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ON DESIGN OF OPTIMAL AND TUNED INVERSE MODELS FOR THE MINIMUM-PHASE SYSTEM

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Abstract. The aim of the given paper is the development of optimal and tuned models and ordinary well-known on-line procedures of unknown parameter estimation for inverse systems (IS) using current observations to be processed. Such models of IS are worked out in the case of correlated additive noise acting on the output of the initial direct system (DS). The results of numerical investigation by means of computer (Table 1) are given.

Key words: optimal model, inverse system, parameter estimation, signal reconstruction, recursive algorithms.

1. Statement of the problem. By identification and parameter estimation of real objects it is often assumed that an additive noise affecting the output of the initial DS is formed by a filter with some fractional transfer function (Åström and Eykhoff, 1971). Therefore various parameter estimation approaches and algorithms, based on this assumption are worked out in Åström and Eykhoff (1971), Isermann (1974), Cypkin (1984). However, in measurement engineering there often appears an inverse problem requiring for the input determination using the known both the noisy output and the DS operator. Such a problem also arises in time delay of DS estimation when a corrective operator is used and the estimates of its parameter may be obtained (Pupeikis, 1985). On the other hand, it is known (Avedjan and Cypkin, 1987) that the optimal IS is not able to exactly reconstruct the unobserved input even in the case of a minimum-phase DS. Therefore it is worthwhile to investigate the possibilities of developing such models of IS that could be used for reconstruction of an unobserved input signal with the least error in the mean square sense. That will also allow us to apply, in such a case, the recursive algorithms, used for unknown parameter estimation of DS by observations.

Assume that the observed k-th meaning of the output sequence U_k of a linear discrete system is

$$u_k = y_k + N_k, \tag{1}$$

where y_k is the meaning of a useful component of the given sequence, related with the input sequence by the difference equation

$$y_k = \frac{B(z^{-1})}{1 + A(z^{-1})} x_k, \tag{2}$$

and the noise N_k is a stationary process of the shape

$$N_k = W_N(z^{-1})\xi_k, \tag{3}$$

$W_N(z^{-1})$ is the transfer function of the filter which is forming the additive noise N_k from a sequence of independent Gaussian variables $\xi_k \sim N(0,1)$; z^{-1} is the backward shift operator defined by $z^{-1}\ x_k = x_{k-1}$.

The input signal is assumed to be a persistent excitation of an arbitrary order according to Åström and Eykhoff (1971) and N_k to be independent of x_k; the polynomials

$$A(z^{-1}) = \sum_{i=1}^{n} a_i z^{-i}, \qquad B(z^{-1}) = \sum_{i=0}^{m} b_i z^{-i} \tag{4}$$

have no common roots and they are outside the unit circle of the z^{-1} plane. The orders n and m are known beforehand and $b_0 \neq 0$.

2. Optimal models of IS. Eq. 1 can be rewritten in such a form

$$u_k = K_x(z^{-1})x_k + W_N(z^{-1})\xi_k, \tag{5}$$

where

$$K_x(z^{-1}) = \frac{B(z^{-1})}{1 + A(z^{-1})}. \tag{6}$$

Suppose that for $K_x(z^{-1})$ there exists such a transfer function of the inverse system

$$K_0(z^{-1}) = \frac{B_0(z^{-1})}{1 + A_0(z^{-1})}, \tag{7}$$

for which the equality

$$K_x(z^{-1})K_0(z^{-1}) = 1 \tag{8}$$

is satisfied.

In Eq. 7

$$B_0(z^{-1}) = \sum_{i=l_1}^{m_0} b_{0i}z^{-i}, \quad A_0(z^{-1}) = \sum_{i=l_2}^{n_0} a_{0i}z^{-i}. \tag{9}$$

Then from Eq. 5 it is easy to determine the k-th input meaning in such a way

$$x_k = K_0(z^{-1})u_k - K_0(z^{-1})W_N(z^{-1})\xi_k. \tag{10}$$

As optimal models of an IS we can apply here the equation of the shape

$$x_k^{(1)} = B_0(z^{-1})u_k - A_0(z^{-1})x_k, \tag{11}$$

in which the observed meanings x_{k-i}, u_{k-j} $(i = \overline{l_2, n_0})$, $(j = \overline{l_1, m_0})$ are used for prediction of the current input meaning as well as the equation

$$x_k^{(2)} = \widetilde{K}_0(z^{-1})u_k = \frac{\widetilde{B}_0(z^{-1})}{1 + \widetilde{A}_0(z^{-1})}u_k, \tag{12}$$

in which only the current meaning u_k and some of its previous meanings are required for the same purpose.

In Eq. 12

$$\widetilde{B}_0(z^{-1}) = \sum_{i=l_1}^{\widetilde{m}_0} \widetilde{b}_{0i}z^{-i}, \quad \widetilde{A}_0(z^{-1}) = \sum_{i=l_2}^{\widetilde{n}_0} \widetilde{a}_{0i}z^{-i}. \tag{13}$$

Model (11) approximately corresponds to the statical model and model (12) to the dynamic one according to Cypkin (1984).

The designing of the optimal inverse models of the DS is reduced to the solution of the problem of minimizing the variance of residuals

$$e_k = x_k - x_k^{(1)}, \qquad M\{e_k\} = 0, \tag{14}$$

$$\varepsilon_k = x_k - x_k^{(2)}, \qquad M\{\varepsilon_k\} = 0, \tag{15}$$

which are functionals

$$M\{e_k^2\} = J(B_0, A_0) \longrightarrow \min_{B_0, A_0}, \quad (16)$$

$$M\{\varepsilon_k^2\} = J(\widetilde{K}_0) \longrightarrow \min_{\widetilde{K}_0}, \quad (17)$$

and depend on transfer functions B_0, A_0 and $\widetilde{K}_0$, respectively. Here $B_0 \equiv B_0(z^{-1})$, $A_0 \equiv A_0(z^{-1})$, $\widetilde{K}_0 \equiv \widetilde{K}_0(z^{-1})$, $M\{\cdot\}$ is the mean value.

If equations (10), (11), (14) and (10), (12), (15) are taken into account then equations (14) and (15) can be rewritten in the form

$$e_k = x_k - x_k^{(1)} = D_x(z^{-1})u_k + \widetilde{W}_\xi(z^{-1})\xi_k, \quad (18)$$

$$\varepsilon_k = x_k - x_k^{(2)} = x_k - \widetilde{K}_0(z^{-1})u_k; \quad (19)$$

or

$$e_k = W_x(z^{-1})x_k + W_\xi(z^{-1})\xi_k, \quad (20)$$

$$\varepsilon_k = W_x^*(z^{-1})x_k + W_\xi^*(z^{-1})\xi_k, \quad (21)$$

by replacing u_k from Eq. 5.

Here

$$W_x(z^{-1}) = D_x(z^{-1})K_x(z^{-1}), \quad (22)$$

$$W_\xi(z^{-1}) = D_x(z^{-1})W_N(z^{-1}) + \widetilde{W}_\xi(z^{-1}), \quad (23)$$

$$D_x(z^{-1}) = K_0(z^{-1}) - B_0(z^{-1}) + A_0(z^{-1})K_0(z^{-1}), \quad (24)$$

$$\widetilde{W}_\xi(z^{-1}) = -\widetilde{W}_N(z^{-1})\widetilde{A}_0(z^{-1}), \quad (25)$$

$$\widetilde{W}_N(z^{-1}) = K_0(z^{-1})W_N(z^{-1}), \quad (26)$$

$$W_x^*(z^{-1}) = 1 - \widetilde{K}_0(z^{-1})K_x(z^{-1}), \quad (27)$$

$$W_\xi^*(z^{-1}) = -\widetilde{K}_0(z^{-1})W_N(z^{-1}), \quad (28)$$

$$\widetilde{A}_0(z^{-1}) = 1 + A_0(z^{-1}). \quad (29)$$

Functionals (16) and (17) can be expressed through the transfer functions $W_x(z^{-1})$, $W_\xi(z^{-1})$ and $W_x^*(z^{-1})$, $W_\xi^*(z^{-1})$ as well as through spectral density functions $S_x(z^{-1})$, $S_\xi(z^{-1})$, respectively (Cypkin, 1984). Thus, (16) and

(17) can be presented in the form

$$J(B_0, A_0) = M\{e_k^2\} = \frac{1}{2\pi j} \oint\limits_L \Big[W_x(z^{-1})W_x(z)S_x(z^{-1}) + W_\xi(z^{-1})W_\xi(z)S_\xi(z^{-1})\Big] \frac{dz^{-1}}{z^{-1}}, \quad (30)$$

$$J(\widetilde{K}_0) = M\{\varepsilon_k^2\} = \frac{1}{2\pi j} \oint\limits_L \Big[W_x^*(z^{-1})W_x^*(z)S_x(z^{-1}) + W_\xi^*(z^{-1})W_\xi^*(z)S_\xi(z^{-1})\Big] \frac{dz^{-1}}{z^{-1}}, \quad (31)$$

where the contour of integration L is the circle of unit radius with the centre at the beginning of the coordinates; the spectral density function $S_\xi(z^{-1}) = \sigma_\xi^2$; $S_x(z^{-1})$ depends on the statistical properties of x_k and is different for different inputs.

Let us require that the structure of IS models (11) and (12) not depended on x_k. It occurs when

$$W_x(z^{-1}) = \big[K_0(z^{-1}) - B_0(z^{-1}) + A_0(z^{-1})K_0(z^{-1})\big]K_x(z^{-1}) = 0 \quad (32)$$

for model (11) and

$$W_x^*(z^{-1}) = 1 - \widetilde{K}_0(z^{-1})K_x(z^{-1}) = 0 \quad (33)$$

for model (12). Then functionals (30) and (31) get simplified and take such forms

$$J(B_0, A_0) = \frac{\sigma_\xi^2}{2\pi j} \oint\limits_L \widetilde{W}_\xi(z^{-1})\widetilde{W}_\xi(z) \frac{dz^{-1}}{z^{-1}}, \quad (34)$$

$$J(K_0) = \frac{\sigma_\xi^2}{2\pi j} \oint\limits_L W_\xi^*(z^{-1})W_\xi^*(z) \frac{dz^{-1}}{z^{-1}}, \quad (35)$$

or

$$J(B_0, A_0) = \frac{\sigma_\xi^2}{2\pi j} \oint\limits_L \widetilde{W}_N(z^{-1})\widetilde{W}_N(z)\widetilde{A}_0(z^{-1})\widetilde{A}_0(z) \frac{dz^{-1}}{z^{-1}}, \quad (36)$$

$$J(K_0) = \frac{\sigma_\xi^2}{2\pi j} \oint\limits_L \widetilde{K}_0(z^{-1})\widetilde{K}_0(z)W_N(z^{-1})W_N(z) \frac{dz^{-1}}{z^{-1}}, \quad (37)$$

by taking $\widetilde{W}_\xi(z^{-1})$ and $W_\xi^*(z^{-1})$ from (25) and (28), respectively.

The integrands in (36), (37) are equal to a unit, while circulation integrals – to $2\pi j$, and hence

$$M\{e_k^2\} = \sigma_\xi^2,$$
$$M\{\varepsilon_k^2\} = \sigma_\xi^2,$$

if

$$B_0(z^{-1}) = -W_N^{-1}(z^{-1}),$$
$$A_0(z^{-1}) = -1 - K_x(z^{-1})W_N^{-1}(z^{-1}),$$

for model (11), and

$$\widetilde{K}_0(z^{-1}) = K_x^{-1}(z^{-1}),$$
$$K_x(z^{-1}) = -W_N(z^{-1}),$$

for (12).

In view of that we can rewrite equations (11) and (12) in such a way

$$x_k^{(1)} = -\,W_N^{-1}(z^{-1})u_k + \left[1 + K_x(z^{-1})W_N^{-1}(z^{-1})\right] x_k, \tag{38}$$
$$x_k^{(2)} = K_x^{-1}(z^{-1})u_k = -W_N^{-1}(z^{-1})u_k. \tag{39}$$

IS models (38) and (39) are optimal, because they guarantee the minimal values of functionals (16) and (17) to be minimized in the case of certain parameter groups of DS and IS.

Now we define the transfer function of a digital correction filter used for reconstruction of x_k in such a way

$$\widehat{x}_k = \widetilde{K}_0(z^{-1})u_k = \frac{\widetilde{B}_0(z^{-1})}{1 + \widetilde{A}_0(z^{-1})}u_k. \tag{40}$$

Then for the minimum-phase system

$$\widetilde{K}_0(z^{-1}) = \widetilde{B}_0(z^{-1})\left[1 + \widetilde{A}_0(z^{-1})\right]^{-1} = B^{-1}(z^{-1})\left[1 + A(z^{-1})\right]. \tag{41}$$

If the transfer function of $W_N(z^{-1})$ has the form

$$W_N(z^{-1}) = -\frac{B(z^{-1})}{1 + A(z^{-1})}, \tag{42}$$

then the measure of the deviation $\widehat{x}_k$ from x_k assumes the form

$$J(\widetilde{K}_0) = M\left\{(x_k - \widehat{x}_k)^2\right\} = \sigma_\xi^2, \tag{43}$$

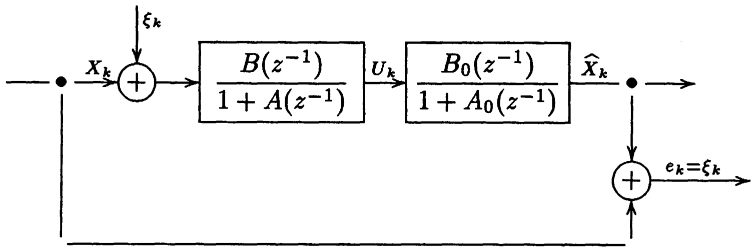

Fig. 1. Optimal digital correction filter.

ensuring the boundary accuracy of x_k reconstruction by processing u_k in the presence of additive noise. It can be mentioned that $W_N(z^{-1})$ of the shape (42) leads to ξ_k at the input of the initial DS (see Fig. 1).

3. Design of tuned models of the IS. We will use here difference equation (11) as a model of an IS and further we will consider different cases of additive noise acting on the output. Then the residual between current meanings of input x_k and its reconstructed values $\widehat{x}_k^{(1)}$ can be calculated according to Fig. 2. Considering equations (2) and (3) we rewrite expression (14) in such a way:

$$\begin{aligned} e_k &= \left[1 + A_0(z^{-1})\right] x_k - B_0(z^{-1}) u_k \\ &= \left[1 + A_0(z^{-1})\right] x_k - B_0(z^{-1}) \left\{ \frac{B(z^{-1})}{1 + A(z^{-1})} x_k + W_N(z^{-1}) \xi_k \right\}, \quad (44) \end{aligned}$$

or otherwise

$$e_k = \left[1 + A_0(z^{-1})\right] x_k - \frac{B_0(z^{-1}) B(z^{-1})}{1 + A(z^{-1})} x_k - B_0(z^{-1}) W_N(z^{-1}) \xi_k. \quad (45)$$

Since equality (8) is valid and accordingly

$$e_k = \xi_k,$$

in this case the mean-square error criterion

$$Q(\beta_0) = \min_{\beta} M\{e_k^2\} = \sigma_\xi^2, \quad (46)$$

if

$$W_N(z^{-1}) = -B_0^{-1}(z^{-1}), \quad (47)$$

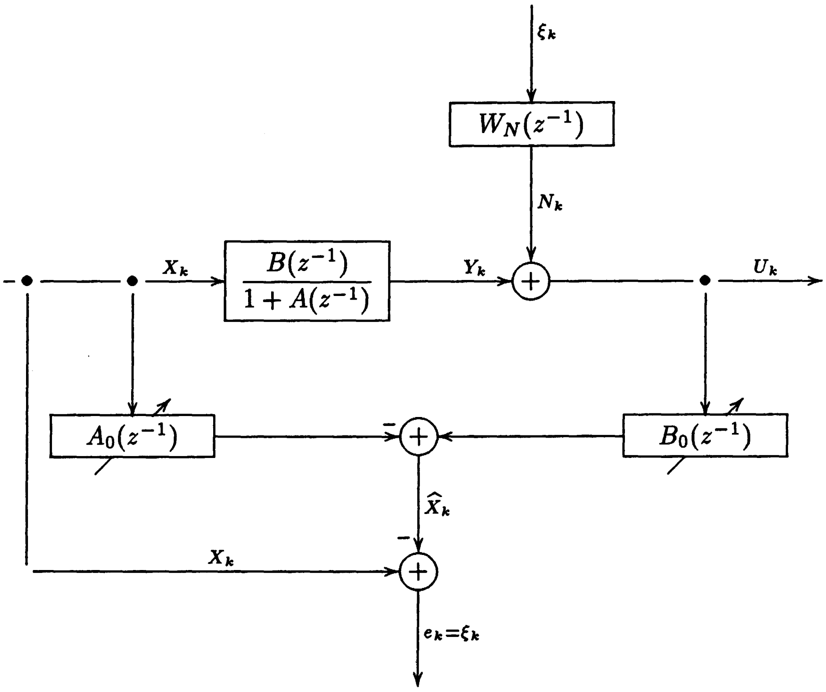

Fig. 2. Tuned inverse model of an DS.

where

$$\begin{aligned} \beta^T &= (\mathbf{a}_0^T, \mathbf{b}_0^T), \\ \mathbf{a}_0^T &= (a_{0l_2}, \dots, a_{0n_0}), \\ \mathbf{b}_0^T &= (b_{0l_1}, \dots, b_{0m_0}) \end{aligned}$$

are parameter vectors of the IS model to be estimated; n_0, m_0 are orders of the polynomials $A_0(z^{-1})$ and $B_0(z^{-1})$, respectively.

Model (11), (47) is the optimal one, because in the presence of certain parameter meanings of IS and DS the minimal value of functional (16) is guaranteed. For current calculation of the estimates

$$\begin{aligned} \widehat{\beta}^T &= \left(\widehat{\mathbf{a}}_0^T, \widehat{\mathbf{b}}_0^T\right), \\ \widehat{\mathbf{a}}_0^T &= (\widehat{a}_{0l_2}, \dots, \widehat{a}_{0n_0}), \end{aligned}$$

$$\widehat{\mathbf{b}}_0^T = \left(\widehat{b}_{0l_1}, \ldots, \widehat{b}_{0m_0}\right),$$

it is worthwhile to apply the recursive least squares (RLS) algorithm of the shape

$$\widehat{\beta}_{k+1} = \widehat{\beta}_k + \mathbf{\Gamma}_{k+1} \nabla_\beta e_{k+1} e_{k+1}, \tag{48}$$

$$\mathbf{\Gamma}_{k+1} = \mathbf{\Gamma}_k - \frac{\mathbf{\Gamma}_k \nabla_\beta e_{k+1} \nabla_\beta^T e_{k+1} \mathbf{\Gamma}_k}{1 + \nabla_\beta^T e_{k+1} \mathbf{\Gamma}_k \nabla_\beta e_{k+1}}, \tag{49}$$

where

$$e_{k+1} = x_{k+1} - \widehat{x}_{k+1}^{(1)}, \tag{50}$$

when

$$\widehat{x}_{k+1}^{(1)} = \widehat{B}_0(z^{-1}) u_{k+1} - \widehat{A}_0(z^{-1}) x_{k+1}, \tag{51}$$

and

$$\nabla_\beta e_{k+1} = \left(-x_{k+1-l_2}, \ldots, -x_{k+1-n_0},\ u_{k+1-l_1}, \ldots, u_{k+1-m_0}\right)^T. \tag{52}$$

Here

$$\widehat{A}_0(z^{-1}) = \sum_{i=l_2}^{n_0} \widehat{a}_{0i} z^{-i}, \qquad \widehat{B}_0(z^{-1}) = \sum_{i=l_1}^{m_0} \widehat{b}_{0i} z^{-i}. \tag{53}$$

Condition (47) strictly limits the application of recursive procedures used to calculate the parameter estimates of IS. Therefore, by analogy with recursive approaches used for parameter estimation of DS, we present here the residual (50) in the form

$$e_{k+1} = x_{k+1}^* - \widehat{x}_{k+1}^{(1)*}, \tag{54}$$

where

$$\widehat{x}_{k+1}^{(1)*} = B_0(z^{-1}) u_{k+1}^* - A_0(z^{-1}) x_{k+1}^*, \tag{55}$$

$$u_k^* = \left[1 + G(z^{-1})\right] u_k, \quad x_k = \left[1 + G(z^{-1})\right] x_k, \tag{56}$$

if

$$W_N(z^{-1}) = -B_0^{-1}(z^{-1}) \left[1 + G(z^{-1})\right]^{-1}, \tag{57}$$

or

$$u_k^* = \left[1 + F(z^{-1})\right]^{-1} u_k, \qquad x_k^* = \left[1 + F(z^{-1})\right]^{-1} x_k, \tag{58}$$

if

$$W_N(z^{-1}) = -B_0^{-1}(z^{-1})\left[1 + F(z^{-1})\right]. \tag{59}$$

Then we rewrite (54) in the form

$$\begin{aligned} e_k =& [1 + A_0(z^{-1})]x_k^* - B_0(z^{-1})u_k^* = [1 + A_0(z^{-1})]x_k^* \\ &- B_0(z^{-1})\left\{\left[\frac{B(z^{-1})}{1 + A(z^{-1})}x_k + W_N(z^{-1})\xi_k\right][1 + G(z^{-1})]\right\}, \end{aligned} \tag{60}$$

if $(u_k^*,\ x_k^*)$ are calculated according to (56) or

$$\begin{aligned} e_k =& [1 + A_0(z^{-1})]x_k^* - B_0(z^{-1})u_k^* = [1 + A_0(z^{-1})]x_k^* \\ &- B_0(z^{-1})\left\{\left[\frac{B(z^{-1})}{1 + A(z^{-1})}x_k + W_N(z^{-1})\xi_k\right][1 + F(z^{-1})]\right\}, \end{aligned} \tag{61}$$

if $(u_k^*,\ x_k^*)$ are obtained using the expression (58). It can be mentioned that in both cases

$$e_k = \xi_k, \tag{62}$$

because equality (8) is satisfied.

Further, for estimation of the parameters $\beta_0^T = (\mathbf{a}_0^T,\ \mathbf{b}_0^T)$ we can use the recursive generalized least squares (RGLS) algorithm, if $W_N(z^{-1})$ is of the shape (57), and that of the recursive maximum likelihood (RML) one if $W_N(z^{-1})$ is of the shape (59).

4. The relation between DS and IS. For minimum-phase systems Eq. 8 is satisfied, if

$$K_0(z^{-1}) = K_x^{-1}(z^{-1}). \tag{63}$$

Therefore

$$B_0(z^{-1}) = 1 + A(z^{-1}), \tag{64}$$

$$1 + A_0(z^{-1}) = B(z^{-1}), \tag{65}$$

or

$$B_0(z^{-1}) = \left[1 + A(z^{-1})\right]/b_0, \tag{66}$$

$$1 + A_0(z^{-1}) = B(z^{-1})/b_0, \tag{67}$$

if $b_0 \neq 0$. Then Eq. 11 can be rewritten in such a way

$$x_k^{(1)} = b_0^{-1}\left[1 + A(z^{-1})\right]u_k - \left[b_0^{-1}B(z^{-1}) - 1\right]x_k. \quad (68)$$

In this case

$$e_k = x_k - \widehat{x}_k^{(1)} = -b_0^{-1}\left[1 + A(z^{-1})\right]u_k + B^*(z^{-1})x_k, \quad (69)$$

where

$$B^*(z^{-1}) = \sum_{i=1}^{m} b_i^* z^{-i},$$

besides, $b_i^* = b_0^{-1}b_i,\ i = \overline{1,m}$. The transfer function of the additive noise filter takes the form

$$W_N(z^{-1}) = -b_0\left[1 + A(z^{-1})\right]^{-1}, \quad (70)$$

for the optimal IS model and

$$W_N(z^{-1}) = -b_0\left[1 + A(z^{-1})\right]^{-1}\left[1 + G(z^{-1})\right]^{-1}, \quad (71)$$

$$W_N(z^{-1}) = -b_0\left[1 + A(z^{-1})\right]^{-1}\left[1 + F(z^{-1})\right], \quad (72)$$

for the other mentioned models, respectively.

5. Simulation results. The numerical investigation of the above mentioned models and recursive algorithms was provided by means of IBM PC/AT. The sequence y_k of initial DS was generated by the equation

$$y_k = \frac{b_0}{1 + a_1 z^{-1}} x_k, \quad (73)$$

where $b_0,\ a_1$ are parameters of DS transfer function: $b_0 = 0.0364$; $a_1 = 0.9574$.

The realization of independent Gaussian variables ν_k with zero mean and unitary dispersion was used as the input sequence x_k.

The realization of independent Gaussian variables ξ_k with zero mean and unitary dispersion

$$N_k = \xi_k, \quad (74)$$

and the sequence of the second order AR model of the form

$$N_k = -\frac{1}{\widehat{b}_{00} + \widehat{b}_{01} z^{-1}} \xi_k, \quad (75)$$

was used as the sequences of additive noise N_k.

Here $\widehat{b}_{00}$, $\widehat{b}_{01}$ are parameters of the numerator of the IS transfer function calculated using the parameters of the initial system in the absence of N_k at the output of DS. We obtaine: $\widehat{b}_{00} = 27.47$; $\widehat{b}_{01} = 26.30$.

Ten experiments were carried out with different realizations of additive noise N_k at the DS noise level $\sigma_N^2/\sigma_y^2 = (0.01; 0.1)$. In each experiment the estimates of parameters of the IS using model (11) were obtained by the RLS algorithm processing the realizations (x_k, u_k) $k = \overline{1, 500}$. Further, the input x_k of DS was reconstructed according to the formula

$$\widehat{x}_k^{(1)} = \sum_{i=0}^{1} \widehat{b}_{0i} u_{k-i} - \sum_{i=1}^{2} \widehat{a}_{0i} x_{k-i}, \tag{76}$$

using different realizations of U_k and $\mathbf{x}_k$ with sample sizes s=800. Here $b_{00} = 26.56$; $b_{01} = 25.51$; $a_{01} = 0.017$; $a_{02} = -0.006$.

Then, for different numbers of the observation $k = \overline{1, s}$ of the previously reconstructed signal $\mathbf{x}_k$ we calculate such variables averaged by 10 experiments:

1) the relative square error

$$\delta_{\overline{x}} = \frac{1}{10} \sum_{i=1}^{10} \delta_{\widehat{x}_i}, \qquad \delta_{\widehat{x}_i} = \frac{\|x_k - \widehat{x}_k^{(1)}\|}{\|x_k\|} 100\%; \tag{77}$$

2) the variance

$$\sigma_{\overline{x}}^2 = \frac{1}{9} \sum_{i=1}^{9} \left(\delta_{\widehat{x}_i} - \delta_{\overline{x}} \right)^2; \tag{78}$$

3) the confidence intervals

$$\Delta_x = \frac{2.26}{3} \sigma_{\overline{x}}. \tag{79}$$

It follows from the simulation and input reconstruction results, presented in Table 1, that the quality of reconstruction of a stochastic signal acting on input of initial DS depends on the shape of N_k and on its intensity. It ought to be mentioned that when the structure of N_k is of the shape (75) (that corresponds to (47), in general), the errors of x_k remarkably decrease.

6. Conclusions. Recursive DS estimation approaches and procedures for unknown IS parameter estimation by processing the realizations of x_k and U_k

Table 1. The averaged by 10 experiments relative square errors (77) and the confidence intervals (79)

Observations number	Noise of (74)	Noise of (75)
	Noise level $\sigma_N^2/\sigma_y^2 = 0.01$	
100	$46.0251 + 0.7388$	$13.5198 + 0.6562$
300	$46.9696 + 0.4323$	$12.6880 + 0.1536$
600	$46.2928 + 0.2510$	$12.4264 + 0.0505$
800	$46.1916 + 0.2572$	$12.4040 + 0.0209$
	Noise level $\sigma_N^2/\sigma_y^2 = 0.1$	
100	$83.7824 + 0.5379$	$37.5898 + 1.2407$
300	$85.0392 + 0.5170$	$36.1096 + 0.2734$
600	$84.8452 + 0.3780$	$35.9733 + 0.1297$
800	$84.5594 + 0.1557$	$35.7101 + 0.0483$

can be used. The results of numerical simulation carried out by computer prove the efficiency of tuned models (11) and (47) used for the reconstruction of the current meaning of the unknown input x_k (see Fig. 2). The boundary accuracy of signal reconstruction can be achieved using model (12) when additive noise is acting on the input of DS (see Fig. 1).

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OPTIMALAUS IR DERINAMO ATVIRKŠTINIŲ MODELIŲ SINTEZĖ MINIMALIOS FAZĖS SISTEMAI

Rimantas PUPEIKIS

Analitinio tyrimo būdu sudaryti optimalūs ir derinami atvirkštiniai dinaminių sistemų modeliai, kurių nežinomus parametrus galima įvertinti, taikant žinomus rekurentinius algoritmus, skirtus tiesioginių sistemų parametrų įverčiams gauti. Pateikti skaitinio modeliavimo rezultatai IPM PC/AT (Lentelė 1).