings, Gou *et al.* (2017) and Pang *et al.* (2016) proposed double hierarchy linguistic term set (DHLTS) and probabilistic linguistic term set (PLTS). Gou *et al.* (2021) later integrated the benefits of both approaches and introduced the PDHLTS. It consisted of multiple DHLTSs and corresponding probabilities. Its linguistic expression of “adverb+adjective” not only delicately described attribute evaluation information, but also conformed to people’s expression habits. In addition, the probability associated with it can more deeply express the DM’s preference information. Therefore, PDHLTS was more suitable for information expression in practical decision-making processes and was currently an important tool for dealing with MADM problems. Based on it, Gou *et al.* (2017) were the first to define PDHLTS along with its scoring function and distance metric. Furthermore, they adapted the traditional VIKOR approach for the PDHL context and introduced a PDHL-VIKOR method to address a real-world MADM issue related to intelligent healthcare. Subsequently, Lei *et al.* (2021) introduced a novel PDHLTS distance measure, which encompasses both Euclidean and Hamming distance measures, along with a new probability completion method. Furthermore, they integrated the prospect theory (Kahneman and Tversky, 1979) with CODAS method, extending a new distance measurement formula to the PDHL environment to construct a MADM model that can reflect DMs’ loss aversion psychology. The research on the expected function, score function, and distance measurement formula of PDHLTS by the two scholars mentioned above lays the foundation for identifying more classical methods to solve MADM problems in the PDHL environment in the future. For example, Wang (2022) proposed the PDHL-TOPSIS method and applied it to evaluate the teaching level of a teacher. Liu, P. *et al.* (2023a) combined WASPAS method with weight determination method and extended it to PDHLTSs environment to solve the risk assessment problem in PDHL environment.

In order to scientifically and effectively solve MADM problems, DMs need to choose appropriate decision information measurement tools and construct scientific and efficient MADM methods. The TODIM method proposed by Gomes and Lima (1991) in 2009 is a validated and effective approach for handling MADM. This method measures the degree of advantage of each alternative solution relative to other alternative solutions by using the overall value, and then evaluates and ranks the alternative solutions based on their overall advantage. In addition, compared with other classic MADM methods (FOR example: TOPSIS method (Hwang and Yoon, 1981), GRA method (Deng, 1989), SWARA method (Zavadskas *et al.*, 2012), EDAS method (Keshavarz Ghorabae *et al.*, 2015), MABAC

method (Pamucar and Cirovic, 2015), etc.), only the TODIM method is unique in that it takes into account the psychological factors affecting DMs, marking a significant advancement in addressing irrational behaviour in uncertain situations. Therefore, it has been applied in many fields (Hong *et al.*, 2019; Liu and Teng, 2015; Wei and Wu, 2019; Zhang *et al.*, 2017; Zindani *et al.*, 2020) and has achieved significant results. Among them, Zhang *et al.* (2019) applied the improved intuitionistic fuzzy set (IFS) score function and exact function to the TODIM method, and constructed an IF-TODIM model for evaluating supplier production capacity while comparing the differences between IFSs. Sun *et al.* (2019) extended the classic TODIM method to interval IFSs and constructed a new MADM model, which made significant progress in solving emergency problems in hydraulic engineering. Wang *et al.* (2021) integrated the TODIM method with binary semantic Pythagorean fuzzy sets to develop innovative models and concepts for the scientific and comprehensive assessment of rural scientific and technological talent. Guo *et al.* (2020) first defined a new Probabilistic linguistic Hamming distance and improved the classical TODIM method, making the improved TODIM method more accurate and effective in industrial detection of carbon dioxide storage locations. Rosli *et al.* (2023) first aggregated expert evaluation information expressed in binary semantic sets using the LAMA (Linguistic Aggregation Majority Additive Operator) operator, and then ranked six candidate partners using the TODIM method, selecting the candidate partner with the highest score. Wu *et al.* (2020) first integrated DEMATEL method with entropy weight method to determine attribute weights, and then evaluated and ranked eight candidate hilly sites by combining PDHLTS and TODIM, and selected the best hilly site selection. Furthermore, to assess the rationality of product recycling channels from a scientific perspective, Hong *et al.* (2021) first established a complete evaluation index system, then used interval type-2 fuzzy numbers to represent decision information, and finally used TODIM method for sorting. Although the classic TODIM is built on the basis of PT (the value function and probability weight function in prospect theory function effectively reflect the loss aversion psychology of DMs and reveal the irrational factors of DM's behaviour), the core idea of PT has not been reflected in it. To this end, Tian *et al.* (2019) introduced the PT theory to improve the TODIM method and constructed an enhanced TODIM method. This study introduces behavioural theory into decision theory, achieving further improvement of decision theory. This article will improve the traditional TODIM method from a new perspective by introducing a new behavioural theory-regret theory which are proposed by Loomes and Sugden (1982) and Bell (1982) to enhance the TODIM method. The improved TODIM method retains both the traditional TODIM method's ability to represent DMs' loss aversion attitudes and their regret aversion psychology.

Regret theory (RT) (Bleichrodt *et al.*, 2010) is another important behavioural theory after PT, which combines emotional and motivational factors into the expectancy structure to express DM's regret and disgust psychology. RT believes that DMs not only focus on its own chosen options, but also on other unselected options. When the outcomes of other unchosen options surpass those of the chosen option, DMs may experience regret; conversely, they may feel happiness if the selected option performs better. Currently, robust techniques have been utilized in the decision-making domain to tackle numerous real-world decision-making challenges (Hu, 2023; Li *et al.*, 2021; Liao *et al.*, 2022; Mondal

et al., 2023; Wang *et al.*, 2018). For instance, Liu *et al.* (2020) developed a comprehensive model for sustainable supplier selection that combines RT and QUALIFLEX methods within a two-dimensional uncertain linguistic context, significantly enhancing the reliability of DM's evaluations. Jia and Wang (2020) introduced the PROMETHEE II method, grounded in RT, to address the selection of cloud services for universities in a PLTS. Hu (2023) noted that many existing MADM methods neglect the optimal efficiency of the MADM process and the preferences of DMs. To remedy this gap, the MADM method approach based on RT was suggested. Liang and Wang (2020) enhanced urban emergency response capabilities by developing interval hesitant fuzzy satisfaction models. Qian *et al.* (2019) incorporated RT into the MADM process and proposed a novel grey risk MADM method to tackle the challenge of selecting mall locations. Liu *et al.* (2023) introduced RT to establish an overall merit model for the urgent relief system of mines, addressing the complexity, ambiguity, and systematicity of the evaluation. Zhang *et al.* (2021) proposed a case retrieval method for handling multiple attributes and incomplete weight information based on RT and LINMAP methods. Zhao *et al.* (2023) incorporated RT into an uncertain linguistic context and introduced the interaction between doctors and patients in an online setting, along with the concept of stable matching model, conducting thorough research on the issue with incomplete information. Due to the complexity of construction projects, selecting capable managers is crucial for their success. Yan *et al.* (2023) integrated the RT with the fuzzy DEMATEL approach to create a MADM model for selecting construction project managers. Furthermore, to assess the specific combat capabilities of a particular fighter jet, Zhang *et al.* (2023a) adapted the RT and CRITIC methods for use in a Pythagorean hesitant fuzzy (PHF) context, introducing an enhanced method to address random issues within this environment. Additionally, to evaluate five renewable energy options, Ding *et al.* (2023) developed an improved hybrid MADM method that combines DEMATEL and RT. Considering the influence of fuzzy information and irrational behaviour in reality, DM often finds it difficult to make a decision. Therefore, Zhang *et al.* (2023b) introduced the VIKOR method utilizing RT to tackle the MADM issue in situations where the weight information is entirely unknown. In summary, utilizing PDHLTS to express decision information and establishing the TODIM method that considers DMs' behavioural psychology based on RT in the PDHL environment has certain significance for the development of the decision-making field.

The organization of this article: the Section 2 offers a brief overview of essential theoretical concepts relevant to our research, including PDHLTS, RT theory, and traditional TODIM methods. In Section 3, we develop a weight calculation method utilizing the PDHLTS distance measure and PDHLWA operator for the PDHL context, referred to as the CCSD method. Additionally, we expand the perceived utility function in RT to assess the regret and joy associated with the overall advantage flow of alternatives determined by the TODIM method in the PDHL setting, leading to the creation of the MADM method that captures the regret and aversion feelings of DMs, called the RT-PDHL-TODIM method. Section 4 presents specific examples for information system investment project selection to demonstrate the effectiveness of the CCSD and RT-PDHL-TODIM methods proposed in this paper, along with a parameter analysis to further con-

firm their effectiveness, superiority, and innovation by comparing them to existing methods like PDHL-WASPAS and PDHL-TOPSIS. Finally, Section 5 concludes the article with a summary.

2. Preliminary Knowledge

Fuzzy (linguistic) sets are an effective carrier for expressing evaluation information in the MADM process. The definition, conversion function, expectation, and standard deviation of PDHLTS set are introduced in this section.

DEFINITION 1 (Gou *et al.*, 2017). Let $\mathfrak{S} = \{\mathfrak{S}_\mu | \mu = -\mathfrak{A}, \dots, -1, 0, 1, \dots, \mathfrak{A}\}$ and $\mathfrak{R} = \{\mathfrak{R}_\nu | \nu = -\mathfrak{A}, \dots, -1, 0, 1, \dots, \mathfrak{A}\}$ be two independent LTSs, where $\mathfrak{S}, \mathfrak{R}$ are the first and second hierarchy LTSs, respectively. The DHLTS is defined as follows:

$$\Theta = \{\mathfrak{S}_{\mu(\mathfrak{R}_\nu)} | \mu = -\mathfrak{A}, \dots, -1, 0, 1, \dots, \mathfrak{A}; \nu = -\mathfrak{A}, \dots, -1, 0, 1, \dots, \mathfrak{A}\}.$$

DEFINITION 2 (Gou *et al.*, 2021). Let $\mathfrak{S} = \{\mathfrak{S}_\mu | \mu = -\mathfrak{A}, \dots, -1, 0, 1, \dots, \mathfrak{A}\}$ and $\mathfrak{R} = \{\mathfrak{R}_\nu | \nu = -\mathfrak{A}, \dots, -1, 0, 1, \dots, \mathfrak{A}\}$ be two independent LTSs, where ξ, ζ are the first and second hierarchy LTSs, respectively. The PDHLTS is defined as follows:

$$\mathbb{Z} = \left\{ \mathfrak{S}_{\mu(\mathfrak{R}_\nu)}^{(h)}(p^{(h)}) \mid \mathfrak{S}_{\mu(\mathfrak{R}_\nu)}^{(h)} \in \Theta, p^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}} p^{(h)} \leq 1 \right\}. \quad (1)$$

For the convenience of writing, remembered $\mathfrak{S}_{\mu(\mathfrak{R}_\nu)}^{(h)}(p^{(h)})$ as $\mathbb{Z}^{(k)}$ and the mathematical expression of PDHLTS is as follows:

$$\mathbb{Z} = \left\{ \mathbb{Z}^{(h)}(p^{(h)}) \mid \mathbb{Z}^{(h)} \in \Theta, p^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}} p^{(h)} \leq 1 \right\}, \quad (2)$$

where, $\mathbb{Z}^{(h)}(p^{(h)})$ represents the h th probabilistic double hierarchy linguistic elements (PDHLE), and each PDHLE is sorted in ascending order of $f(\mathbb{Z}^{(h)})$. $\#\mathbb{Z}(P)$ denotes the quantity of PDHLEs present in $\mathbb{Z}(P)$, and $p^{(h)}$ is the probability corresponding to $\mathbb{Z}^{(h)}$.

DEFINITION 3 (Gou *et al.*, 2021). Let $\mathbb{Z} = \{\mathbb{Z}^{(h)}(p^{(h)}) | \mathbb{Z}^{(h)} \in \Theta, p^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}} p^{(h)} \leq 1\}$ be a PDHLTS, and the mathematical expression for the equivalent transformation function between the linguistic index (μ, ν) of $\mathbb{Z}(P)$ and the real value χ is as follows:

$$f : (\mu, \nu) \rightarrow [0, 1], \quad f(\mu, \nu) = \frac{\nu + (\mathfrak{A} + \mu)\mathfrak{A}}{2\mathfrak{A}\mathfrak{A}} = \chi, \quad (3)$$

$$f^{-1} : [0, 1] \rightarrow (\mu, \nu),$$

$$f^{-1}(\chi) = [2\mathfrak{A}\chi - \wp]_{\langle \mathfrak{A}((2\mathfrak{A}\chi - \mathfrak{A}) - [2\mathfrak{A}\chi - \mathfrak{A}]) \rangle} \text{ or} \\ [2\mathfrak{A}\chi - \mathfrak{A}] + 1_{\langle \mathfrak{A}((2\mathfrak{A}\chi - \mathfrak{A}) - [2\mathfrak{A}\chi - \mathfrak{A}]) - \mathfrak{A} \rangle}. \quad (4)$$

DEFINITION 4 (Gou et al., 2021). Let $\mathbb{Z} = \{\mathbb{Z}^{(h)}(p^{(h)}) | \mathbb{Z}^{(h)} \in \Theta, p^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}} p^{(h)} \leq 1\}$ be a PDHLTS, and the mathematical expression for defining the expectation and standard deviation of PDHLTS are as follows:

$$E = \frac{\sum_{h=1}^{\#\mathbb{Z}} f(\mathbb{Z}^{(h)}) p^{(h)}}{\sum_{h=1}^{\#\mathbb{Z}} p^{(h)}}, \quad (5)$$

$$D = \sqrt{\frac{\sum_{h=1}^{\#\mathbb{Z}} (f(\mathbb{Z}^{(h)}) p^{(h)} - E)^2}{\sum_{h=1}^{\#\mathbb{Z}} p^{(h)}}}. \quad (6)$$

Using equations (5)–(6), the subsequent equations are formulated to ascertain the order of the two PDHLTSs.

if $E(\mathbb{Z}_1) > E(\mathbb{Z}_2)$, then $\mathbb{Z}_1 > \mathbb{Z}_2$;

if $E(\mathbb{Z}_1) = E(\mathbb{Z}_2)$, then if $D(\mathbb{Z}_1) = D(\mathbb{Z}_2)$, then $\mathbb{Z}_1 = \mathbb{Z}_2$;

then if $D(\mathbb{Z}_1) > D(\mathbb{Z}_2)$, then $\mathbb{Z}_1 < \mathbb{Z}_2$.

DEFINITION 5 (Gou et al., 2021). Let $\mathbb{Z} = \{\mathbb{Z}^{(h)}(p^{(h)}) | \mathbb{Z}^{(h)} \in \Theta, p^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}} p^{(h)} < 1\}$ represent PDHLTS, in which the total probabilities for all PDHLEs do not add up to 1, the mathematical expression for normalized PDHLTS is defined as follows:

$$\tilde{\mathbb{Z}} = \left\{ \mathbb{Z}^{(h)}(\tilde{p}^{(h)}) \mid \mathbb{Z}^{(h)} \in \Theta, \tilde{p}^{(h)} \geq 0, \sum_{h=1}^{\#\tilde{\mathbb{Z}}} \tilde{p}^{(h)} = 1 \right\}, \quad \text{where } \tilde{p}^{(h)} = \frac{p^{(h)}}{\sum_{h=1}^{\#\mathbb{Z}} p^{(h)}}. \quad (7)$$

DEFINITION 6 (Lei et al., 2021). Let $\mathbb{Z}_j = \{\mathbb{Z}_j^{(h)}(p_j^{(h)}) | \mathbb{Z}_j^{(h)} \in \Theta, p_j^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}_j} p_j^{(h)} \leq 1\}$, ($j = 1, 2$) be two PDHLTSs, where the $\#\mathbb{Z}_1$ and the $\#\mathbb{Z}_2$ denote the amounts of \mathbb{Z}_1 and \mathbb{Z}_2 . Adding $\#\mathbb{Z}_1 - \#\mathbb{Z}_2$ DHLTSs to $\mathbb{Z}_2(P)$ while $\#\mathbb{Z}_1 > \#\mathbb{Z}_2$, Furthermore, the newly introduced DHLTS must be the smallest DHLTS among $\mathbb{Z}_2(P)$, and the associated probability may be zero.

DEFINITION 7 (Lei et al., 2021). Let $\mathbb{Z}_j = \{\mathbb{Z}_j^{(h)}(p_j^{(h)}) | \mathbb{Z}_j^{(h)} \in \Theta, p_j^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}_j} p_j^{(h)} \leq 1\}$, ($j = 1, 2$) be two normalized PDHLTSs with equal numbers of elements, and the mathematical expression for the Hamming distance of PDHLTS is defined as follows:

$$HD(\tilde{\mathbb{Z}}_1, \tilde{\mathbb{Z}}_2) = \frac{\sum_{h=1}^{\#\tilde{\mathbb{Z}}} |f(\mathbb{Z}_1^{(h)}) \tilde{p}_1^{(h)} - f(\mathbb{Z}_2^{(h)}) \tilde{p}_2^{(h)}|}{\#\tilde{\mathbb{Z}}}, \quad \text{where } \#\mathbb{Z}_1 = \#\mathbb{Z}_2 = \#\mathbb{Z}. \quad (8)$$

3. The PDHL CCSD and PDHL-RT-TODIM Method for MADM Problem

Next, based on the RT theory, PDHLTS theory, and classic TODIM method, we will construct the PDHL-RT-TODIM method. The mathematical symbols required are described:

let $\Lambda = \{\Lambda_1, \Lambda_2, \dots, \Lambda_s\}$, $\Upsilon = \{\Upsilon_1, \Upsilon_2, \dots, \Upsilon_t\}$ represent sets of alternatives and evaluation attributes, and $\omega = (\omega_1, \omega_2, \dots, \omega_t)$ represent the weight vector of the evaluation attribute, satisfying $\omega_j \geq 0$,

$\sum_{j=1}^t \omega_j = 1$. The PDHL evaluation information of the attribute Υ_j for the alternative Λ_i is represented $\mathbb{Z}_{ij} = \{\mathbb{Z}_{ij}^{(h)}(p_{ij}^{(h)}) | \mathbb{Z}_{ij}^{(h)} \in \Theta, p_{ij}^{(h)} > 0, \sum_{h=1}^{\#\mathbb{Z}_{ij}} p_{ij}^{(h)} \leq 1\}$ and $G = \{\mathbb{Z}_{ij}\}_{s \times t}$ is the decision matrix.

3.1. Calculation of the Weight of Attributes

The following will provide the detailed process of extending the classical CCSD method (Wang and Luo, 2010) to the PDHL environment and using it to determine the weights of attributes.

Step 1. By Definition 6, supplement the elements of PDHLTS, so that each supplemented PDHLTS has the same number of elements. According to Definition 6, normalize the evaluation information, and the cost details are converted into benefit details as follows: $\mathbb{Z}_{ij}(P) = \{\mathbb{Z}_{ij}^{(h)}(p_{ij}^{(h)}) | \mathbb{Z}_{ij}^{(h)} \in \Theta, p_{ij}^{(h)} > 0, \sum_{h=1}^{\#\mathbb{Z}_{ij}} p_{ij}^{(h)} \leq 1\}$ was changed to evaluation information $\mathbb{Z}_{ij}h = \{-\mathbb{Z}_{ij}^{(h)}(p_{ij}^{(h)}) | -\mathbb{Z}_{ij}^{(h)} \in \Theta, p_{ij}^{(h)} > 0, \sum_{h=1}^{\#\mathbb{Z}_{ij}} p_{ij}^{(h)} \leq 1\}$.

Step 2. Determine the total evaluation score for each alternative by omitting the evaluation score for the attribute Υ_j using the PDHLWA operator.

$$\begin{aligned} \mathbb{Z}'_{il} &= PDHLWA(\tilde{\mathbb{Z}}_{i1}, \dots, \tilde{\mathbb{Z}}_{il}, \dots, \tilde{\mathbb{Z}}_{it}) \\ &= \left\{ f^{-1} \left(\bigcup_{\substack{\mathbb{Z}_{il}^{(k)} \\ \tilde{p}_{il}^{(k)}}} \left(1 - \prod_{l=1, l \neq j}^t (1 - f(\mathbb{Z}_{il}^{(h)}))^{\omega_l} \right) \right) \left(\sum_{l=1, l \neq j}^t \tilde{p}_{il}^{(h)} / t \right) \right\} \\ &= \left\{ \mathbb{Z}'_i^{(h)}(\tilde{p}_i^{(h)}) \mid \mathbb{Z}'_i^{(h)} \in \Theta, \tilde{p}_i^{(h)} \geq 0, \sum_{h=1}^{\#\mathbb{Z}'_i} \tilde{p}_i^{(h)} = 1 \right\}. \end{aligned} \quad (9)$$

Among them, ω_l represents the weight of the attribute Υ_l , which satisfies $\omega_l \in [0, 1]$ and $\sum_{l=1}^t \omega_l = 1$.

Step 3. Compute the CC between the attribute Υ_j and the overall assessment data.

$$\begin{aligned} CC(\Upsilon_j) &= \frac{\sum_{i=1}^s \left(\left(\sum_{k=1}^{\#\tilde{\mathbb{Z}}_{ij}} (f(\mathbb{Z}_{ij}^{(h)}) \tilde{p}_{ij}^{(h)} - f(\tilde{\mathbb{Z}}_j^{(h)}) \tilde{p}_j^{(h)}) \right) \right. \\ &\quad \left. \times \left(\sum_{k=1}^{\#\mathbb{Z}'_i} (f(\mathbb{Z}'_i^{(h)}) \tilde{p}_i^{(h)} - f(\tilde{\mathbb{Z}}_j^{(h)}) \tilde{p}_j^{(h)}) \right) \right)}{\sqrt{\sum_{i=1}^s \left(\sum_{r=1}^{\#\tilde{\mathbb{Z}}_{ij}} (f(\mathbb{Z}_{ij}^{(h)}) \tilde{p}_{ij}^{(h)} - f(\tilde{\mathbb{Z}}_j^{(h)}) \tilde{p}_j^{(h)}) \right)^2}}}, \quad j = 1, 2, \dots, t. \\ &\quad \times \sqrt{\sum_{i=1}^s \left(\sum_{h=1}^{\#\mathbb{Z}'_i} (f(\mathbb{Z}'_i^{(k)}) \tilde{p}_i^{(h)} - f(\tilde{\mathbb{Z}}_j^{(h)}) \tilde{p}_j^{(h)}) \right)^2} \end{aligned} \quad (10)$$

where,

$$\begin{aligned}\bar{Z}_j &= \{\bar{Z}_j^{(h)}(\bar{p}_j^{(h)})\} = f^{-1}\left(\sum_{i=1}^s f(\mathbb{Z}_{ij}^{(h)})/s\right)\left(\sum_{i=1}^s \bar{p}_{ij}^{(h)}/s\right), \\ \bar{Z}'_j &= \{\bar{Z}'_j^{(h)}(\bar{p}'_j^{(h)})\} = f^{-1}\left(\sum_{i=1}^s f(\mathbb{Z}'_{il}{}^{(h)})/s\right)\left(\sum_{i=1}^s \bar{p}'_{il}{}^{(h)}/s\right).\end{aligned}\quad (11)$$

Among them, $CC(\Upsilon_j)$, \bar{Z}_j and \bar{Z}'_j , respectively, represents the CC between the attribute Υ_j and the overall assessment data, as well as the average value of the attribute Υ_j and the average value of the overall evaluation value excluding attribute Υ_j .

- i) In the above equation, $CC(\Upsilon_j)$ varies from -1 to 1 . When $CC(\Upsilon_j)$ is near 1 , that means the absence/presence of attribute did not contribute much to the change in evaluation value and final alternatives' ranking. At this point, attributes Υ_j should have less importance, and should be weighted with a smaller value.
- ii) As $CC(\Upsilon_j)$ nears -1 , it suggests that the removal of attribute Υ_j significantly affected the score and the ranking of the options. Therefore, attribute Υ_j should be assigned more importance.
- iii) Furthermore, it is important to take into account how deviations affect the overall evaluation score and the final ranking of options. If an attribute provides the same utility across all alternatives, eliminating it from the attribute set will not influence the decision. That is to say, attributes with higher SD should carry more significance, while those with lower standard deviations should be given less importance.

Step 4. Based on the above analysis, calculate the weights of the attributes.

$$w_j = \frac{D(\Upsilon_j)\sqrt{1-CC(\Upsilon_j)}}{\sum_{l=1}^t D(\Upsilon_l)\sqrt{1-CC(\Upsilon_l)}}, \quad (12)$$

where, $D(\Upsilon_j) = \sqrt{\frac{1}{s}(\sum_{i=1}^s (\sum_{h=1}^{\#\mathbb{Z}_{ij}} (f(\mathbb{Z}_{ij}^{(h)})\bar{p}_{ij}^{(h)} - f(\bar{Z}_{ij}^{(h)})\bar{p}_{ij}^{(h)}))^2)}$ was the standard deviation of the evaluation information under attribute.

Equation (12) represented a nonlinear system of equations with t equations that uniquely identify t weight variables. To address this equation, we reformulated it into the subsequent nonlinear optimization model:

$$J = \sum_{l=1}^t \left(\omega_j - \frac{D(B_j)\sqrt{1-CC(B_j)}}{\sum_{l=1}^t D(B_l)\sqrt{1-CC(B_l)}} \right)^2. \quad (13)$$

The optimal solution is ω_j , which minimized the value of equation J. Among them, ω_j represents the weight of the Υ_j and satisfies $\omega_j \in [0, 1]$, $\sum_{j=1}^t \omega_j = 1$.

The nonlinear optimization model mentioned can be addressed by Microsoft Excel with macros, MATLAB, or LINGO software to find the best possible value of the objective function while adhering to the given constraints.

3.2. The PDHL-RT-TODIM Method for Ranking Alternatives

The RT theory is introduced into PDHLTS, and the constructed PDHL-RT-TODIM method is as follows.

Step 1. Calculate the relative weight of each attribute.

$$\varpi_j = \frac{\omega_j}{\omega_l}, \quad (14)$$

where ϖ_j represents the relative weight of attribute Υ_j , and $\omega_l = \max\{\omega_1, \dots, \omega_j, \dots, \omega_t\}$.

Step 2. Calculate the advantage of alternative Λ_i over other alternatives under attribute Υ_j .

$$\varphi_j(\Lambda_i, \Lambda_d) = \begin{cases} \sqrt{\frac{\varpi_j HD(\tilde{Z}_{ij} - \tilde{Z}_{dj})}{\sum_{j=1}^t \varpi_j}}, & \text{if } \tilde{Z}_{ij} \geq \tilde{Z}_{dj}, \\ -\frac{1}{\theta} \sqrt{\frac{(\sum_{j=1}^t \varpi_j) HD(\tilde{Z}_{ij} - \tilde{Z}_{dj})}{\varpi_j}}, & \text{if } \tilde{Z}_{ij} < \tilde{Z}_{dj}, \end{cases} \quad (15)$$

where $HD(\tilde{Z}_{ij} - \tilde{Z}_{dj}) = \frac{\sum_{h=1}^{\#\tilde{Z}_{ij}} |f(\mathbb{Z}_{ij}^{(h)})\tilde{P}_{ij}^{(h)} - f(\mathbb{Z}_{dj}^{(h)})\tilde{P}_{dj}^{(h)}|}{\#\tilde{Z}_{ij}}$, and θ ($\theta > 0$) represents the loss attenuation coefficient.

Step 3. Calculate the overall advantage of alternative A_i relative to other alternatives.

$$\varphi(\Lambda_i, \Lambda_d) = \sum_{j=1}^t \varphi_j(\Lambda_i, \Lambda_d), \quad i, d = 1, 2, \dots, s. \quad (16)$$

Step 4. Calculate the comprehensive advantage of each alternative.

$$\mathbb{N}(\Lambda_i) = \sum_{d=1}^s \varphi(\Lambda_i, \Lambda_d). \quad (17)$$

Step 5. Calculate the standardized comprehensive advantages of each alternative.

$$Z(A_i) = \frac{\mathbb{N}(\Lambda_i) - \min_i \{\mathbb{N}(\Lambda_i)\}}{\max_i \{\mathbb{N}(\Lambda_i)\} - \min_i \{\mathbb{N}(\Lambda_i)\}}. \quad (18)$$

Step 6. Determine the utility value and the regret-joy value for each alternative.

$$F(A_i) = UV(A_i) + RV(A_i). \quad (19)$$

The utility value for each option was determined in the following manner

$$UV(A_i) = (Z(A_i))^\alpha. \quad (20)$$

The regret-joy value for each alternative was determined in the following manner:

$$RV(A_i) = (1 - e^{-\vartheta(UV(A_i) - UV^+)}) + (1 - e^{-\vartheta(UV(A_i) - UV^-)}), \quad (21)$$

where $UV^+ = \max\{UV(A_1), \dots, UV(A_2), \dots, UV(A_s)\}$, $UV^- = \min\{UV(A_1), \dots, UV(A_2), \dots, UV(A_s)\}$, α ($\alpha \in (0, 1)$) represents the risk attitude coefficient, and ϑ ($\vartheta > 0$) represents the regret aversion coefficient.

Step 7. Rank the alternatives based on their utility value and regret-joy value, and the higher the evaluation value, the better the ranking.

4. Numerical Example, Sensitivity Analysis and Comparative Analysis

4.1. Numerical Example for Information System Investment Project Selection Decision-Making

The decision-making process for information system investment project selection is exceptionally difficult. Many enterprises know that the application of information systems can improve operational efficiency, yet they are unclear about exactly how much improvement will be achieved and in which specific aspects. Such uncertainty about value makes enterprises highly hesitant when making decisions in advance. A survey once showed that 35% of enterprises had to abandon projects because they could not determine the value brought by Information Technology (IT) initiatives. Among the projects already invested in, most enterprises based their decisions on Faith on IT—subjective confidence—while only 16% adopted rigorous methods to evaluate IT investment decisions. However, investment in information system projects is characterized by high risk, making prudent advance decision-making particularly crucial. The consequences of blind IT investment are striking. On the one hand, immature investment decisions lead to inaccurate grasp of project objectives and scope, causing projects to deviate from their intended direction and fail to deliver expected outcomes. On the other hand, without evaluation, enterprises cannot rationally estimate and predict project costs and potential problems, resulting in weak risk control and problem-solving capabilities, which greatly increases the likelihood of project failure. There are many causes of IT project failure. Studies indicate that the lack of proper decision-making and evaluation for information system investment project selection is one of the main reasons. Any omissions or errors in advance decision-making and evaluation will bring severe losses to enterprises. Therefore, enterprises should act prudently and avoid making decisions blindly. Now a certain company will choose the best information system investment project selection project from five alternative investment projects for investment. Experts are required to evaluate five information system investment project selection projects $Ip = \{Ip_1, Ip_2, \dots, Ip_5\}$ based on four evaluation attributes $\Upsilon = \{\Upsilon_1, \Upsilon_2, \Upsilon_3, \Upsilon_4\}$, and Υ_1 (earning rate), Υ_2 (investment cost), Υ_3 (safety performance), Υ_4 (sustainable development), $\omega = (\omega_1, \omega_2, \dots, \omega_4)$ represents the weight vector of the evaluation attribute, satisfying $\omega_j \geq 0$, $\sum_{j=1}^4 \omega_j = 1$. Except

Table 1
The expert decision matrix.

	Υ_1	Υ_2
Ip_1	$\{\mathfrak{S}_{0<\mathfrak{M}_0>}(0.47), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.27), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.26)\}$	$\{\mathfrak{S}_{-3<\mathfrak{M}_0>}(0.42), \mathfrak{S}_{-3<\mathfrak{M}_1>}(0.35), \mathfrak{S}_{-3<\mathfrak{M}_2>}(0.23)\}$
Ip_2	$\{\mathfrak{S}_{1<\mathfrak{M}_0>}(0.43), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.25), \mathfrak{S}_{2<\mathfrak{M}_1>}(0.32)\}$	$\{\mathfrak{S}_{1<\mathfrak{M}_{-1}>}(0.55), \mathfrak{S}_{2<\mathfrak{M}_{-2}>}(0.23), \mathfrak{S}_{2<\mathfrak{M}_{-1}>}(0.22)\}$
Ip_3	$\{\mathfrak{S}_{-2<\mathfrak{M}_1>}(0.19), \mathfrak{S}_{-2<\mathfrak{M}_2>}(0.23), \mathfrak{S}_{-1<\mathfrak{M}_1>}(0.58)\}$	$\{\mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.67), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.33)\}$
Ip_4	$\{\mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.38), \mathfrak{S}_{2<\mathfrak{M}_{-1}>}(0.62)\}$	$\{\mathfrak{S}_{-1<\mathfrak{M}_{-2}>}(0.24), \mathfrak{S}_{-1<\mathfrak{M}_0>}(0.38), \mathfrak{S}_{-1<\mathfrak{M}_2>}(0.38)\}$
Ip_5	$\{\mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.17), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.83)\}$	$\{\mathfrak{S}_{-3<\mathfrak{M}_1>}(0.66), \mathfrak{S}_{-3<\mathfrak{M}_2>}(0.23), \mathfrak{S}_{-2<\mathfrak{M}_0>}(0.11)\}$
	Υ_3	Υ_4
Ip_1	$\{\mathfrak{S}_{1<\mathfrak{M}_0>}(0.45), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.55)\}$	$\{\mathfrak{S}_{-1<\mathfrak{M}_0>}(0.77), \mathfrak{S}_{1<\mathfrak{M}_0>}(0.23)\}$
Ip_2	$\{\mathfrak{S}_{-2<\mathfrak{M}_2>}(0.32), \mathfrak{S}_{-1<\mathfrak{M}_0>}(0.33), \mathfrak{S}_{-1<\mathfrak{M}_2>}(0.35)\}$	$\{\mathfrak{S}_{-3<\mathfrak{M}_0>}(0.23), \mathfrak{S}_{-3<\mathfrak{M}_2>}(0.24), \mathfrak{S}_{-2<\mathfrak{M}_1>}(0.53)\}$
Ip_3	$\{\mathfrak{S}_{-1<\mathfrak{M}_0>}(0.28), \mathfrak{S}_{-1<\mathfrak{M}_1>}(0.31), \mathfrak{S}_{-1<\mathfrak{M}_2>}(0.41)\}$	$\{\mathfrak{S}_{-2<\mathfrak{M}_1>}(0.31), \mathfrak{S}_{-2<\mathfrak{M}_2>}(0.28), \mathfrak{S}_{-1<\mathfrak{M}_0>}(0.41)\}$
Ip_4	$\{\mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.56), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.44)\}$	$\{\mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.45), \mathfrak{S}_{2<\mathfrak{M}_0>}(0.55)\}$
Ip_5	$\{\mathfrak{S}_{2<\mathfrak{M}_0>}(0.34), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.19), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.47)\}$	$\{\mathfrak{S}_{3<\mathfrak{M}_{-2}>}(0.17), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.27), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.56)\}$

Table 2
The normalized expert decision matrix for benefit.

	Υ_1	Υ_2
Ip_1	$\{\mathfrak{S}_{0<\mathfrak{M}_0>}(0.47), \mathfrak{S}_{1<\mathfrak{M}_0>}(0.27), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.26)\}$	$\{\mathfrak{S}_{3<\mathfrak{M}_{-2}>}(0.23), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.35), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.42)\}$
Ip_2	$\{\mathfrak{S}_{1<\mathfrak{M}_0>}(0.43), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.25), \mathfrak{S}_{2<\mathfrak{M}_1>}(0.32)\}$	$\{\mathfrak{S}_{-2<\mathfrak{M}_1>}(0.22), \mathfrak{S}_{-2<\mathfrak{M}_2>}(0.23), \mathfrak{S}_{-1<\mathfrak{M}_1>}(0.55)\}$
Ip_3	$\{\mathfrak{S}_{-2<\mathfrak{M}_1>}(0.19), \mathfrak{S}_{-2<\mathfrak{M}_2>}(0.23), \mathfrak{S}_{-1<\mathfrak{M}_1>}(0.58)\}$	$\{\mathfrak{S}_{-3<\mathfrak{M}_0>}(0.00), \mathfrak{S}_{-3<\mathfrak{M}_0>}(0.33), \mathfrak{S}_{-3<\mathfrak{M}_1>}(0.67)\}$
Ip_4	$\{\mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.00), \mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.38), \mathfrak{S}_{2<\mathfrak{M}_{-1}>}(0.62)\}$	$\{\mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.38), \mathfrak{S}_{1<\mathfrak{M}_0>}(0.38), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.24)\}$
Ip_5	$\{\mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.00), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.17), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.83)\}$	$\{\mathfrak{S}_{2<\mathfrak{M}_0>}(0.11), \mathfrak{S}_{3<\mathfrak{M}_{-2}>}(0.23), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.66)\}$
	Υ_3	Υ_4
Ip_1	$\{\mathfrak{S}_{1<\mathfrak{M}_0>}(0.00), \mathfrak{S}_{1<\mathfrak{M}_0>}(0.45), \mathfrak{S}_{1<\mathfrak{M}_2>}(0.55)\}$	$\{\mathfrak{S}_{-1<\mathfrak{M}_0>}(0.00), \mathfrak{S}_{-1<\mathfrak{M}_0>}(0.77), \mathfrak{S}_{1<\mathfrak{M}_0>}(0.23)\}$
Ip_2	$\{\mathfrak{S}_{-2<\mathfrak{M}_2>}(0.32), \mathfrak{S}_{-1<\mathfrak{M}_0>}(0.33), \mathfrak{S}_{-1<\mathfrak{M}_2>}(0.35)\}$	$\{\mathfrak{S}_{-3<\mathfrak{M}_0>}(0.23), \mathfrak{S}_{-3<\mathfrak{M}_2>}(0.24), \mathfrak{S}_{-2<\mathfrak{M}_1>}(0.53)\}$
Ip_3	$\{\mathfrak{S}_{-1<\mathfrak{M}_0>}(0.28), \mathfrak{S}_{-1<\mathfrak{M}_1>}(0.31), \mathfrak{S}_{-1<\mathfrak{M}_2>}(0.41)\}$	$\{\mathfrak{S}_{-2<\mathfrak{M}_1>}(0.31), \mathfrak{S}_{-2<\mathfrak{M}_2>}(0.28), \mathfrak{S}_{-1<\mathfrak{M}_0>}(0.41)\}$
Ip_4	$\{\mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.00), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.56), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.44)\}$	$\{\mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.00), \mathfrak{S}_{1<\mathfrak{M}_{-2}>}(0.45), \mathfrak{S}_{2<\mathfrak{M}_0>}(0.55)\}$
Ip_5	$\{\mathfrak{S}_{2<\mathfrak{M}_0>}(0.34), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.19), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.47)\}$	$\{\mathfrak{S}_{3<\mathfrak{M}_{-2}>}(0.17), \mathfrak{S}_{3<\mathfrak{M}_{-1}>}(0.27), \mathfrak{S}_{3<\mathfrak{M}_0>}(0.56)\}$

for attribute Υ_2 , all other attributes are benefit attributes. The evaluation information of the attribute Υ_j for the alternative Λ_i is represented as $\mathbb{Z}_{ij} = \{\mathbb{Z}_{ij}^{(h)}(p_{ij}^{(h)}) | \mathbb{Z}_{ij}^{(h)} \in \Theta, p_{ij}^{(h)} > 0, \sum_{h=1}^{\#\mathbb{Z}_{ij}} p_{ij}^{(h)} \leq 1\}$ and $G = \{\mathbb{Z}_{ij}\}_{5 \times 4}$ is the decision matrix. The first and second hierarchy LTSs are as follows:

$$\begin{aligned} \mathfrak{S} &= \{\mathfrak{S}_{-3} = \textit{extremely poor}, \mathfrak{S}_{-2} = \textit{very poor}, \mathfrak{S}_{-1} = \textit{poor}, \mathfrak{S}_0 = \textit{medium}, \\ &\quad \mathfrak{S}_1 = \textit{good}, \mathfrak{S}_2 = \textit{very good}, \mathfrak{S}_3 = \textit{extremely good}\}, \\ \mathfrak{R} &= \{\mathfrak{R}_{-3} = \textit{far from}, \mathfrak{R}_{-2} = \textit{only a little}, \mathfrak{R}_{-1} = \textit{a little}, \mathfrak{R}_0 = \textit{just right}, \\ &\quad \mathfrak{R}_1 = \textit{much}, \mathfrak{R}_2 = \textit{very much}, \mathfrak{R}_3 = \textit{totally}\}. \end{aligned}$$

The following outlines the detailed procedures for addressing MADM issues using the PDHL-RT-TODIM approach. The expert decision matrix appears in Table 1.

Step 1. According to Definition 6, supplement the elements of PDHLTS so that each supplemented PDHLTS has the same number of elements. Normalize the evaluation information according to the Definition 5, as presented in Table 2.

Table 3
The overall evaluation value which eliminating the value under Υ_1 .

Υ_1	Υ_2
Ip_1 $\{\mathfrak{S}_0^{(0.2,58)}(0.08), \mathfrak{S}_1^{(0.9,91)}(0.52), \mathfrak{S}_3^{(0.0,00)}(0.40)\}$	$\{\mathfrak{S}_{-1}^{(0.0,66)}(0.16), \mathfrak{S}_{-1}^{(0.0,50)}(0.50), \mathfrak{S}_0^{(0.1,69)}(0.34), \}$
Ip_2 $\{\mathfrak{S}_{-3}^{(0.2,38)}(0.26), \mathfrak{S}_{-2}^{(0.0,41)}(0.27), \mathfrak{S}_{-2}^{(0.2,08)}(0.47)\}$	$\{\mathfrak{S}_{-2}^{(0.1,93)}(0.32), \mathfrak{S}_{-1}^{(0.0,64)}(0.27), \mathfrak{S}_{-1}^{(0.2,98)}(0.41)\}$
Ip_3 $\{\mathfrak{S}_{-3}^{(0.2,27)}(0.20), \mathfrak{S}_{-3}^{(0.2,80)}(0.31), \mathfrak{S}_{-2}^{(0.0,63)}(0.49)\}$	$\{\mathfrak{S}_{-2}^{(0.0,19)}(0.26), \mathfrak{S}_{-2}^{(0.0,93)}(0.27), \mathfrak{S}_{-2}^{(0.1,97)}(0.47)\}$
Ip_4 $\{\mathfrak{S}_0^{(0.2,30)}(0.13), \mathfrak{S}_0^{(0.2,91)}(0.46), \mathfrak{S}_3^{(0.0,0)}(0.41)\}$	$\{\mathfrak{S}_0^{(0.1,76)}(0.00), \mathfrak{S}_0^{(0.1,76)}(0.46), \mathfrak{S}_3^{(0.0,00)}(0.54)\}$
Ip_5 $\{\mathfrak{S}_1^{(0.1,85)}(0.21), \mathfrak{S}_2^{(0.0,49)}(0.23), \mathfrak{S}_3^{(0.0,00)}(0.56)\}$	$\{\mathfrak{S}_1^{(0.2,20)}(0.17), \mathfrak{S}_2^{(0.0,37)}(0.21), \mathfrak{S}_3^{(0.0,00)}(0.62)\}$
Υ_3	Υ_4
Ip_1 $\{\mathfrak{S}_0^{(0.2,38)}(0.23), \mathfrak{S}_1^{(0.1,23)}(0.46), \mathfrak{S}_3^{(0.0,00)}(0.31)\}$	$\{\mathfrak{S}_0^{(0.2,98)}(0.23), \mathfrak{S}_1^{(0.1,66)}(0.36), \mathfrak{S}_3^{(0.0,00)}(1.41)\}$
Ip_2 $\{\mathfrak{S}_{-2}^{(0.2,26)}(0.29), \mathfrak{S}_{-1}^{(0.1,04)}(0.24), \mathfrak{S}_0^{(0.0,50)}(0.37)\}$	$\{\mathfrak{S}_{-2}^{(0.2,98)}(0.32), \mathfrak{S}_{-1}^{(0.1,50)}(0.27), \mathfrak{S}_0^{(0.0,85)}(0.41)\}$
Ip_3 $\{\mathfrak{S}_{-3}^{(0.2,08)}(0.17), \mathfrak{S}_{-3}^{(0.2,64)}(0.28), \mathfrak{S}_{-2}^{(0.0,81)}(0.55)\}$	$\{\mathfrak{S}_{-3}^{(0.2,23)}(0.16), \mathfrak{S}_{-3}^{(0.2,74)}(0.29), \mathfrak{S}_{-2}^{(0.0,86)}(0.55)\}$
Ip_4 $\{\mathfrak{S}_{-1}^{(0.2,76)}(0.13), \mathfrak{S}_0^{(0.0,60)}(0.40), \mathfrak{S}_1^{(0.1,13)}(0.47)\}$	$\{\mathfrak{S}_0^{(0.2,24)}(0.13), \mathfrak{S}_0^{(0.2,86)}(0.44), \mathfrak{S}_3^{(0.0,00)}(0.43)\}$
Ip_5 $\{\mathfrak{S}_2^{(0.0,13)}(0.09), \mathfrak{S}_2^{(0.0,89)}(0.23), \mathfrak{S}_3^{(0.0,00)}(0.68)\}$	$\{\mathfrak{S}_1^{(0.2,39)}(0.15), \mathfrak{S}_2^{(0.0,41)}(0.30), \mathfrak{S}_3^{(0.0,00)}(0.65)\}$

Table 4
The CC.

	Υ_1	Υ_2	Υ_3	Υ_4
CC	0.5922	0.7205	0.7951	0.7712

Table 5
The weights of attributes.

	Υ_1	Υ_2	Υ_3	Υ_4
ω	0.2349	0.3194	0.2014	0.2442

Use the CCSD method in the PDHL environment to calculate the weights of attributes. The specific steps are shown in Tables 3–5.

Step 2. Equation (9) excluded the assessed value of alternatives under attribute and calculated the total appraisalment value of alternatives; excluded the assessed value of each alternative under attribute Υ_1 , listed out the c total appraisalment value of alternatives in Table 3. Similarly, we can list out the total appraisalment value of alternatives after eliminating the assessed value of alternatives under attribute Υ_2, Υ_3 or Υ_4 .

Step 3. Calculate the CC between attribute Υ_1 and overall evaluation information using equation (10), as presented in Table 4.

Step 4. Calculate the e weights of attributes by equations (12) to (13), as presented in Table 5.

Use PDHL-RT-TODIM method for ranking alternatives. The specific steps were shown in Tables 6–12.

Step 1. The relative weights of attributes are determined by equation (14) and are presented in Table 6.

Table 6
The relative weights of attributes.

	Υ_1	Υ_2	Υ_3	Υ_4
ϖ	0.6937	1.0000	0.5170	0.7687

Table 7
The overall benefit of alternative Ip_i relative to the other alternatives.

	Ip_1	Ip_2	Ip_3	Ip_4	Ip_5
Ip_1	0.0000	-1.0950	-1.6134	0.2070	0.3074
Ip_2	0.3644	0.0000	-0.5038	0.2167	0.9626
Ip_3	0.8050	0.0102	0.0000	0.8777	0.9728
Ip_4	-0.9113	-1.1531	-1.8202	0.0000	0.0164
Ip_5	-1.2212	-1.9763	-1.9609	-0.8486	0.0000

Step 2. Calculate the advantage of alternative Ip_i over other alternatives under attribute Υ_j by equation (15), where $\theta = 2$. The calculation results are shown in matrix φ_j ($j = 1, 2, 3, 4$).

$$\varphi_1 = \begin{pmatrix} 0.0000 & 0.1073 & -0.3448 & 0.2059 & 0.2588 \\ -0.2304 & 0.0000 & -0.3941 & -0.4235 & 0.2563 \\ 0.1606 & 0.1835 & 0.0000 & 0.1861 & 0.2401 \\ -0.4421 & 0.1972 & -0.3996 & 0.0000 & 0.1715 \\ -0.5557 & -0.5505 & -0.5156 & -0.3683 & 0.0000 \end{pmatrix},$$

$$\varphi_2 = \begin{pmatrix} 0.0000 & -0.3950 & -0.4774 & -0.2806 & -0.3314 \\ 0.2651 & 0.0000 & -0.2681 & 0.2059 & 0.2575 \\ 0.3204 & 0.1800 & 0.0000 & 0.2621 & 0.3142 \\ 0.1884 & -0.3067 & -0.3904 & 0.0000 & 0.2602 \\ 0.2224 & -0.3836 & -0.4680 & -0.3876 & 0.0000 \end{pmatrix},$$

$$\varphi_3 = \begin{pmatrix} 0.0000 & -0.5144 & -0.4989 & 0.1181 & 0.1606 \\ 0.1785 & 0.0000 & 0.0491 & 0.2141 & 0.1829 \\ 0.1732 & -0.1415 & 0.0000 & 0.2096 & 0.1762 \\ -0.3403 & -0.6168 & -0.6039 & 0.0000 & -0.5642 \\ -0.4628 & -0.5270 & -0.5076 & 0.1958 & 0.0000 \end{pmatrix},$$

$$\varphi_4 = \begin{pmatrix} 0.0000 & -0.2929 & -0.2923 & 0.1637 & 0.2193 \\ 0.1511 & 0.0000 & 0.1093 & 0.2202 & 0.2658 \\ 0.1508 & -0.2118 & 0.0000 & 0.2200 & 0.2423 \\ -0.3173 & -0.4268 & -0.4264 & 0.0000 & 0.1489 \\ -0.4251 & -0.5152 & -0.4696 & -0.2886 & 0.0000 \end{pmatrix}.$$

Step 3. The overall benefit of each alternative compared to the other alternatives is obtained with equation (16) and is presented in Table 7.

Table 8
The comprehensive advantage of each alternative.

	I_{p1}	I_{p2}	I_{p3}	I_{p4}	I_{p5}
$\mathbb{N}(A_i)$	-0.9631	-4.2141	-5.8983	0.4529	2.2592

Table 9
The normalized comprehensive advantage of alternatives.

	I_{p1}	I_{p2}	I_{p3}	I_{p4}	I_{p5}
$Z(A_i)$	0.6050	0.2065	0.0000	0.7786	1.0000

Table 10
The utility value of alternatives.

	I_{p1}	I_{p2}	I_{p3}	I_{p4}	I_{p5}
$UV(A_i)$	0.6426	0.2495	0.0000	0.8023	1.0000

Table 11
The regret-joy value of alternatives.

Alternative	$RV^-(A_i)$	$RV^+(A_i)$
I_{p1}	-0.1132	0.1753
I_{p2}	-0.2525	0.0721
I_{p3}	-0.3499	0.0000
I_{p4}	-0.0611	0.2139
I_{p5}	0.0000	0.2592

Table 12
The utility value and regret-joy value of alternatives.

Alternative	Value	Ranking
I_{p1}	0.7048	
I_{p2}	0.0691	
I_{p3}	-0.3499	$I_{p5} > I_{p4} > I_{p1} > I_{p2} > I_{p3}$
I_{p4}	0.9551	
I_{p5}	1.2592	

Step 4. The comprehensive advantage of each alternative is obtained with equation (17) and is presented in Table 8.

Step 5. The normalized comprehensive advantage of alternatives is determined with equation (18) and is presented in Table 9.

Step 6. Calculate the utility value and regret-joy value of alternatives with equations (19)–(21), as shown in Tables 10–12. Calculate the utility value of alternatives with equation (19), as presented in Table 10.

Calculate the regret-joy value of alternatives with equation (20), as presented in Table 11.

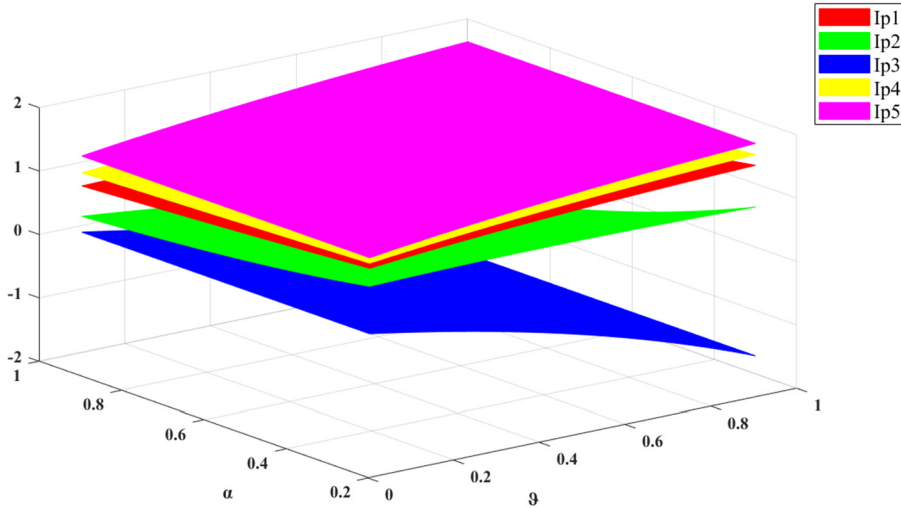


Fig. 1. Analysis of the impact of parameters α and ϑ change in the PDHL-RT-TODIM method on alternative evaluation values.

Calculated the utility value and regret-joy value of alternatives by equation (21) and presented in Table 12.

From Table 12 the Ip_5 is the best alternative and Ip_3 is the worst alternative.

4.2. Parameter Analysis

Due to the influence of parameters α , ϑ and θ on DMs' risk preferences, it was necessary to explore the impact of changes in these three parameter values on the ranking of alternatives.

i) Set the parameter $\theta = 2$, while changing the values of parameters α , ϑ , and increasing them from 0 to 1. Fig. 1 displays the specific results to analysis how various parameter values affect the evaluation outcomes.

α and ϑ represent the risk attitude and regret aversion coefficient, as the values of α and ϑ increase, the likelihood of DM being at risk also rises. From Fig. 1, it can be seen that when parameters α , $\vartheta \rightarrow 0$, the evaluation values of the alternatives are most concentrated. As the value of parameter α or ϑ increases, the values of all alternatives increase beyond for alternative Ip_3 . In addition, when the parameters α , $\vartheta \rightarrow 1$, the evaluation value of the alternatives has the highest degree of dispersion, and the alternative A_3 achieves the minimum evaluation value. But the sorting result always remains $Ip_5 > Ip_4 > Ip_1 > Ip_2 > Ip_3$. It is evident that the PDHL-RT-TODIM method constructed in this section has strong sensitivity and stability to changes in parameters α and ϑ .

ii) Set the parameter $\alpha = 0.88$, $\vartheta = 0.3$, while changing the values of parameter θ , and increase θ from 0.2 with intervals of 0.02 units to 5. Fig. 2 displays the specific results to analysis how various parameter values affect the evaluation outcomes.

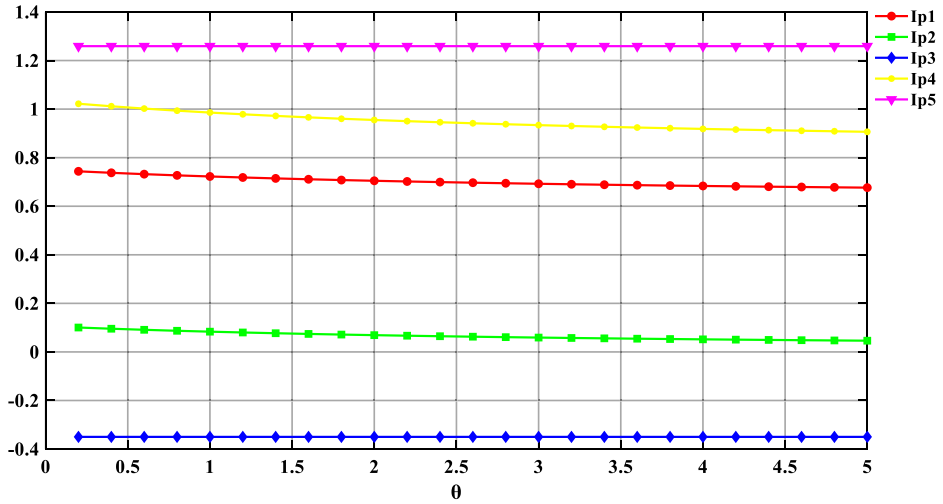


Fig. 2. Analysis of the impact of parameter θ change in the PDHL-RT-TODIM method on compromise solution of the alternative.

θ represents the DM’s sensitive attitude towards losses, and as the parameter value increases, the DM becomes more conservative when facing risks. From Fig. 2, it can be seen that as the parameter θ changes, the PDHL-RT-TODIM method constructed in this section exhibits weak flexibility, and the sorting results always maintain $Ip_5 > Ip_4 > Ip_1 > Ip_2 > Ip_3$. The PDHL-RT-TODIM method constructed in this section has weak sensitivity to parameter changes.

In summary, with different parameter values of $\alpha, \vartheta, \theta$, the PDHL-RT-TODIM method constructed in this section exhibits strong flexibility and stability, and the parameters in this method have a strong moderating effect on the model, effectively expressing the DM’ preference attitude. Therefore, in practical decision-making, DMs can choose different parameter values based on their preferences and the needs of the actual problem.

4.3. Comparative Analysis

The PDHL-RT-TODIM method is compared with the existing TODIM method (Gomes et al., 2009), PDHL-WASPAS method (Liu, P. et al., 2023a) and PDHL-VIKOR method (Gou et al., 2021), as shown in Tables 13 to 15.

To facilitate comparison, Tables 16 and 17 present the rankings and features of various methods.

Table 13
Value and ranking of alternatives obtained by PDHL-TODIM method.

	Ip_1	Ip_2	Ip_3	Ip_4	Ip_5
Value	0.6050	0.2065	0.3211	0.7786	0.8943
Ranking	$Ip_5 > Ip_4 > Ip_1 > Ip_3 > Ip_2$				

Table 14
Value and ranking of alternatives obtained by PDHL-WASPAS method.

	I_{p1}	I_{p2}	I_{p3}	I_{p4}	I_{p5}
Value	0.4369	0.2914	0.2392	0.5366	0.6192
Ranking	$I_{p5} > I_{p4} > I_{p1} > I_{p2} > I_{p3}$				

Table 15
Value and ranking of alternatives obtained by PDHL- VIKOR method.

	I_{p1}	I_{p2}	I_{p3}	I_{p4}	I_{p5}
Value	0.5690	0.7853	0.8778	0.3801	0.4167
Ranking	$I_{p4} > I_{p5} > I_{p1} > I_{p2} > I_{p3}$				

Table 16
Ranking summary of different methods.

Method	Ranking
TODIM method (Gomes <i>et al.</i> , 2009)	$I_{p5} > I_{p4} > I_{p1} > I_{p3} > I_{p2}$
PDHL-WASPAS method (Liu, P. <i>et al.</i> , 2023a)	$I_{p5} > I_{p4} > I_{p1} > I_{p2} > I_{p3}$
PDHL-VIKOR method (Gou <i>et al.</i> , 2021)	$I_{p4} > I_{p5} > I_{p1} > I_{p2} > I_{p3}$
PDHL-RT-TODIM method	$I_{p5} > I_{p4} > I_{p1} > I_{p2} > I_{p3}$

Table 17
Characteristics of different methods.

Method	Can the evaluation information of DMs be expressed	Can information be flexibly aggregated based on parameters	Can the psychological behaviour of DMs be captured	Method for determining attribute weights
TODIM(Gomes <i>et al.</i> , 2009)	Yes	No	No	–
PDHL-WASPAS (Liu, P. <i>et al.</i> , 2023a)	Yes	No	No	SNA method
PDHL-VIKOR (Gou <i>et al.</i> , 2021)	Yes	No	No	–
PDHL-RT-TODIM	Yes	Yes	Yes	CCSD method

From Table 16–17, it can be seen that except for the PDHL-RT-TODIM method, the ranking results of alternative solutions obtained by the PDHL-RT-TODIM method are the same as other existing methods. However, the worst alternative to the PDHL-RT-TODIM method is consistent with the PDHL taxonomy method. The reason for the difference in ranking results between the PDHL classification method and other methods is that in the PDHL environment, this method is relatively rough and cannot effectively determine the superiority between alternative solutions with similar expected values (from the evaluation or expected values of the above methods, it is clear that the overall evaluation information value of the alternative is nearly identical to that of the other alternatives). The PDHL-WASPAS method determines the relative importance and initial priority of each

alternative based on decision information, and then ranks the alternative based on this. The PDHL classification method mainly ranks alternative solutions by measuring their differences through utility functions. The PDHL-RT-TODIM method retains the core idea of the classical TODIM method, while also making some improvements. Specifically, considering that the regret theory in behavioural psychology can reflect the regret and disgust psychology of DMs, accurately reflecting their emotions and motivations throughout the decision-making process, the perceived utility function of RT is extended to the TODIM method to compensate for the shortcomings of the TODIM method in representing DMs' regret and disgust psychology. Obtain the ranking results that best match the perpetrator's regret and disgust psychology. Therefore, it is known that the PDHL-RT-TODIM model proposed in this article is scientifically effective.

Based on the information provided, this paper offers three key contributions in comparison to current methods:

- (1) As indicated in Section 4.2 and Table 17, it is evident that the approach suggested in this article is the only one capable of achieving flexible information aggregation through parameter adjustments. Specifically, the method proposed in this article contains three parameters α , ϑ and θ , where parameter θ comes from the TODIM method and is used to reflect the DMs sensitive attitude towards losses. The parameters α , ϑ are derived from regret theory and are used to reflect the DMs risk attitude coefficient and regret aversion coefficient. This makes the method proposed in this article more flexible than the other methods mentioned above.
- (2) The approach suggested in this article is the only one that considers the regret and disgust psychology of the DMs, in order to fully simulate their emotions and motivations throughout the entire decision-making process.
- (3) When dealing with MADM, the PDHL-RT-TODIM method is more reasonable and scientific in determining attribute weights than the method of directly assigning them by experts mentioned above. The specific content is as follows: This article enhances the CCSD method by integrating the standard deviation approach with the similarity method, introducing the PDHL-CCSD method, which considers both the standard deviation of attributes and their interrelationships within the PDHL environment. A non-linear optimization model was developed to determine the target weights of attributes using this method. By solving this model to derive the objective weights of attributes, it expands the available techniques for determining attribute weights in the PDHL environment.

5. Conclusion

Since RT in behavioural psychology can accurately depict the emotions and motivations of DMs during the MADM process, it is integrated into this process. The traditional TODIM method is improved for the PDHLTs environment, resulting in the creation of the PDHL-RT-TODIM method, which seeks to thoroughly capture the emotions and motivations of DMs throughout their decision-making journey. Furthermore, the CCSD method is refined to effectively and reasonably assess attribute weights in the PDHL environment

when these weights are completely unknown. The practicality of the proposed method is demonstrated through numerical examples for information system investment project selection, and its stability, effectiveness, and benefits are further confirmed through sensitivity analysis of parameters and comparisons with existing methods. As a result, the method presented in this section not only accurately reflects the psychological and behavioural characteristics of DMs with different risk attitudes when assessing gains and losses but also accommodates their preferences by allowing for parameter adjustments.

This article integrates RT into the decision-making process, which can accurately describe the emotions and motivations of DM in the MADM process, making the decision-making results more accurate and effective. In future research, the method established in this article has certain guiding significance for other practical MADM problems (He and Jiang, 2025; Liao *et al.*, 2023; Zhang *et al.*, 2025). However, the discussion in this article about the psychological factors of decision-makers in the decision-making process is incomplete. If the impact of decision-makers' preference attitudes towards profit loss on decision-making results is not discussed, we will focus on exploring this aspect in the future.

Compliance with ethical standards

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

The data used to support the findings of this study are included within the article.

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