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A Novel MCDM Method: The Integrative Reference Point Approach

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Abstract. This study proposes a novel method called the "Integrative Reference Point Approach (IRPA)" as an alternative method to existing MCDM methods. The basis of the newly proposed method is the satisfaction function and the reference set approach. Three different applications are performed to verify the validity of the proposed method from the perspective of optimal alternative rankings and sensitivity to changes in criteria weights. All results of comparative and sensitivity analyses show that the novel method is moderately sensitive to changes in criteria weights and compatible with other methods.

Key words: multi-criteria decision-making, satisfaction function, integrative reference point approach, simulation.

1. Introduction

People face decision-making problems in various fields, such as technology, finance, marketing, production, and environmental issues, while evaluating product and service alternatives (Guitouni and Martel, 1998; Nordin and Ravald, 2023; Lopes *et al.*, 2024). Many studies in the literature cover solution methods for decision-making problems, which are divided into two main parts: Multi-Attribute Decision-Making (MADM) and Multi-Criteria Decision-Making (MCDM). The current study will focus on MCDM, which is used to rank or make a selection by considering the characteristics of the products or services (Bardos *et al.*, 2001; Yalcin *et al.*, 2022; Taherdoost and Madanchian, 2023; Yüksel *et al.*, 2023).

The MCDM process generally covers defining the problem, determining the alternatives and criteria, solving the problem with the appropriate solution method, and obtaining the results. The criteria are chosen depending on the problem facing the decision maker, the purpose of the evaluation, and the alternatives (Podvezko *et al.*, 2020). Criteria types can be cost or benefit, depending on their characteristics. Also, criteria weights can be a part of the decision-making process, as they reflect the criteria priorities (O'Brien and

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Brugha, 2010; Kentli and Kar, 2011; Chaube *et al.*, 2024), and they significantly affect the MCDM process outcome (Podvezko *et al.*, 2020). On the other hand, many MCDM methods with different characteristics have been proposed in the literature. Most of these methods aim to maximise the utility level of the decision-maker in terms of all criteria. In addition, utilisation levels are generally considered linearly. However, the utility levels of the decision-makers may be restricted due to budget, capacity, and so on. They may not always be at the maximum level. A value that exceeds the maximum utility level that the decision-maker can get will not benefit the decision-maker more; on the contrary, it will cause the decision-maker to bear more costs. From this point of view, the utility level that the decision-maker gets should be taken into account in the methods when comparing alternatives, and this should be performed based on reference sets of criteria defined by decision-makers. These situations are the primary motivation of this study. From this point of view, we propose a novel MCDM method called the *Integrative Reference Point Approach (IRPA)* to overcome the shortcomings of existing MCDM methods whose reference sets may change (Özçil, 2020).

The IRPA method addresses the nonlinear utility level assessed by the satisfaction function approach. Our method allows decision-makers to make more realistic and practical decisions against daily life decision-making problems. Numerical applications are conducted to demonstrate the efficiency and applicability of the IRPA method. *Firstly* the problem adopted by Keshavarz Ghorabaee *et al.* (2015) is solved with the IRPA methods and 15 pioneering methods in the literature; Simple Additive Weighting (SAW) (Fishburn, 1967), Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang and Yoon, 1981), Grey Relational Analysis (GRA) (Ju-long, 1982), Interactive and Multicriterial Decision-Making (TOmada de Decisao Interativa Multicriterio – TODIM) (Gomes and Lima, 1991), COmplex PRoportional ASsessment (COPRAS) (Zavadskas and Kaklauskas, 1996), Multi-Criteria Optimisation and Compromise Solution (VlseKriterijumska Optimizacijia I Kompromisno Resenje – VIKOR) (Opricovic, 1998), Multi-Objective Optimization on the basis of Ratio Analysis (MOORA I and II) (Brauers and Zavadskas, 2006), Additive Ratio ASsessment (ARAS) (Zavadskas and Turskis, 2010), Weighted Aggregated Sum Product ASsesment (WASPAS) (Zavadskas *et al.*, 2012), Multi-Attributive Ideal-Real Comparative Analysis (MAIRCA) (Pamučar *et al.*, 2014), Evaluation based on Distance from Average Solution (EDAS) (Keshavarz Ghorabaee *et al.*, 2015), Reference Ideal Method (RIM) (Cables *et al.*, 2016), COmbinative Distance-based ASsessment (CODAS) (Keshavarz Ghorabaee *et al.*, 2016) and Double Normalization Based Multi Aggregation (DNBMA) (Liao *et al.*, 2018). As a result of this application, the proposed method is compared with other methods, and the methods' sensitivity against changes in criteria weights is analysed. In addition, the rank reversal problem of the IRPA method is examined for this case study. *Secondly,* a large number of decision problems with different sizes is generated by simulation analysis to investigate the performance of the IRPA method. We use simulation analysis to compare alternative rankings and scores obtained through various methods using Spearman and Pearson correlation coefficients. *Lastly,* the encountered computer selection problem is handled, and computer alternative rankings from the previously mentioned methods are found and

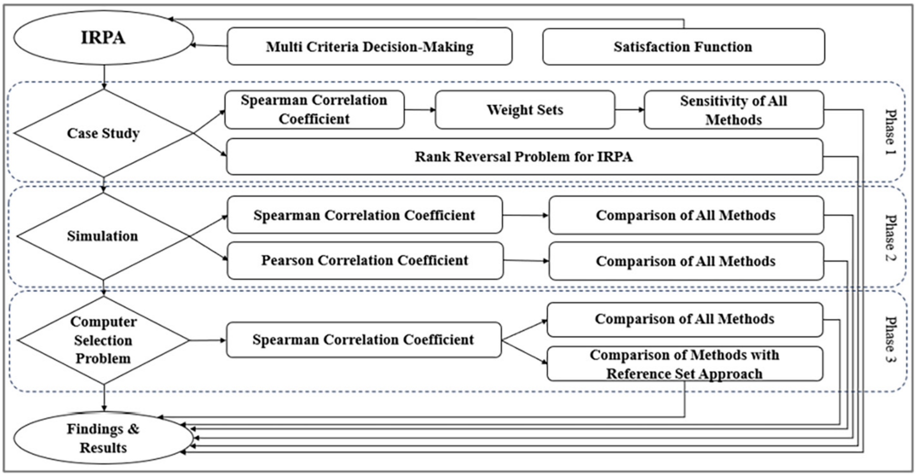

Fig. 1. Summative flow chart.

compared. Also, the methods considering the reference set approach, ARAS, DNBMA, GRA, MOORA – II, and RIM, are compared separately regarding Spearman correlation coefficients, and the results are discussed. In addition, the flowchart summarising the comparisons and analyses conducted in this study is shown in Fig. 1. MATLAB R2020b and Microsoft Excel Professional Plus 2013 programs perform all computations and simulation applications. The authors use MATLAB libraries to provide the necessary codes for the methods, correlation coefficients, and simulation. The application steps of the methods are followed as: SAW (Fishburn, 1967; Memariani *et al.*, 2009), TOPSIS (Hwang and Yoon, 1981; Rao, 2013), GRA (Ju-long, 1982; Wu, 2002; Chen, 2005; Lin *et al.*, 2005; Chan, 2008), TODIM (Gomes and Lima, 1991; Gomes and Rangel, 2009; Gomes *et al.*, 2009; Zindani *et al.*, 2017), COPRAS (Zavadskas and Kaklauskas, 1996; Banaitiene *et al.*, 2008), VIKOR (Opricovic, 1998; Opricovic and Tzeng, 2002, 2004, 2007), MOORA I–II (Brauers and Zavadskas, 2006; Zavadskas *et al.*, 2013), ARAS (Zavadskas and Turskis, 2010), WASPAS (Zavadskas *et al.*, 2012; Chakraborty and Zavadskas, 2014), MAIRCA (Pamučar *et al.*, 2014), EDAS (Keshavarz Ghorabaee *et al.*, 2015), RIM (Cables *et al.*, 2016), CODAS (Keshavarz Ghorabaee *et al.*, 2016), and DNBMA (Liao *et al.*, 2018). The MATLAB codes of each method are prepared separately, and the applications in the referenced studies are tested. The codes of the methods used in this study can be accessed in the file specified as a footnote.[1]

The main contributions of the novel method in this study are explained as follows:

1. The satisfaction functions are employed in the decision-making process whereby the IRPA method is more reasonable and efficient in addressing the reference sets of the alternatives in terms of all criteria. In this way, it is aimed to adapt the nonlinear relationship approach of the satisfaction function to MCDM. The similarity of the reference

[1] https://drive.google.com/file/d/1u7uEXykpCqgDNilKlTzELE6PqlYXzLAH/view?usp=sharing

set approach in MCDM methods and the threshold value approaches in the satisfaction function are discussed. It is assumed that the satisfaction level of the decision-maker increases non-linearly after the threshold values. The reference set approach is essential for maximising the level of utility that the decision-maker can obtain. The decision-maker will obtain more realistic solutions to the decision-making problems in daily life with the reference set approach.

2. The decision-maker can specify any value between the maximum and minimum values as the reference set. Two versions of the IRPA method are employed by changing the reference set of the criteria. On one of them, the averages are taken as the reference set. Conversely, the reference set is determined as maximum or minimum values according to the criteria characteristics. In this way, it is aimed to show the effects of reference set changes in alternative rankings. Different reference set versions are compared with other methods. In addition, different versions of the methods with the reference set approach in the literature are compared with the IRPA method. In this way, the superiority of the reference set approach of the IRPA method is analysed.
3. The number of decision problems of different sizes generated through simulation to compare methods is relatively high. To the best of our knowledge, this study is the first in the MCDM literature to assess the applicability of the proposed method to such a large number of decision problems. There are studies in the literature comparing MCDM methods. The number of decision matrices created by simulation will be an example for similar studies. Comparisons with a large number of decision problems are more generalisable than compared with a small number of decision problems. In this way, the advantages of the methods can be compared better. The results of the simulation application show the similarity of the IRPA method with other methods and the superiority of the reference set approach.
4. This study is a powerful alternative to traditional decision-making methods to solve decision problems more effectively. Decision-makers can make more appropriate decisions for themselves with the reference set approach. Decision-makers can model that they can benefit more from values close to the reference set values with the IRPA method. Decision-makers can think of reference values as threshold values. The benefit of values greater or less than the reference values will not be linear; however, the methods with reference set approach in the literature deal with this relationship linearly. Obtaining the reference value is the primary goal of the decision-maker. For the benefit criterion, the same amount of increase or decrease of the reference value will not be the same benefit or cost to the decision maker. In other words, reaching the reference value is more important for the decision-maker than the amount exceeding the reference value. This approach is mathematically modelled for the decision-maker with the IRPA method.

The rest of this paper is organised as follows. In Section 2, a novel IRPA method and satisfaction function, which is the underlying approach of the proposed method, are presented in detail, and the application steps of the IRPA method are described. In Section 3, three different numerical applications are performed to demonstrate the efficiency of the proposed method. In the next section, discussion and managerial implications are discussed. Lastly, conclusions and suggestions are presented for further studies.

2. Integrative Reference Point Approach

The utility or benefit level can be defined as the profit that the decision-maker gains from his or her choice. The decision-maker wants to move into a better situation with the choices they will make. The utility or benefit level that the decision-maker can get from an alternative may be below the maximum level or above the minimum level. This situation will cause the decision-maker to bear more costs. One way to eliminate this situation is the reference set approach, which will produce more appropriate solutions to daily life problems. The reference set approach is the basis of the IRPA method. In this method, decision-makers can determine reference sets for the criteria they consider according to their purposes, needs, and preferences. Any value between an alternative's maximum and minimum performance scores concerning each criterion is determined as a reference value in the IRPA method. With this crucial feature, the IRPA method is similar to ARAS, DNBMA, GRA, MOORA-II and RIM methods in terms of the reference point approach. However, it differs from these methods in terms of the non-linear weighting. On the other hand, although WASPAS and TODIM methods use non-linear weighting, the reference point approach is not performed in these methods. In other words, the IRPA method combines non-linear weighting with the reference point approach. In addition, the non-linear weighting methods in IRPA, WASPAS, and TODIM are different from each other. Namely, exponential weighting and expectation theory structure are used in non-linear weighting for the WASPAS and TODIM methods, respectively. However, the satisfaction function is used in non-linear weighting for the IRPA method. Thanks to the satisfaction function approach, the IRPA method also nonlinearly evaluates the positive and negative differences from the reference set. The power of the IRPA method to distinguish the alternatives decreases in values close to the reference set, whereas it increases in values that are not close.

2.1. *Satisfaction Function*

It was demonstrated by Martel and Aouni (1990) that the decision-maker's preferences can be integrated into the objective function of the goal programming and solution process by the satisfaction function. With the help of the *satisfaction function*, decision-makers can clearly express their preferences for any deviation from the desired success level of each goal (Allouche *et al.*, 2009). Depending on the threshold values of satisfaction functions, positive and negative deviations are rewarded or punished differently, and thus, the probability of reaching the goals can be changed (Aouni *et al.*, 2013). It aims to maximise the decision-maker's satisfaction level through the satisfaction function. They are also used in modelling uncertainty regarding the values of the goals. It is expressed in intervals where the decision-maker determines the upper and lower limits (Aouni *et al.*, 2005). The satisfaction function does not require being linear and symmetrical like the membership function used in fuzzy goal programming and the penalty function used in interval goal programming (Cherif *et al.*, 2008). The general form of the satisfaction function, including the threshold values, is given in Fig. 2 (Cherif *et al.*, 2008). $S(x)$ shows the satisfaction function related to the deviation amount of x, and a_{iv}, a_{i0}, and a_{id} show the indifference, dissatisfaction, and rejection threshold values, respectively.

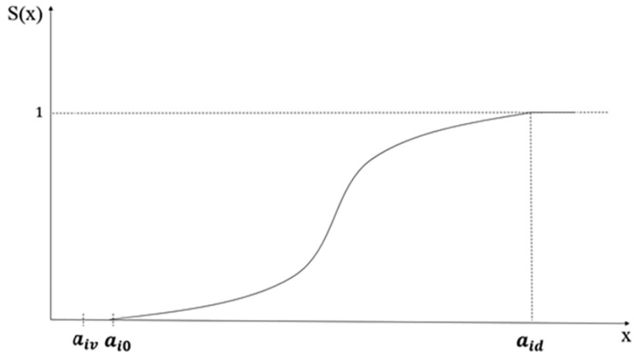

Fig. 2. Satisfaction function with threshold values.

According to Fig. 2, the decision-maker will be completely satisfied if the deviation value (d_i) is above a_{id}. If the deviation value is in the interval of $[a_{i0}, a_{id}]$, the satisfaction level of the decision-maker will increase rapidly. Moreover, if the deviation value d_i is in the interval of $[0, a_{iv}]$, the decision-maker will remain completely indifferent (Abhishek *et al.*, 2017). Eq. (1) shows the *S(x)* function, where x is the difference in preferability between the two alternatives. The standard deviation is determined by the information from the alternative distribution (Brans and Vincke, 1985).

$$S(x) = \begin{cases} 0, & x \leqslant 0, \\ 1 - e^{-\frac{x^2}{2\sigma^2}}, & x > 0. \end{cases} \tag{1}$$

Martel and Aouni (1990) effectively compared the criteria with nonlinear goal programming using Gaussian comparison methods given in Eq. (1). Studies utilising satisfaction function in the literature can be summarised in Table 1.

2.2. *Application Steps of Integrative Reference Point Approach Method*

In this section, the application steps of the IRPA method are explained in detail. The core idea of the IRPA method is the distance of the alternatives from the reference set, which is similar to the threshold approach of the satisfaction function. It is assumed that the distance of the alternatives from the reference set is non-linearly related to the degree of satisfaction. The threshold approach in the satisfaction function is performed in the IRPA method's weighting step. It is assumed that there are m alternatives ($i = 1, 2, \dots, m$) and n criteria ($j = 1, 2, \dots, n$) in the MCDM problem. Under this assumption, the following steps are necessary for the IRPA method:

Step 1. Construct the decision matrix (X) as shown in Eq. (2):

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}, \quad (i = 1, 2, 3, \dots, m;\ j = 1, 2, \dots, n), \tag{2}$$

where x_{ij} presents the performance of alternative i with respect to criterion j.

Table 1
Some studies related to the satisfaction function.

Method(s)	Brief description about studies
Taguchi method	Process performance optimization with multiple quality responses (Al-Refaie, 2014), machine performance optimization with fuzzy inference system (Abhishek *et al.*, 2017), weld variables optimization (Bandhu *et al.*, 2021), alloy materials optimization (Sharma *et al.*, 2022).
Goal programming	Portfolio management (Mansour *et al.*, 2007), quality control system design (Cherif *et al.*, 2008), modelling the decision-maker preferences (Aouni *et al.*, 2009), location selection for fire and emergency service facilities (Kanoun *et al.*, 2010), risk management and optimal portfolio diversification (Maggis and La Torre, 2012), reproduction planning (Mezghani and Loukil, 2012), planning the investments for the sustainability targets of the sectors (Jayaraman *et al.*, 2015), sustainable development management (Nechi *et al.*, 2020; Ali *et al.*, 2021), lake pollution control (Cheng *et al.*, 2022), evaluation of patient flow and reducing waiting time (Ltaif *et al.*, 2022).
Stochastic goal programming	Modelling decision-maker preferences (Aouni *et al.*, 2005), solutions to media selection and planning problems (Aouni *et al.*, 2012), making a venture capital investment decision (Aouni *et al.*, 2013), strategic planning for sustainable development decisions (Jayaraman *et al.*, 2017).
Other methods	Ordering fuzzy values (Lee *et al.*, 1994), taking into account the decision-maker preferences with fuzzy goal programming (Martel and Aouni, 1996), solution of scheduling flow problem with compromise programming (Allouche *et al.*, 2009), minimizing the waiting time for flow shop scheduling problems with genetic algorithm (Keskin and Engin, 2021), inventory classification with scatter search algorithm with multi-objective optimization (Saracoglu, 2022).

Step 2. Determine the Reference Point (value) (*RP*) for each criterion to compare the alternatives and construct the *RP* matrix as shown in Eq. (3):

$$RP = [rp_j]_{1\times n} = \begin{bmatrix} rp_1 & rp_2 & \cdots & rp_n \end{bmatrix}. \tag{3}$$

rp_j shown in Eq. (3) is the reference value for criterion *j*. It can vary depending on the decision-maker. The rp_j can be the average of the values in the decision matrix for each criterion. Also, the rp_j can be set as the minimum or maximum value of the criterion values or any value depending on the decision-maker's knowledge, experience, and personal optimization constraints.

Step 3. Normalize the decision matrix using the vector normalization method and obtain the normalized decision matrix (*N*), as shown in Eq. (4):

$$N = [n_{ij}]_{m\times n} = \begin{bmatrix} n_{11} & \cdots & n_{1n} \\ \vdots & \ddots & \vdots \\ n_{m1} & \cdots & n_{mn} \end{bmatrix}, \tag{4}$$

where n_{ij} is the normalised decision matrix element, which is computed by Eq. (5):

$$n_{ij} = \frac{x_{ij}}{\sqrt{\left(\sum_{i=1}^{m} x_{ij}^2\right) + rp_j^2}}. \tag{5}$$

Step 4. Normalize the Reference Point (*RP*) matrix by utilising Euclidean distances and obtain the Normalized Reference Point (*NRP*) matrix shown in Eq. (6):

$$NRP = [nrp_j]_{1\times n} = \left[nrp_1 \ nrp_2 \ \cdots \ nrp_{1n}\right], \tag{6}$$

where nrp_j is a normalised reference value as shown in Eq. (7):

$$nrp_j = \frac{rp_{1j}}{\sqrt{\left(\sum_{i=1}^{m} x_{ij}^2\right) + rp_j^2}}. \tag{7}$$

Step 5. Construct the Difference Matrix (*DF*) shown in Eq. (8):

$$DF = [df_{ij}]_{m\times n} = \begin{bmatrix} df_{11} & \cdots & df_{1n} \\ \vdots & \ddots & \vdots \\ df_{m1} & \cdots & df_{mn} \end{bmatrix}, \tag{8}$$

where df_{ij} is the difference between normalised performance values and normalised reference points, they are calculated as shown in Eq. (9):

$$df_{ij} = n_{ij} - nrp_j. \tag{9}$$

Step 6. Construct the Positive and Negative Difference matrices (DF^+ and DF^-) shown in Eqs. (10) and (13), respectively. At this step, criteria types are taken into account. The elements of the positive difference matrix (df_{ij}^+) are determined by applying Eqs. (11) and (12) for benefit and cost criteria, respectively. Similarly, the elements of the negative difference matrix (df_{ij}^-) are determined using Eqs. (14) and (15) for benefit and cost criteria, respectively.

$$DF^+ = \left[df_{ij}^+\right]_{m\times n} = \begin{bmatrix} df_{11}^+ & \cdots & df_{1n}^+ \\ \vdots & \ddots & \vdots \\ df_{m1}^+ & \cdots & df_{mn}^+ \end{bmatrix}, \tag{10}$$

$$df_{ij}^+ = \begin{cases} \frac{df_{ij}}{nrp_j}, & df_{ij} > 0, \\ 0, & df_{ij} \leqslant 0, \end{cases} \tag{11}$$

$$df_{ij}^+ = \begin{cases} 0, & df_{ij} \geqslant 0, \\ \left|\frac{df_{ij}}{nrp_j}\right|, & df_{ij} < 0, \end{cases} \tag{12}$$

$$DF^- = \left[df_{ij}^-\right]_{m\times n} = \begin{bmatrix} df_{11}^- & \cdots & df_{1n}^- \\ \vdots & \ddots & \vdots \\ df_{m1}^- & \cdots & df_{mn}^- \end{bmatrix}, \tag{13}$$

$$df_{ij}^{-} = \begin{cases} \left|\frac{df_{ij}}{nrp_j}\right|, & df_{ij} < 0, \\ 0, & df_{ij} \geqslant 0, \end{cases} \tag{14}$$

$$df_{ij}^{-} = \begin{cases} 0, & df_{ij} \leqslant 0, \\ \frac{df_{ij}}{nrp_j}, & df_{ij} > 0. \end{cases} \tag{15}$$

Step 7. Calculate the Weighted Difference matrices (*WDF*). The Weighted Positive Difference matrix (WDF^{+}) shown in Eq. (16) and the Weighted Negative Difference matrix (WDF^{-}) shown in Eq. (18) are calculated. While calculating these matrix elements, the criteria are weighted exponentially and multiplicatively, as shown in Eqs. (17) and (19).

$$WDF^{+} = \left[wdf_{ij}^{+}\right]_{m\times n} = \begin{bmatrix} wdf_{11}^{+} & \cdots & wdf_{1n}^{+} \\ \vdots & \ddots & \vdots \\ wdf_{m1}^{+} & \cdots & wdf_{mn}^{+} \end{bmatrix}, \tag{16}$$

$$wdf_{ij}^{+} = \left(w_j \times DF^{+}\right)^{(1-w_j)}, \tag{17}$$

$$WDF^{-} = \left[wdf_{ij}^{-}\right]_{m\times n} = \begin{bmatrix} wdf_{11}^{-} & \cdots & wdf_{1n}^{-} \\ \vdots & \ddots & \vdots \\ wdf_{m1}^{-} & \cdots & wdf_{mn}^{-} \end{bmatrix}, \tag{18}$$

$$wdf_{ij}^{-} = \left(w_j \times DF^{-}\right)^{(1-w_j)}. \tag{19}$$

w_j is the weight (importance degree) of the criterion j where $0 < w_j < 1$, ($j = 1, 2, \dots, n$) and $\sum_{j=1}^{n} w_j = 1$. The main difference between the IRPA and other MCDM methods is utilizing a similar structure to the satisfaction function in weighting the criteria. Figure 3 shows a hypothetical example of the weighting of the IRPA method and the satisfaction function with the threshold approach. For this example, any criterion weight in the decision problem and the satisfaction function's standard deviation value are assumed to be 0.5. The changes of different matrix elements in the IRPA method and the change of the x value shown in Eq. (1) for the satisfaction function are shown on the horizontal axis in Fig. 3. In contrast, the performance scores of the difference matrix elements in the IRPA method and the performance scores between the alternatives for the satisfaction function are shown on the vertical axis. Based on the hypothetical example, it can be seen in Fig. 3 that the weighting process of the IRPA method and the structure of the satisfaction function are similar.

Step 8. Calculate the Positive Distance (*PD*) matrix shown in Eq. (20) and the Negative Distance (*ND*) matrix shown in Eq. (22). pd_i and nd_i are calculated by performing Eq. (21) and (23) for each alternative, respectively.

$$PD = [pd_i]_{m\times 1} = \begin{bmatrix} pd_1 \\ pd_2 \\ \vdots \\ pd_m \end{bmatrix}, \tag{20}$$

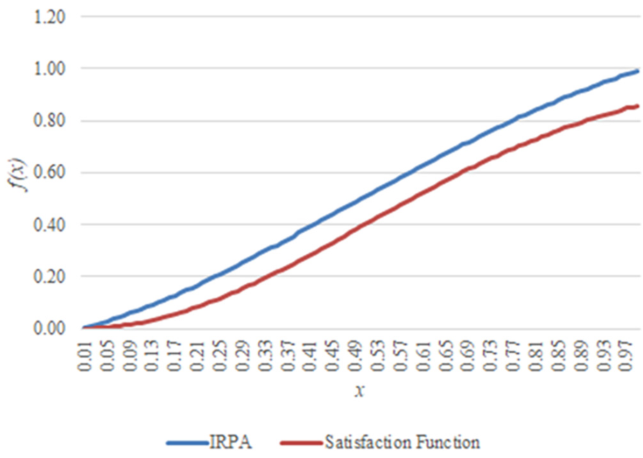

Fig. 3. The similarity of satisfaction function and the weighting of distance in the IRPA method.

$$pd_i = \sum_{j=1}^{n} wdf_{ij}^{+}, \tag{21}$$

$$ND = [nd_i]_{m\times 1} = \begin{bmatrix} nd_1 \\ nd_2 \\ \vdots \\ nd_m \end{bmatrix}, \tag{22}$$

$$nd_i = \sum_{j=1}^{n} wdf_{ij}^{-}. \tag{23}$$

Step 9. Obtain the Ranking Values (RV) of the alternatives shown in Eq. (24) by considering the alternatives' positive and negative distances.

$$RV = [rv_i]_{m\times 1} = \begin{bmatrix} rv_1 \\ rv_2 \\ \vdots \\ rv_m \end{bmatrix}, \tag{24}$$

where rv_i is the ranking value of alternative i and is calculated as in Eq. (25):

$$rv_i = \frac{pd_i - nd_i}{2}. \tag{25}$$

Step 10. Rank the alternatives in descending order based on their ranking values (rv_i). In the IRPA method, the highest and smallest rv_i values indicate the best and worst alternatives, respectively.

3. Numerical Applications

This section presents the application of the IRPA method in the previous section and a comparative analysis of the IRPA method with other methods. In this sense, this section is designed as three subsections:

- An example taken from the literature is solved with the IRPA method.
- Decision problems of different sizes are generated by simulation analysis, and the performance of the IRPA method is tested.
- The computer selection problem encountered daily is addressed, and a solution is sought with the IRPA method.

3.1. *Case Study Adopted From Literature*

This section includes a comparison of the alternative rankings of the proposed method with other MCDM methods, as well as a sensitivity analysis employing variations of criteria weights. For these purposes, the decision problem of Keshavarz Ghorabaee *et al.* (2015) is solved with the IRPA and other methods (EDAS, VIKOR, TOPSIS, SAW, COPRAS, GRA, TODIM, MOORA – I, MOORA – II, ARAS, WASPAS, MAIRCA, RIM, CODAS, and DNBMA). Then, the results are compared.

The problem adopted by Keshavarz Ghorabaee *et al.* (2015) includes seven criteria ($C_1, C_2, \ldots, C_7$) and ten alternatives ($A_1, A_2, \ldots, A_{10}$). C_1, C_2, and C_3 are the benefit criteria, whereas C_4, C_5, C_6, and C_7 are the cost criteria. The decision matrix, which includes the performance values of alternatives concerning each criterion, is given in Table 2 (Keshavarz Ghorabaee *et al.*, 2015).

Keshavarz Ghorabaee *et al.* (2015) specified seven different weight sets for the criteria. In addition to seven weight sets, we also consider the situation of assigning equal weight to all criteria in this study. Thus, eight weight sets shown in Table 3 are used for sensitivity analysis.

Figure 4 shows the graph of the weight sets in Table 3. As can be seen, values are assigned to each criterion in order from minimum to maximum. For example, the impor-

Table 2
Decision matrix.

Alternatives	Criteria						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	23	264	2.37	0.05	167	8900	8.71
A_2	20	220	2.2	0.04	171	9100	8.23
A_3	17	231	1.98	0.15	192	10800	9.91
A_4	12	210	1.73	0.2	195	12300	10.21
A_5	15	243	2	0.14	187	12600	9.34
A_6	14	222	1.89	0.13	180	13200	9.22
A_7	21	262	2.43	0.06	160	10300	8.93
A_8	20	256	2.6	0.07	163	11400	8.44
A_9	19	266	2.1	0.06	157	11200	9.04
A_{10}	8	218	1.94	0.11	190	13400	10.11

Table 3
Criteria weight sets.

Weight sets	Criteria weights						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Set 1	0.25	0.214	0.179	0.143	0.107	0.071	0.036
Set 2	0.182	0.212	0.182	0.152	0.121	0.091	0.061
Set 3	0.139	0.167	0.194	0.167	0.139	0.111	0.083
Set 4	0.108	0.135	0.162	0.189	0.162	0.135	0.108
Set 5	0.083	0.111	0.139	0.167	0.194	0.167	0.139
Set 6	0.061	0.091	0.121	0.152	0.182	0.212	0.182
Set 7	0.036	0.071	0.107	0.143	0.179	0.214	0.25
Set 8	0.143	0.143	0.143	0.143	0.143	0.143	0.143

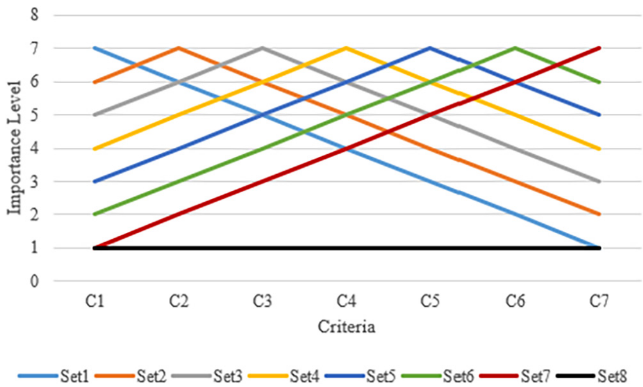

Fig. 4. Determining the importance levels of the criteria for different weight sets.

tance level of the C_1 is 7 for Weight Set 1. The sum of the importance levels of all criteria for Weight Set 1 is 28. Therefore, the weight of the C_1 is 0.25 (7/28) for Weight Set 1. Similarly, the importance level of the C_1 is 6 for Weight Set 2. The sum of the importance levels of all criteria for Weight Set 2 is 33. Therefore, the weight of the C_1 is 0.182 (6/33) for Weight Set 2. The same calculation method was used to calculate the weight values for intermediate values. Figure 4 shows the importance level assignment of each criterion.

The solutions to the problem with the IRPA and other MCDM methods are enforced by using MATLAB. Although the primary inputs are decision matrix and weight sets in all methods, the parameters required by some methods are taken as follows:

- In ARAS, GRA, MOORA-II, and DNBMA methods, the reference sets are determined as the maximum and minimum values of the benefit and cost criteria, respectively.
- In the GRA method, the xi value (ξ) is 0.5.
- In the TODIM method, the theta value (θ) is 1.
- In the VIKOR method, the v value is 0.5.
- In the WASPAS method, the lambda value (λ) is 0.5.
- In the RIM method, reference set ranges have been tested as 5%, 10%, and 20%. The 10% range value is chosen as the reference set range since it gives the highest correlation

Table 4
Ranking results of IRPA (avg) for different weight sets.

Alternatives	Weight sets							
	Set1	Set2	Set3	Set4	Set5	Set6	Set7	Set8
A_1	1	1	1	1	1	1	1	1
A_2	5	4	4	3	2	2	2	3
A_3	6	6	6	6	6	6	6	6
A_4	10	10	10	10	10	10	10	10
A_5	7	7	7	7	7	7	8	7
A_6	8	8	8	8	8	8	7	8
A_7	2	2	2	2	3	3	3	2
A_8	3	3	3	4	4	4	4	4
A_9	4	5	5	5	5	5	5	5
A_{10}	9	9	9	9	9	9	9	9

Table 5
Ranking results of IRPA (min/max) for different weight sets.

Alternatives	Weight sets							
	Set1	Set2	Set3	Set4	Set5	Set6	Set7	Set8
A_1	1	1	1	2	2	2	2	1
A_2	3	2	2	1	1	1	1	2
A_3	7	9	9	9	9	9	9	9
A_4	10	10	10	10	10	10	10	10
A_5	6	6	8	8	8	8	8	7
A_6	8	7	7	7	7	7	6	6
A_7	2	3	3	3	3	3	3	3
A_8	4	4	4	5	5	5	4	4
A_9	5	5	5	4	4	4	5	5
A_{10}	9	8	6	6	6	6	7	8

with other methods. Accordingly, the maximum and 90% of the maximum values are used as reference set ranges for the benefit criteria, and the minimum and 110% of the minimum values are used as reference set ranges for the cost criteria.

- In the CODAS method, the threshold value (τ) is 0.02.
- In the DNBMA method, the phi coefficient (ϕ) is 0.5.
- In the IRPA method, the reference sets are taken as averages and shown as "IRPA (Avg)". Secondly, the reference sets are taken as the maximum and minimum values, and this version is similarly named "IRPA (Min/Max)" in the current study.

The problem of Keshavarz Ghorabaee *et al.* (2015) is solved with the IRPA and other methods for eight weight sets shown in Table 3. Because of the page constraint, only ranking results the IRPA (Avg) and IRPA (Min/Max) are shown in Tables 4 and 5, respectively.

The correlations for the change in rankings for alternatives between each weight set are calculated for each method. In other words, seven Spearman correlation coefficients are calculated for each method since the number of weight sets is eight. For example, eight different rankings are obtained for the IRPA (Min/Max) method, and the correlations between sets (Set1–Set2, Set2–Set3, Set3–Set4, Set4–Set5, Set5–Set6, Set6–Set7,

Table 6
Spearman correlation coefficient means for weight set variation of all methods.

Method	Correlation means	Sensitivity	Method	Correlation means	Sensitivity
MOORA II	0.9437	Highest	IRPA (Avg)	0.9896	Middle
VIKOR	0.9619	Very High	MOORA I	0.9913	Low
RIM	0.9654	Very High	EDAS	0.9913	Low
DNBMA	0.9688	Very High	SAW	0.9931	Low
GRA	0.9688	Very High	CODAS	0.9948	Very Low
TOPSIS	0.9706	High	WASPAS	0.9948	Very Low
IRPA (Min/Max)	0.9758	High	ARAS	0.9965	Lowest
MAIRCA	0.981	Middle	TODIM	0.9965	Lowest
COPRAS	0.9879	Middle	–	–	–

Table 7
Spearman correlation coefficients for eight weight sets of all methods.

Method	Mean	Ranking	Method	Mean	Ranking
ARAS	0.9524	8	IRPA (Min/Max)	0.8978	17
CODAS	0.9466	10	IRPA (Avg)	0.959	3
COPRAS	0.9586	4	RIM	0.9387	13
DNBMA	0.9433	12	SAW	0.9535	7
EDAS	0.9621	2	TODIM	0.955	5
GRA	0.9498	9	TOPSIS	0.9144	15
MAIRCA	0.9449	11	VIKOR	0.9324	14
MOORA I	0.9626	1	WASPAS	0.9541	6
MOORA II	0.9013	16	–	–	–

Set7–Set8) are calculated. Then, the mean of Spearman correlation coefficients is calculated for each method, and the results are shown in Table 6. Thus, the sensitivities of the methods against weight changes are tried to be measured, and the methods with the lowest and highest correlation values are labelled as "Lowest" and "Highest". The other 15 methods are classified into five degrees (Very High, High, Middle, Low and Very Low). Methods with the same correlation value are labelled with the same degree.

Table 6 shows that the IRPA (Avg) method has moderate variability compared to the other methods. Excessive sensitivity and insensitivity to change of criteria weight values in method selection are undesirable situations. In this sense, the IRPA method should be preferred by decision-makers.

The methods are compared with each other in terms of Spearman correlation coefficient averages. Correlations of the methods with each other are calculated for eight weight sets, and Spearman correlation coefficient averages calculated from these eight cross-correlation tables are given in Table 7.

Table 7 shows that MOORA – I, EDAS, and IRPA (Avg) methods have the highest average correlation values for eight weight sets, respectively. Similarly, MOORA – II, TODIM, and IRPA (Min/Max) methods have the lowest correlation averages. The IRPA method differs in weighting and relative distance to the reference set. As a result of the comparisons made with the problem of Keshavarz Ghorabaee *et al.* (2015), it is understood that the IRPA method shows a high level of similarity with other methods in terms of Spearman correlation coefficient results. Moreover, the IRPA (Avg) method is one of the methods with the highest three correlation averages when all weight sets are considered. This result proves that the IRPA (Avg) method is very similar to other methods.

The fact that the IRPA (Min/Max) is in the last rank as the correlation proves the method's sensitivity to the reference set. The decision-maker who chooses to use the reference set approach should consider the results of the IRPA due to the difference between the versions of the IRPA method. The decision-maker can determine a different value for each criterion between the maximum/minimum values. In this way, she/he can make the most appropriate decision for himself.

3.1.1. *Rank Reversal Problem*

The problem presented by Keshavarz Ghorabaee *et al.* (2015) is solved by considering two different approaches for the ranking reversal problem, utilizing the IRPA (Avg) and IRPA (Min/Max) methods. These approaches can be explained as:

- In the first approach, alternatives are sequentially excluded from the analysis. The rankings of the remaining alternatives are compared with the previous ranking.
- In the second approach, each alternative is removed from the analysis respectively, and the rankings of the remaining nine alternatives are obtained.

The IRPA (Avg) method's rankings are compared with both approaches. It is observed that there is no rank reversal problem for eight weight sets in the rankings obtained with IRPA (Avg). The IRPA (Avg) method's rankings obtained with weight set 1 for the first and second approaches are given in Tables 8 and 9.

On the other hand, the rankings of some alternatives change in the IRPA (Min/Max) method for both approaches. When only the two best alternatives are excluded from the analysis, there are changes in some alternative rankings contrary to expectations. However, this situation is normal for a decision-maker who uses the maximum/minimum value as a reference. This result is valid for all weight sets. It can be concluded that the rank reversal problem depends on the reference set selection in the IRPA method.

The values of the decision matrix can be in different measurements, such as quantitative or scale, in MCDM problems. When reference sets appropriate to the data structure

Table 8
IRPA (Avg) rankings for the first approach (weight set 1).

Alternatives	Extracted alternative								
	All	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
A_1	1								
A_2	3	2							
A_3	6	5	4						
A_4	10	9	8	7					
A_5	7	6	5	4	4				
A_6	8	7	6	5	5	4			
A_7	2	1	1	1	1	1	1		
A_8	4	3	2	2	2	2	2	1	
A_9	5	4	3	3	3	3	3	2	1
A_{10}	9	8	7	6	6	5	4	3	2

Table 9
IRPA (Avg) rankings for the second approach (weight set 1).

Alternatives	Extracted alternative										
	All	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}
A_1	1		1	1	1	1	1	1	1	1	1
A_2	3	2		3	3	3	3	2	3	3	3
A_3	6	5	5		6	6	6	5	5	5	6
A_4	10	9	9	9		9	9	9	9	9	9
A_5	7	6	6	6	7		7	6	6	6	7
A_6	8	7	7	7	8	7		7	7	7	8
A_7	2	1	2	2	2	2	2		2	2	2
A_8	4	3	3	4	4	4	4	3		4	4
A_9	5	4	4	5	5	5	5	4	4		5
A_{10}	9	8	8	8	9	8	8	8	8	8	

are selected, the rank reversal problem does not occur. Therefore, to avoid the reverse order problem in MCDM problems, reference sets can be determined with the values shown in Eqs. (26), (27), or (28):

$$rp_j = \frac{\sum_{i=1}^{m} x_{ij}}{m}, \tag{26}$$

$$rp_j = \frac{\max_j x_{ij} - \min_j x_{ij}}{2}, \tag{27}$$

$$rp_j = \frac{\max_j x_{ij}}{2}. \tag{28}$$

3.2. *Simulation*

In the literature, the changes in the alternative's rankings of the methods against the changes in the weight sets have been examined by the authors. However, the effect of the change in the number of alternatives on the ranking was investigated by Keshavarz Ghorabaee *et al.* (2018) with the simulation comparing TOPSIS and EDAS methods. Similarly, in this study, the IRPA method is compared with other MCDM methods using simulation to present the randomness and comprehensiveness of the method's performance. Uniformly distributed weight sets and decision matrices are used as input data. The necessary data, random and equal probability values in the interval (0, 1), are derived with the "RAND ()" command in MATLAB. In order to present the similarities and differences between the IRPA method and the other MCDM methods, Spearman or Pearson correlation coefficients are performed.

3.2.1. *Comparative Analysis with the Spearman Correlation Coefficient*

Firstly, problems involving binary combinations of different numbers of alternatives ($m = 3, 4, \ldots, 29$) and criteria ($n = 3, 4, \ldots, 29$) are generated. These binary combinations are repeated 10,000 times so that the rankings' correlation means obtained from 7 290 000 ($27 \times 27 \times 10000$) comparisons converge to the normal distribution. The number of iterations is determined according to the 99.9% similarity of the different results obtained by running the codes more than once. The generated problems are solved by IRPA

Table 10
Spearman correlation means of all methods.

Method	Mean	Ranking	Method	Mean	Ranking
ARAS	0.7102	14	IRPA (Min/Max)	0.4432	17
CODAS	0.7193	13	IRPA (Avg)	0.826	2
COPRAS	0.8147	5	RIM	0.8003	8
DNBMA	0.8004	7	SAW	0.7561	11
EDAS	0.8255	3	TODIM	0.4687	16
GRA	0.7925	9	TOPSIS	0.8021	6
MAIRCA	0.8201	4	VIKOR	0.7333	12
MOORA I	0.8288	1	WASPAS	0.783	10
MOORA II	0.4884	15	–	–	–

(Avg), IRPA (Min/Max), and 15 other different MCDM methods. Based on the solution results of these methods, Spearman correlation coefficients are calculated, and the mean values of these correlation coefficients are presented in Table 10.

When the methods are examined in terms of correlation means which is shown in Table 10, the methods with the highest correlation means are MOORA I, IRPA (Avg), EDAS, MAIRCA, and COPRAS, respectively. CODAS, ARAS, MOORA II, TODIM, and IRPA (Min/Max) methods have the lowest correlation means, respectively. ARAS, DNBMA, GRA, MOORA II, and RIM methods, whose reference sets may vary, are seen to be on the 14th, 7th, 9th, 15th, and 8th rank in terms of correlation means, respectively. Two versions of the IRPA method have the highest and lowest mean values among the methods whose reference set can vary.

Table 10 is examined in detail in terms of each method for decision-makers who choose a method. As a result of this examination, the methods with the most and the least similarity of each method in terms of Spearman correlation coefficient are determined, and Table 11 is formed. For example, the ARAS method is similar to the WASPAS, SAW, and CODAS methods. The least similar methods with ARAS are TODIM, IRPA (Min/Max), and MOORA – II methods, respectively. Most and least similarities for all methods are shown in Table 11.

3.2.2. *Comparative Analysis with the Pearson Correlation Coefficients*

The simulation is repeated by solving the decision problems generated randomly and uniformly distributed. The ranking results of the IRPA and the other 15 methods are compared using the Pearson correlation coefficients. For comparisons, alternative scores of different methods are normalized by the Linear (Min-Max) normalization method, and Pearson correlations of the methods with each other are calculated. The binary combinations of alternatives ($m = 30, 31, \dots, 100$) and criterion variables ($n = 30, 31, \dots, 100$) are repeated 100 times. Thus, it is aimed that the correlation results of 504,100 ($71 \times 71 \times 100$) comparisons converge to the normal distribution. The number of iterations is determined according to the maximum similarity of 99.9% between the different results obtained by running the codes more than once. The Pearson correlation coefficient means of the problem results generated with different alternatives, criteria, and iteration numbers are calculated, and the results are given in Table 12.

Table 11
Similarities of all methods for Spearman correlation coefficients.

Method	Similarity	
	The most	The least
ARAS	WASPAS, SAW & CODAS	TODIM, IRPA (Min/Max) & MOORA II
CODAS	SAW, WASPAS & ARAS	TODIM, MOORA II & IRPA (Min/Max)
COPRAS	EDAS, MOORA I & IRPA (Avg)	TODIM, IRPA (Min/Max) & MOORA II
DNBMA	MAIRCA, RIM & MOORA I	MOORA II, IRPA (Min/Max) & TODIM
EDAS	MOORA I, IRPA (Avg) & COPRAS	MOORA II, IRPA (Min/Max) & TODIM
GRA	MAIRCA, MOORA I & IRPA (Avg)	TODIM, IRPA (Min/Max) & MOORA II
MAIRCA	MOORA I, IRPA (Avg) & EDAS	TODIM, MOORA II & IRPA (Min/Max)
MOORA I	EDAS, IRPA (Avg) & MAIRCA	MOORA II, TODIM & IRPA (Min/Max)
MOORA II	VIKOR, DNBMA & TOPSIS	ARAS, IRPA (Min/Max) & TODIM
IRPA (Min/Max)	EDAS, MOORA I & COPRAS	CODAS, MOORA II & TODIM
IRPA (Avg)	EDAS, MOORA I & COPRAS	MOORA II, IRPA (Min/Max) & TODIM
RIM	DNBMA, TOPSIS & IRPA (Avg)	MOORA II, IRPA (Min/Max) & TODIM
SAW	WASPAS, CODAS & ARAS	TODIM, MOORA II & IRPA (Min/Max)
TODIM	WASPAS, SAW & MAIRCA	VIKOR, IRPA (Min/Max) & MOORA II
TOPSIS	IRPA(Avg), EDAS & MOORA I	MOORA II, IRPA (Min/Max) & TODIM
VIKOR	DNBMA, RIM & MAIRCA	ARAS, IRPA (Min/Max) & TODIM
WASPAS	SAW, CODAS & ARAS	TODIM, MOORA II & IRPA (Min/Max)

Table 12
Pearson correlation means of all methods for simulation application.

Method	Mean	Ranking	Method	Mean	Ranking
ARAS	0.4699	14	IRPA (Min/Max)	0.2942	17
CODAS	0.6198	13	IRPA (Avg)	0.7911	1
COPRAS	0.7858	5	RIM	0.7783	6
DNBMA	0.7498	9	SAW	0.6957	11
EDAS	0.7905	3	TODIM	0.3902	15
GRA	0.7688	8	TOPSIS	0.7776	7
MAIRCA	0.7900	4	VIKOR	0.6890	12
MOORA I	0.7909	2	WASPAS	0.7363	10
MOORA II	0.3435	16	–	–	–

According to Table 12, in terms of Pearson correlation coefficients means, the methods with the highest means are IRPA (Avg), MOORA – I, and EDAS, respectively, while those with the lowest means are TODIM, MOORA – II, and IRPA (Min/Max) methods, respectively.

3.3. *Computer Selection Problem*

People use computers daily for several reasons, such as communication, financial transactions, learning, etc. A decision-maker who wants to buy a computer faces the problem of selecting many alternatives. Computer selection problem has been considered in many studies in the literature, and some of studies are summarized in Table 13. The list of criteria used in these studies is given below:

Table 13
Literature review on computer selection problem.

Author	Method	Weighting
Goswami *et al.* (2022)	ARAS, COPRAS	SMART[1], SWARA[2]
Doğan and Borat (2021)	TOPSIS	AHP[3]
Sönmez Çakır and Pekkaya (2020)	–	DEMATEL, AHP & Fuzzy AHP
Mitra and Goswami (2019)	TOPSIS	AHP
Aytaç Adalı and Tuş Işık (2017)	MOOSRA[4], MULTIMOORA[5]	AHP
Lakshmi *et al.* (2015)	TOPSIS	–
Pekkaya and Aktogan (2014)	DEA[6], TOPSIS, VIKOR	AHP, AHP-DEA
Srichetta and Thurachon (2012)	Fuzzy AHP	Fuzzy AHP
Kasim *et al.* (2011)	SAW	ROC[7]
Sumi and Kabir (2010)	AHP	AHP

[1]Simple multi-attribute rating technique; [2]Step-wise weight assessment ratio analysis; [3]Analytic hierarchy process; [4]Multi-objective optimization on the basis of simple ratio analysis; [5]Multi-objective optimization by ratio analysis plus the full multiplicative form; [6]Data envelopment analysis; [7]Rank order centroid.

- Goswami *et al.* (2022): Processor, RAM, Screen Size, Storage Capacity, Brand, Operating System, Color;
- Doğan and Borat (2021): Processor Speed, Ram Capacity, Warranty Period, Hard Disk Capacity, Cost, and the Number of Service Networks;
- Sönmez Çakır and Pekkaya (2020): Price, Processor Speed, RAM Speed, Card Speed, RAM Capacity, HDD/SDD Capacity, Graphics Card-Memory, Processor-cache, Resolution, Size, Touch Screen, Other, Ports, Weight, Battery Properties, Drivers, Service Quality, Design, Eco-friendly, Hardware Quality, Durability;
- Mitra and Goswami (2019): Processor, Brand, Screen Size, Hard Disk Capacity, RAM;
- Aytaç Adalı and Tuş Işık (2017): Processor Speed, Cache Memory, Storage/Hard Drive, Display Card Memory, RAM, Screen Resolution, Screen Size, Brand Reliability, Weight, Cost;
- Lakshmi *et al.* (2015): Cost, Specification, Warranty, Size, Battery Life, With or Without OS, Weight, Keyboard and Touchpad, WiFi;
- Pekkaya and Aktogan (2014): Processor Type, Processor Speed, Hard Drive Speed, Part Quality, Design, Technical Service, Hard Drive, RAM, Graphics Card, Resolution, Sizes, Card Reader, Battery, CD/DVD, Camera, Weight, USB Port, Cost;
- Srichetta and Thurachon (2012): Hard Disk Capacity, RAM Capacity, CPU Speed, Monitor Resolution, Weight, Price, Durability, Beauty;
- Kasim *et al.* (2011): Processor, Hard Drive, Price, Memory, Size, Weight;
- Sumi and Kabir (2010): Memory Capacity, Graphics Capacity, Size and Weight, Price.

In this section, a real computer selection problem is handled. It aims to test the IRPA method's similarity with other methods and its superiority over the methods whose reference set may vary. For these purposes, the results of the IRPA method and other methods are compared. This section assumes that the decision-maker can choose any computer alternatives with online ordering. The main constraints of the decision-maker are as follows:

- The decision-maker does not need any particular computer (home, game, or office).
- The budget of the decision-maker is between 5000–10000 Turkish Liras (TL).
- A computer has an SSD (Solid State Disk) feature.

Table 14
Decision matrix for computer selection problem.

Alternative	Criteria					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	6864.35	1.8	8	256	2	14
A_2	9298.99	2.2	16	512	6	17.3
A_3	9796.62	1.8	16	512	2	13.3
A_4	9583.66	1.8	16	1024	2	14
A_5	7299	1.8	8	512	2	14
A_6	7699	2.6	8	256	4	15.6
A_7	8558.15	1.8	16	256	2	13.3
A_8	9999	2.6	16	512	6	15.6
A_9	8899	1.8	8	512	2	13.3
A_{10}	8023.87	2.6	8	256	4	15.6
A_{11}	8331.94	2.2	8	256	4	15.6
A_{12}	7047.69	1.8	8	256	2	14
A_{13}	7651.86	2.2	8	1024	4	17.3
A_{14}	9735.88	1.8	16	512	2	14

First of all, selection criteria are determined as Price (C_1, TL), Processor Speed (C_2, GHz), RAM (C_3, GB), SSD Capacity (C_4, GB), Graphics Card Capacity (C_5, GB), Screen Size (C_6, inches). C_1 is the cost criterion; other criteria are determined as benefit criteria. The criteria weights are evaluated as equal by assuming that the criteria have no superiority. Fourteen different alternatives belonging to six different brands are determined according to price and SSD feature restrictions.[2] The decision matrix of the computer selection problem, including the alternatives and their values concerning each criterion, is given in Table 14.

3.3.1. *Comparative Analysis of Computer Selection Problem with All Methods*

The computer selection problem is solved with IRPA (Avg), IRPA (Min/Max), and 15 different MCDM methods, and computer alternatives are ranked. The ranking results of computer alternatives with 17 different methods are shown in Table 15. The ranking results of the methods are compared with the Spearman correlation coefficient means given in Table 16.

As Table 16 is examined, the IRPA (Avg) method has the highest correlation mean, followed by EDAS, MOORA – I, and WASPAS methods. The GRA, TODIM, RIM, and MOORA – II methods have the lowest correlation means. ARAS method, one of the references set differentiable methods, has the same order as the CODAS method and is ranked as 6.5. The IRPA (Avg) method is the most similar to others, and has the highest correlation value according to the methods whose reference set may vary.

3.3.2. *Comparative Analysis of Computer Selection Problem with Methods Considering Reference Set Approach*

The computer selection problem is solved with IRPA, ARAS, DNBMA, GRA, MOORA – II, and RIM methods in this section, and the ranking of alternatives is shown in Table 17.

[2] Vatan Computer (2019), 5000-9999 TL Laptop, https://www.vatanbilgisayar.com/5000-10000-tl-arasi/laptop/?opf=p29924634 (Date accessed: 21.11.2019).

Table 15
Ranking results of computer alternatives for all methods.

Alternative	ARAS	CODAS	COPRAS	DNBMA	EDAS	GRA	MAIRCA	MOORA I	MOORA II	IRPA (Min/Max)	IRPA (Avg)	RIM	SAW	TODIM	TOPSIS	VIKOR	WASPAS
A_1	13	12	13	12	13	11	12	13	11.5	12	13	12.5	12	13	13	12	13
A_2	2	2	2	1	1	1	1	1	2.5	1	1	1	2	1	2	1	1
A_3	8	8	8	10	8	9	10	8	6	9	9	11	8	10	6	10	8
A_4	4	4	4	6	4	4	6	4	6	4	4	7	4	4	4	6	4
A_5	10	11	10	11	10	13	11	10	6	11	10	10	11	9	11	11	10
A_6	5	5	5	4	5	5	4	5	11.5	5	5	4	5	5	7	4	5
A_7	11	9	11	9	11	8	9	11	11.5	10	11	9	10	11	10	9	11
A_8	1	1	1	2	2	2	2	2	2.5	2	2	3	1	2	3	2	2
A_9	12	14	12	14	12	14	14	12	6	14	12	14	13	12	12	14	12
A_{10}	6	6	6	5	6	6	5	6	11.5	6	6	5	6	6	8	5	6
A_{11}	9	10	9	7	9	10	7	9	11.5	7	8	6	9	7	9	7	9
A_{12}	14	13	14	13	14	12	13	14	11.5	13	14	12.5	14	14	14	13	14
A_{13}	3	3	3	3	3	3	3	3	1	3	3	2	3	3	1	3	3
A_{14}	7	7	7	8	7	7	8	7	6	8	7	8	7	8	5	8	7

Table 16
Spearman correlation coefficients of all methods.

Method	Mean	Ranking	Method	Mean	Ranking
ARAS	0.9486	6.5	IRPA (Min/Max)	0.9439	8
CODAS	0.9392	9	IRPA (Avg)	0.9516	1
COPRAS	0.9486	6.5	RIM	0.8993	16
DNBMA	0.9278	12	SAW	0.9500	5
EDAS	0.9509	3	TODIM	0.9386	10
GRA	0.9123	14	TOPSIS	0.9050	15
MAIRCA	0.9278	12	VIKOR	0.9278	12
MOORA I	0.9509	3	WASPAS	0.9509	3
MOORA II	0.6129	17	–	–	–

ARAS, DNBMA, GRA, MOORA – II, and RIM are MCDM methods whose reference sets vary. Their reference sets can be determined between maximum and minimum values. In this section, differently from the previous section, reference sets of these methods are changed and determined as follows:

- The reference set is the maximum and minimum values for the benefit and cost criteria, respectively. This method version is shown as "A" in Tables 17 and 18.
- The reference set is the average values for all criteria. This method is shown as "B" in Tables 17 and 18.
- The reference set is determined in a mixed way as the average, maximum, or minimum for each criterion. This version of the IRPA method is shown as "C" in Tables 17 and 18.

Spearman correlation coefficients of the methods with different reference sets shown in Table 18 are performed for comparison purposes. As Table 18 is examined, the following conclusions are reached:

Table 17
Alternative rankings of the methods considering reference set approach.

Method	ARAS		DNBMA		GRA		MOORA – II		IRPA			**RIM**	
Reference set	A	B	A	B	A	B	A	B	A	B	C	A	B
A1	13	13	12	3	11	11	11.5	7.5	12	13	11	12.5	1.5
A2	2	2	1	11	1	8	2.5	11.5	1	1	2	1	13.5
A3	8	8	10	9	9	7	6	3.5	9	9	6	11	9
A4	4	4	6	13	4	14	6	13.5	4	4	3	7	9
A5	10	10	11	5	13	3	6	1.5	11	10	10	10	3
A6	5	5	4	10	5	9	11.5	7.5	5	5	5	4	6.5
A7	11	11	9	6	8	4	11.5	7.5	10	11	7	9	4.5
A8	1	1	2	14	2	13	2.5	11.5	2	2	1	3	13.5
A9	12	12	14	4	14	2	6	1.5	14	12	14	14	4.5
A10	6	6	5	8	6	6	11.5	7.5	6	6	8	5	6.5
A11	9	9	7	2	10	1	11.5	7.5	7	8	12	6	12
A12	14	14	13	1	12	10	11.5	7.5	13	14	13	12.5	1.5
A13	3	3	3	12	3	12	1	13.5	3	3	4	2	11
A14	7	7	8	7	7	5	6	3.5	8	7	9	8	9

- In the ARAS method, there is no change in the rankings as the reference set varies.
- The rankings of DNBMA and RIM methods showed extreme variability, and the correlation values between the different versions of the methods are −0.7843 and −0.7312, respectively.
- The rankings between different versions of GRA and MOORA – II methods are varied at a high level, and the correlation values of these methods are −0.5743 and −0.1447, respectively.
- The IRPA is the method in which the different reference sets have the lowest effects on the rankings. The maximum correlation value between different versions of IRPA method is 0.8644.

As the mean correlation values are examined, it is seen that the method with the most similar ranking to other methods is the IRPA$^{(B)}$. The IRPA$^{(B)}$ method is followed by ARAS$^{(A,B)}$, DNBMA$^{(A)}$, and RIM$^{(A)}$ methods.

4. Discussion and Managerial Implications

MCDM problems are solved with different methods according to the data structure in the problem. In this study, we have focused on MCDM methods, which process quantitative data, and proposed a novel MCDM method called the IRPA method. The main characteristics of the proposed method are being sensitive to weight changes, allowing variation in the reference set, and being similar to the nonlinear satisfaction function. The application steps of the IRPA method have been explained in detail, and different application examples in different sections have been examined to present the validity of the method. Firstly, a case study from the literature (Keshavarz Ghorabaee *et al.*, 2015) has been solved performing the IRPA method. Then, the solution results of 15 different MCDM methods have been

Table 18
Spearman correlation coefficients of the methods considering reference set approach.

Method	Ref.	ARAS		DNBMA		GRA		MOORA – II		IRPA			RIM	
	Set	A	B	A	B	A	B	A	B	A	B	C	A	B
ARAS	A	1	1	0.9388	−0.911	0.9257	−0.452	0.6611	−0.556	0.965	0.9913	0.895	0.8945	−0.815
	B	1	1	0.9388	−0.911	0.9257	−0.452	0.6611	−0.556	0.965	0.9913	0.895	0.8945	−0.815
DNBMA	A	0.9388	0.9388	1	−0.784	0.9475	−0.395	0.4687	−0.635	0.9825	0.9563	0.8469	0.9841	−0.78
	B	−0.911	−0.911	−0.784	1	−0.863	0.602	−0.546	0.625	−0.832	−0.867	−0.937	−0.68	0.6698
GRA	A	0.9257	0.9257	0.9475	−0.863	1	−0.574	0.5168	−0.704	0.9563	0.9257	0.9213	0.8989	−0.73
	B	−0.452	−0.452	−0.395	0.602	−0.574	1	−0.22	0.7562	−0.465	−0.404	−0.605	−0.308	0.1844
MOORA – II	A	0.6611	0.6611	0.4687	−0.546	0.5168	−0.22	1	−0.145	0.5408	0.637	0.5911	0.4539	−0.486
	B	−0.556	−0.556	−0.635	0.625	−0.704	0.7562	−0.145	1	−0.67	−0.573	−0.608	−0.597	0.5419
IRPA	A	0.965	0.965	0.9825	−0.832	0.9563	−0.465	0.5408	−0.67	1	0.9781	0.8819	0.9535	−0.821
	B	0.9913	0.9913	0.9563	−0.867	0.9257	−0.404	0.637	−0.573	0.9781	1	0.8644	0.9251	−0.828
	C	0.895	0.895	0.8469	−0.937	0.9213	−0.605	0.5911	−0.608	0.8819	0.8644	1	0.772	−0.651
RIM	A	0.8945	0.8945	0.9841	−0.68	0.8989	−0.308	0.4539	−0.597	0.9535	0.9251	0.772	1	−0.731
	B	−0.815	−0.815	−0.78	0.6698	−0.73	0.1844	−0.486	0.5419	−0.821	−0.828	−0.651	−0.731	1
Mean		0.426	0.426	0.4208	−0.341	0.3959	−0.103	0.318	−0.163	0.4181	0.4305	0.3743	0.42	−0.328
Ranking		2.5	2.5	4	13	7	10	9	11	6	1	8	5	12

analysed for comparison. Additionally, sensitivity analysis has been conducted by performing eight weight sets. The results of each method have been compared according to the weight set change. Spearman correlation values between the rankings of each method have been calculated. Then, the mean of the seven correlation values of each method has been calculated. The results have shown that IRPA (Min/Max) and MOORA – II methods are the most sensitive to weight change compared to other methods. If the decision-maker wants the weight values to have a high effect on the rankings, she/he can choose one of the MOORA-II, IRPA (Min/Max), or TOPSIS methods. Otherwise, if she/he wants the weight values to have the lowest effect on the rankings, she/he can choose one of the CODAS or WASPAS methods. If the decision-maker wants the weight values to be moderately sensitive to the rankings, she/he can choose one of the IRPA (Avg), COPRAS, or MAIRCA methods. Therefore, the sensitivity of different IRPA method versions has been measured with different weight sets. The results show why the IRPA method should be preferred according to the decision-maker's request. In addition, the rank reversal problem related to this case study taken from the literature has been examined with two approaches for the IRPA method. According to both approaches, there is no rank reversal problem for the IRPA (Avg) method. It is seen that there will be no rank reversal problem for the IRPA method by selecting the reference set according to the distribution of the decision matrix data. Alternative reference set preferences that can be preferred to avoid the rank reversal problem are presented. A decision-maker who wants to avoid the rank reversal problem can make choices that will not be affected by this problem with the IRPA method.

As a second analysis, many large-scale decision problems have been generated by simulation analysis, and these problems have been solved with the IRPA method and other MCDM methods. Spearman and Pearson correlation coefficients are used as the tools for comparison purposes. They have been calculated for each simulated decision matrix. The average of all correlation values of each method with other methods has been calculated for each decision matrix. Then, the mean of these correlation values has been calculated for each method. According to the comparison results with the Spearman correlation coefficient, it is seen that MOORA – I and IRPA (Avg) methods are the most similar to other methods. Similarly, according to the comparison results with the Pearson correlation coefficient, IRPA (Avg) and MOORA – I methods are the most similar to other methods, respectively. As can be seen from the results of the simulation application, if the reference point in the IRPA method is chosen as averages, it is the method that shows the most similarity to other methods. Thus, this proves that the IRPA method is an alternative to other methods.

The input variables of the simulation application have a Uniform distribution, and yet the correlation value of the IRPA method is the highest, especially for Pearson correlations. These results show that the IRPA method is highly distinctive compared to other methods. Likely, daily life applications will not have a uniform distribution. This is also seen in the computer selection application. Despite this, the IRPA method should be chosen as an alternative method instead of all methods because of its similarity to other methods and its high distinctiveness.

As the last analysis, the computer selection problem encountered in daily life has been addressed, and computer alternatives have been ranked with the IRPA method and other

MCDM methods. Besides, in this part, the comparative analysis related to the methods covering reference sets has been performed. As a result of this analysis, the IRPA (Avg) method is the most similar method to other methods in solving ordinary daily life problems. Spearman correlation values prove this for methods whose reference set can take different values. If the decision-maker wants to choose a method for similar problems, it is reasonable to prefer the IRPA (Avg) method because of the correlation means. Similarly, the decision-maker who wants to choose among the methods whose reference point can change should prefer the IRPA (Avg) method because of the correlation means. These results reveal that the IRPA (Avg) method should be preferred over other methods. In this way, the decision-maker can determine the reference set according to her/his wishes and obtain the most beneficial ranking for herself/himself.

All analyses have shown that the IRPA method has the highest correlation with other methods, so the IRPA method is compatible with the existing methods in the literature. Also, it has some advantages over other methods. The IRPA method considers the nonlinear utility level assessed by the satisfaction function approach. This feature allows decision-makers to make more realistic and practical decisions against daily life decision-making problems. On the other hand, the methods from the literature such as CODAS, COPRAS, MAIRCA, MOORA-I, SAW, TODIM, TOPSIS, VIKOR, and WASPAS use a reference set as the maximum or minimum value according to the criterion type. However, this approach is only valid for some situations. Hence, some methods like ARAS, DNBMA, GRA, MOORA-II, and RIM consider different values as reference sets. In addition to these methods, it is expected that IRPA will find a place in the literature as a method that can solve decision problems by considering different reference sets. From the perspective of application steps, the IRPA method is similar to other methods. Hence, it is easy to apply and understand. One can easily observe from the simulation results that it effectively ranks high or low numbers of alternatives. In addition, the IRPA method evaluates the distances from the reference set similarly for both benefit and cost criteria. If the positive differences from the reference set increase, the satisfaction level increases as a nonlinear function. So, the IRPA method, based on the satisfaction function approach, will allow a more realistic choice in decision-making problems whose reference set can change. In this way, the IRPA method can be used in areas that do not contain linear relationships, such as marketing, product selection, career choice, machine working conditions or outputs, finance, etc.

5. Conclusion

This study proposes a novel MCDM method -IRPA- as an alternative to existing MCDM methods. The key features of the IRPA method are its satisfaction function and reference set considerations. To demonstrate its efficiency and applicability, various decision problems have been solved. Specifically, a problem from the literature (Keshavarz Ghorabaee *et al.*, 2015) was solved using the IRPA method, and its results were compared with those obtained from other MCDM methods. The average Spearman correlation between the

IRPA method and other methods was remarkable. In terms of correlation averages, the IRPA (Avg) method ranked third, while the IRPA (Min/Max) method ranked seventeenth. Additionally, the sensitivity of the IRPA method to changes in criteria weights was analysed. The results indicated that the IRPA method exhibited high and moderate sensitivity to weight variations. Multiple decision problems of varying sizes were generated through simulation analysis to evaluate further the IRPA method's performance in large-scale decision problems. These problems were then solved using IRPA and other MCDM methods, and necessary comparisons were made using Spearman and Pearson correlation coefficients. In the simulation analysis, the IRPA (Avg) method ranked second in terms of average Spearman correlation, whereas the IRPA (Min/Max) method ranked seventeenth. The changes in the reference set highlighted the comprehensiveness of the IRPA method. Similar trends were observed in the Pearson correlation coefficient averages; in this case, the IRPA (Avg) method ranked first, while the IRPA (Min/Max) method ranked last. As a final application, a real-life decision-making problem (the computer selection problem) was analysed. The solutions obtained from different MCDM methods, particularly those incorporating the reference set approach, were compared using their Spearman correlation coefficients. When the Spearman correlation averages of all methods were evaluated, the IRPA (Avg) method ranked first, while the IRPA (Min/Max) method ranked eighth. Moreover, the IRPA (Avg) method achieved the highest ranking among the methods that considered the reference set approach. The findings indicate that the IRPA method is a viable alternative to other MCDM methods. This study also has several limitations. The first limitation concerns the number of decision-makers or experts involved in the problem. In this study, it was assumed that the number of decision-makers is odd. However, in real-world applications, decision-making groups can consist of more than one individual. Future research may extend the IRPA method to accommodate group decision-making problems. Secondly, sensitivity analysis was conducted using eight weight sets. Future studies could expand this analysis by further increasing the number of weight sets to assess the IRPA method's sensitivity to weight changes. Additionally, simulations incorporating Pearson and Spearman correlation coefficients could be extended to measure the impact of weight variations and compare the results with other reference set-based methods. Thirdly, the IRPA method is designed for decision problems involving quantitative data. However, real-world problems often involve both qualitative and quantitative data. Future studies could adapt the IRPA method to handle mixed-data decision problems. Fourthly, the reference sets used in this study consisted of maximum, minimum, and average values, meaning that only one reference set was analysed per solution. Future research could explore the impact of gradually increasing the number of reference sets and examine the relationships between different scenarios for methods utilizing the reference set approach. Fifthly, the rank reversal problem could be investigated concerning other reference set-based methods, and the results could be compared with the rankings produced by the IRPA method. Finally, this study introduces the classical IRPA method. Future research could integrate the method with different set theories (such as Fuzzy, Heuristic, Neutrosophic, and Plithogenic approaches) to better model human decision-making behaviour.

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