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A Consensus-Based MULTIMOORA Framework under Probabilistic Hesitant Fuzzy Environment for Manufacturing Vendor Selection

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Abstract. Multi-criteria group decision-making has gained considerable attention due to its ability to aggregate diverse expert opinions and establish a preference order among alternatives. While probabilistic hesitant fuzzy (PHF) sets offer increased flexibility and generality for representing criteria values compared to traditional fuzzy and hesitant fuzzy set theories, existing aggregation techniques often fail to enhance consensus among biased expert judgments. Motivated by the need for more effective consensus-based decision-making, this paper proposes a new framework that integrates PHF set theory with Aczel-Alsina weighted averaging and geometric aggregation operators. These operators, known for their flexibility and the inclusion of an adjustable parameter, are particularly well-suited for addressing real-world decision-making challenges. The framework employs a cross-entropy based model to determine criteria weights and multi-objective optimization by ratio analysis plus the full multiplicative form (MULTIMOORA) method to establish priority orders of alternatives. The proposed framework is demonstrated through a case study on manufacturing outsourcing vendor selection. The results show that Bertrandt is the most suitable vendor, with a score of 0.2390, and resources consumption is identified as the most critical criterion, with a weight of

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0.20. To validate the robustness of the proposed framework, sensitivity and comparison analyses have also been conducted.
Key words: probabilistic hesitant fuzzy set, Aczel-Alsina aggregation, consensus-based MULTIMOORA, fuzzy optimization, group decision-making.

1. Introduction

Advancements in science and technology have highlighted the necessity of managing uncertainty in decision-making processes, a challenge that has become increasingly critical in today's complex and data-driven world. As the volume and complexity of data continue to grow, so does the need for tools and methodologies that can effectively handle ambiguity and incomplete information. Traditional binary logic, which operates on clear-cut true or false values, often falls short in dealing with the nuances of real-world scenarios where data is seldom straightforward. The introduction of fuzzy sets (FSs) by Zadeh (1965) has played a crucial role in addressing data ambiguity. Alongside FSs, hesitant FSs (HF) (Torra, 2010) have emerged, allowing for more flexible membership degrees (MDs) to consider various potential inputs, as perceived in the work of Rodriguez *et al.* (2014). This development not only reduces subjective randomness but also aids in expressing expert preferences and accommodating occurrence probabilities. Building on this foundation, Xu and Zhou (2016) introduced the probabilistic HFSs (PHFSs), which assign occurrence probabilities to elements based on systematic reviews. Integrating PHF into decision-making frameworks, as demonstrated by Li and Wang (2017), has led to the establishment of novel decision models such as those incorporating preference ranking organization method for enrichment evaluation (PROMETHEE) and (qualitative flexible multiple criteria method (QUALIFLEX). Further advancements include distance measures of PHFSs by Ding *et al.* (2017), and development of a density function for investor assessments by Li *et al.* (2019). Reference ideal-based algorithms by He and Xu (2019) provide a means to evaluate projects, linking ideal values with PHF information (PHFI). Additionally, Liu *et al.* (2020) combined PHFI with regret theory and entropy measures for venture capital evaluations. Li *et al.* (2020) introduced an approach based on Organization Rangement EtSynthese De Donnes Relationnelles (ORESTE) employing PHFI. Lin *et al.* (2020a) used a PHFI algorithm for consistency testing in investment projects. Jin *et al.* (2020) suggested a preference relation-based measure under PHFI, and applied to logistical selection. To address rational CO2 storage location selection, Guo *et al.* (2020) proposed Tomada de Decisão Interativa Multicritério (TODIM) method incorporating Choquet Integrals under PHFI. Lin *et al.* (2020b) proposed PHF-Multi-Objective Optimization by Ratio Analysis plus Full Multiplicative Form (MULTIMOORA) method, while Liu *et al.* (2021) defined cross-efficiency model of Data Envelopment Analysis (DEA) using PHF preference relations. Krishankumar *et al.* (2022) developed PHF-Complex Proportional Assessment (COPRAS) method, while Liao *et al.* (2022a) addressed a supplier selection problem based on PHF-CODAS (Combinative Distance based Assessment) model. Liao *et al.* (2022b) employed prospect theory based TODIM method under PHF setting, and Qi (2023) used PHF-Technique for Order of Preference by Similarity to Ideal Solution (PHF-TOPSIS) for quality assessment of public charging services. Jaisankar *et al.* (2023) used a hybrid PHF decision-making approach for assessment of plastic disposal technologies, while Liu *et al.* (2023) developed a modified Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS) method incorporating PHFI.

1.1. *Research Gaps and Motivations*

MCDM is an important sub-set of decision theory, focusing on selecting the optimal alternative from a diverse set. The dynamic and ever-evolving socio-economic environment has significantly increased the complexity of real-world decision-making problems. This complexity arises from the need to consider various, often conflicting, criteria that impact the outcomes of decisions. Over recent decades, various methods have been developed to address the complexities of MCDM problems. These methods aim to enhance both the accuracy and efficiency of MCDM processes, each with distinct advantages and limitations. Some methods offer computational efficiency and ease of implementation, making them suitable for applications with limited resources; however, they may be less effective in managing uncertainty or imprecise data. On the other hand, certain methods are well-suited for handling uncertainty and providing stable solutions under variable conditions, though they often require increased computational resources and complexity.

The significance of each criterion in any decision-making process can vary based on the context, the specific decision to be made, and the stakeholders involved. Criteria weights represent the relative importance of each criterion in the decision-making process. Hence, determining the criteria weights must be done logically and systematically. The pertinent existing methods (Lin *et al.*, 2020b; Krishankumar *et al.*, 2022; Liao *et al.*, 2022a; Qi, 2023; Liu *et al.*, 2023a), often fail to derive these weights systematically, leading to subjectivity and inaccuracies. The difficulty in accurately weighting criteria arises from the complex nature of decision-making environments and varied perspectives of stakeholders. Thus, developing a comprehensive framework for determining criteria weights remains a challenging problem. Multi-criteria group decision-making (MCGDM) addresses this problem by integrating diverse expert perspectives and effectively managing trade-offs among conflicting criteria. By involving experts from various fields, MCGDM ensures that decisions are informed and balanced, enabling proper prioritization of the criteria. MCGDM models systematically evaluate trade-offs, providing clarity on compromises and optimizing outcomes. The process promotes transparency, consistency, and accountability, incorporating both objective data and subjective opinions. MCGDM also enhances decision quality by considering all relevant criteria, fostering collaboration, and increasing stakeholder trust. It helps decision-makers to understand the compromises involved in choosing between alternatives, ensuring that the final decision aligns with the overarching objectives. Furthermore, considering all relevant criteria ensures that no important aspects are overlooked, thereby improving the overall quality of the decision. However, due to their varied experiences and backgrounds, decision-makers often exhibit significant differences in evaluation, making consensus challenging. Existing methods like PHF-COPRAS (Krishankumar *et al.*, 2022), PHF-TODIM (Liao *et al.*,

Table 1
Existing works on Aczel-Alsina AOs.

Reference	AO	Application
Senapati *et al.* (2022a)	Interval-valued Pythagorean fuzzy Aczel-Alsina AOs	Selection of an emerging IT software company
Mahmood *et al.* (2022)	Complex intuitionistic fuzzy Aczel-Alsina AOs	Selection of an advertising administrator
Ali and Naeem (2022)	Complex q-rung orthopair fuzzy Aczel-Alsina AOs	Selection of the most impactful sector effecting the Stock Exchange
Li *et al.* (2023)	Neutrosophic multi-valued Aczel-Alsina AOs	Selection of service robots
Wang *et al.* (2023)	T-spherical fuzzy Aczel-Alsina Hamy Mean AOs	Assessment of investment company plans
Chen *et al.* (2023)	Complex Fermatean fuzzy Aczel-Alsina AOs	Assessment of different bands for solar panel system
Athar Farid and Riaz (2023)	q-rung orthopair fuzzy Aczel-Alsina AOs	Green supplier selection
Liu *et al.* (2023)	Complex intuitionistic fuzzy Aczel-Alsina prioritized AOs	Assessment of business alternatives
Senapati (2024)	Single valued neutrosophic Aczel-Alsina AOs	Assessment of investment opportunities
Gula *et al.* (2024)	Aczel-Alsina linear Diophantine fuzzy AOs	Selection of weather forecasting techniques

2022b), PHF-MARCOS (Liu *et al.*, 2023a), and PHF-TOPSIS (Qi, 2023), may fail to fully capture the ambiguity in expert judgments, especially when biases are present. Therefore, it becomes necessary to implement consensus-building strategies to increase agreement among decision-makers, a factor that previous studies with PHFI have not adequately addressed.

One critical aspect of improving consensus and enhancing decision quality is the effective aggregation of diverse expert inputs. Combining various input data into a unified entity often requires aggregation operators (AOs), which have proven highly effective in data processing, decision-making, pattern recognition, data analytics, and neural networks. While AOs like Archimedean AOs, Hamacher AOs, Einstein AOs, and Dombi AOs have been employed for aggregating PHFI, the potential of Aczel-Alsina AOs has been increasingly recognized in this context (Senapati *et al.*, 2022a, 2023a, 2023b). These studies suggested that Aczel-Alsina AOs offer a promising approach to address these aggregation challenges, providing significantly accurate results in MCGDM environment, thus making Aczel-Alsina AOs a valuable tool for improving both consensus-building process and overall decision-making framework. The existing works on Aczel-Alsina AOs are summarized in Table 1.

The continuous improvement and adaptation of MCDM methods reflect ongoing efforts to bridge theoretical progress with practical application. Efforts to make these methods more responsive to emerging needs, including large-scale data processing and adaptable decision-making in dynamic environments, are steadily advancing. Balancing theoretical rigor with practical usefulness highlights the essential role of MCDM frameworks in modern applications, where solutions must combine accuracy with adaptability to meet the complexities of real-world problems. Brauers and Zavadskas (2006) proposed MOORA model, a well-known and effective MCDM method that combines reference point (RP) and ratio system (RS) models. MULTIMOORA (Brauers and Zavadskas, 2010), an extension of MOORA, was developed based on RS, RP and full multiplicative form (FMF) models. In recent years, under various fuzzy contexts MULTIMOORA method have been utilized for purchasing rental space (Stanujkic *et al.*, 2019), selection of technology for food waste treatment (Rani *et al.*, 2021), charging station selection for electric vehicles (Rani and Mishra, 2021), CNC machine tool selection (Sahin and Aydemir, 2022), solid waste disposal method selection (Mishra *et al.*, 2023), failure mode and effects analysis (Yu *et al.*, 2023), green supplier selection (Gai *et al.*, 2023), welding process selection (Saluja and Singh, 2023), offshore wind power station site selection (Zhou *et al.*, 2024), crop disease detection (Zhang *et al.*, 2024), sustainable supplier selection (Vaezi *et al.*, 2024), car selection through online reviews (Liu *et al.*, 2024), business strategies evaluation (Ghaemi-Zadeh and Eghbali-Zarch, 2024), sustainability of urban mobility evaluation (Yucesan *et al.*, 2024). Consensus-reaching mechanisms for structured group decision-making have not yet been incorporated into MULTIMOORA method. With increasing environmental concerns and regulatory pressures, the importance of sustainable practices in operations and supply chains is being widely acknowledged. Lean, agile, resilient, green, and sustainable approaches are being integrated throughout supply chain and manufacturing processes to address uncertainties and support competitiveness. The selection of manufacturing outsourcing vendors (MOVs) significantly impacts operational efficiency, environmental performance, and resilience to disruptions. Although promising, the PHF-based MULTIMOORA method has not been applied to MOV selection, marking an area with potential for further development.

1.2. *Contributions*

Nowadays, decision makers are increasingly seeking systematic approaches to determine optimal actions. Despite the wealth of literature on PHFSs, there is a notable dearth of research leveraging the complete potential of PHFI, Aczel-Alsina AOs, the consensus-building process, and MULTIMOORA method to address decision-making complexities. This study fills critical gaps in the literature by making significant contributions in these areas.

a) The research introduces two novel PHF Aczel-Alsina Aggregation Operators (PHFAAWA and PHFAAWG) designed specifically to tackle the intricate challenges of group decision-making utilizing PHFI. These adaptable weighted operators provide decision-makers with a resilient tool to efficiently amalgamate varied perspectives within a group context.
b) A comprehensive consensus-based MULTIMOORA model is presented in this research, specifically designed to assess MOVs. This model incorporates an advanced optimization process to calculate criteria weights, ensuring the objectivity and robustness of the assessment process. By utilizing this model, decision-makers are equipped to make well-informed decisions when selecting manufacturing outsourcing vendors.

1.3. *Structural Overview of the Paper*

This paper follows a structured framework: Section 2 provides a comprehensive overview of essential concepts necessary for understanding the subsequent discussions. Section 3 examines the details of Aczel-Alsina operations between PHF elements, explaining the definitions and characteristics while introducing PHFAAWA and PHFAAWG AOs. Building upon this foundation, Section 4 elucidates the consensus-based PHF decision support paradigm. The practical application of these methodologies is exemplified in Section 5, which presents a detailed case study and the corresponding solution. Section 6 provides comprehensive discussions on sensitivity analysis, and comparative study. Finally, Section 7 encapsulates the research outcomes, offering insightful recommendations for future investigations in the field.

2. Preliminaries

Definition 1 (Xu and Zhou, 2016). A PHF set $\aleph$ on a set U is defined as: $\aleph = \{\langle \alpha, \varpi(\alpha)\rangle : \alpha \in U\}$ where $\varpi(\alpha) = \bigcup_t \{\delta_\alpha^{(t)}(p^{(t)})\}$, and $0 \leqslant \sum_t p^{(t)} \leqslant 1$ ($P^{(q)}$ being the probability of $\delta_\alpha^{(t)} \in [0, 1]$). If U is singleton, then $\aleph$ reduces to a PHF element (PHFE) and we write $\aleph = \bigcup_t \{\delta^{(t)}(p^{(t)})\}$ and $s(\aleph) = \sum_t (\delta^{(t)} \times p^{(t)})$ to denote it's a score value. For two PHFEs $\aleph$ and $\Re$, $s(\aleph) > s(\Re) \Rightarrow \aleph \succ \Re$.

Definition 2 (Senapati *et al.*, 2022b). The Aczel-Alsina (AA) t-norm is described as:

$$AA_N(x, y) = \exp\bigl[-\bigl((-\ln x)^\theta + (-\ln y)^\theta\bigr)^{\frac{1}{\theta}}\bigr], \quad (0 < \theta < \infty). \tag{1}$$

The Aczel-Alsina (AA) t-conorm (s-norm) is described as:

$$AA_{CN}(x, y) = 1-\exp\bigl[-\bigl(\bigl(-\ln(1-x)\bigr)^\theta+\bigl(-\ln(1-y)\bigr)^\theta\bigr)^{\frac{1}{\theta}}\bigr], \quad (0 < \theta < \infty). \tag{2}$$

3. Aczel-Alsina-Operations Between PHFEs and Associated Weighted Operators

Let $\aleph_n = \langle \bigcup_t \{\delta_n^{(t)}(p_n^{(t)})\}\rangle$ $(n = 1, 2, \dots, \ell)$ be a collection of *PHFEs*.

Definition 3. For $\sigma > 0$, we define:

$$\text{(i) } \aleph_1 \oplus \aleph_2 = \left\langle \bigcup_t \left\{1 - \exp\left[-\left(\sum_{n=1}^{2}\bigl(-\ln\bigl(1 - \delta_n^{(t)}\bigr)\bigr)^\theta\right)^{\frac{1}{\theta}}\right]\right\}\bigl(p_1^{(t)} p_2^{(t)}\bigr)\right\rangle, \tag{3}$$

$$\text{(ii) } \aleph_1 \otimes \aleph_2 = \left\langle \bigcup_t \left\{\exp\left[-\left(\sum_{n=1}^{2}\bigl(-\ln\bigl(\delta_n^{(t)}\bigr)\bigr)^\theta\right)^{\frac{1}{\theta}}\right]\right\}\bigl(p_1^{(t)} p_2^{(t)}\bigr)\right\rangle, \tag{4}$$

$$\text{(iii) } \sigma\aleph_1 = \left\langle \bigcup_t \{1 - \exp[-(\sigma(-\ln(1 - \delta_1^{(t)}))^\theta)^{\frac{1}{\theta}}]\}\bigl(p_1^{(t)}\bigr)\right\rangle, \tag{5}$$

$$\text{(iv) } \aleph_1^{\sigma} = \left\langle \bigcup_{q} \left\{ \exp\left[-\left(\sigma\left(-\ln\left(\delta_1^{(t)}\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right]\right\} \left(p_1^{(t)}\right)\right\rangle. \tag{6}$$

Theorem 1. *Let* $\aleph_n = \langle \bigcup_t \{\delta_n^{(t)}(p_n^{(t)})\}\rangle$ $(n = 1, 2)$ *be a collection of PHFEs. Then*

(i) $\aleph_1 \oplus \aleph_2 = \aleph_2 \oplus \aleph_1$,

(ii) $\aleph_1 \otimes \aleph_2 = \aleph_2 \otimes \aleph_1$,

(iii) $\sigma_1(\aleph_1 \oplus \aleph_2) = (\sigma_1\aleph_1) \oplus (\sigma_2\aleph_2)$,

(iv) $(\aleph_1 \otimes \aleph_2)^{\sigma_1} = \left(\aleph_1^{\sigma_1}\right) \oplus \left(\aleph_2^{\sigma_1}\right)$,

(v) $(\sigma_1 + \sigma_2)\aleph_1 = (\sigma_1\aleph_1) \oplus (\sigma_2\aleph_2)$,

(vi) $\aleph_1^{\sigma_1+\sigma_2} = \left(\aleph_1^{\sigma_1}\right) \otimes \left(\aleph_2^{\sigma_2}\right)$.

Proof. Follows from Definition 3. □

Definition 4. Suppose W_n denotes the weight of $\aleph_n$ such that $0 \leqslant W_n \leqslant 1$ and $\sum_{n=1}^{\ell} W_n = 1$. Then the *PHF-AA* weighted averaging (*PHFAAWA*) operator is presented as:

$$\mathit{PHFAAWA}(\aleph_n, \aleph_n, \ldots, \aleph_n) = \bigoplus_{n=1}^{\ell}(W_n\aleph_n). \tag{7}$$

Theorem 2. *PHFAAWA*$(\aleph_1, \aleph_2, \ldots, \aleph_\ell)$ *can be expressed as a PHFE and*

$$\begin{aligned}&\mathit{PHFAAWA}(\aleph_1, \aleph_2, \ldots, \aleph_\ell)\\ &\quad= \left\langle \bigcup_{t} \left\{ 1 - \exp\left[-\left(\sum_{n=1}^{\ell} W_n\left(-\ln\left(1-\delta_n^{(t)}\right)\right)^{\theta}\right)^{\frac{1}{\theta}}\right]\right\} \left(\prod_{n=1}^{\ell} p_n^{(t)}\right)\right\rangle.\end{aligned} \tag{8}$$

Proof. Follows from Definition 3 and Theorem 1. □

Several properties of the PHFAAWA operator are presented below.

Theorem 3. *If* $\aleph_0$ ($\neq$ $\aleph_n$ *for any* n) *is a PHFE, then PHFAAWA*$(\aleph_0 \oplus \aleph_1, \aleph_0 \oplus \aleph_2, \ldots, \aleph_0 \oplus \aleph_\ell) = \aleph_0 \oplus$ *PHFAAWA*$(\aleph_1, \aleph_2, \ldots, \aleph_\ell)$.

Theorem 4. *If* $\aleph_0$ ($\neq \aleph_n$ *for any* n) *is a PHFE, then PHFAAWA*$(\aleph_1, \aleph_2, \ldots, \aleph_\ell) = \aleph_0$.

Theorem 5. *If* $(\aleph_n)^- = \left\langle \bigcup_t \{\min_n \delta_n^{(t)}(p_n^{(t)})\}\right\rangle$ *and* $(\aleph_n)^+ = \langle \bigcup_t \{\max_n \delta_n^{(t)}(p_n^{(t)})\}\rangle$, *then* $(\aleph_n)^- \prec$ *PHFAAWA*$(\aleph_1, \aleph_2, \ldots, \aleph_\ell) \prec (\aleph_n)^+$.

Theorem 6. *If* $\Re_n = \left\langle \bigcup_t \{\delta_n'^{(t)}(p_n^{(t)})\}\right\rangle$ $(n = 1, 2, \ldots, \ell)$ *be another collection of PHFEs with* $\delta_n^{(t)} \leqslant \delta_n'^{(t)}$ $\forall n$, *then PHFAAWA*$(\aleph_1, \aleph_2, \ldots, \aleph_\ell) \prec$ *PHFAAWA*$(\Re_1, \Re_2, \ldots, \Re_\ell)$.

Definition 5. Suppose W_n denotes the weight of $\aleph_n$ such that $0 \leqslant W_n \leqslant 1$ and $\sum_{n=1}^{\ell} W_n = 1$. Then the *PHF-AA* weighted geometric (*PHFAAWG*) operator is presented as:

$$PHFAAWG(\aleph_n, \aleph_n, \dots, \aleph_n) = \bigotimes_{n=1}^{\ell} (\aleph_n^{W_n}). \tag{9}$$

Theorem 7. *PHFAAWG*$(\aleph_1, \aleph_2, \dots, \aleph_\ell)$ *can be expressed as a PHFE and*

$$\begin{aligned} &PHFAAWG(\aleph_1, \aleph_2, \dots, \aleph_\ell) \\ &\quad = \left\langle \bigcup_t \left\{ \exp\left[-\left(\sum_{n=1}^{\ell} W_n \left(-\ln(\delta_n^{(t)}) \right)^{\theta} \right)^{\frac{1}{\theta}} \right] \right\} \left(\prod_{n=1}^{\ell} p_n^{(t)} \right) \right\rangle. \end{aligned} \tag{10}$$

Proof. Follows from Definition 5 and Theorem 1. □

Some properties of PHFAAWG operator are presented as follows:

Theorem 8. *If* $\aleph_0$ $(\neq \aleph_n$ *for any* $n)$ *is a PHFE, then* *PHFAAWG*$(\aleph_0 \otimes \aleph_1, \aleph_0 \otimes \aleph_2, \dots, \aleph_0 \otimes \aleph_\ell) = \aleph_0 \otimes$ *PHFAAWG*$(\aleph_1, \aleph_2, \dots, \aleph_\ell)$.

Theorem 9. *If* $\aleph_0$ $(\neq \aleph_n$ *for any* $n)$ *is a PHFE, then* *PHFAAWG*$(\aleph_1, \aleph_2, \dots, \aleph_\ell) = \aleph_0$.

Theorem 10. *If* $(\aleph_n)^- = \langle \bigcup_t \{\min_n \delta_n^{(t)} (p_n^{(t)})\} \rangle$ *and* $(\aleph_n)^+ = \langle \bigcup_t \{\max_n \delta_n^{(t)} (p_n^{(t)})\} \rangle$, *then*

$$(\aleph_n)^- \prec PHFAAWG(\aleph_1, \aleph_2, \dots, \aleph_\ell) \prec (\aleph_n)^+.$$

Theorem 11. *If* $\Re_n = \langle \bigcup_t \{\delta_n'^{(t)} (p_n^{(t)})\} \rangle$ $(n = 1, 2, \dots, \ell)$ *be another collection of PHFEs with* $\delta_n^{(t)} \leqslant \delta_n'^{(t)}$ $\forall n$, *then* *PHFAAWG*$(\aleph_1, \aleph_2, \dots, \aleph_\ell) \prec$ *PHFAAWG*$(\Re_1, \Re_2, \dots, \Re_\ell)$.

4. Group Decision-Making Methodology

Let's consider that the alternatives $N_1, N_2, \dots, N_n$ and factors (criteria) $F_1, F_2, \dots, F_q$ are linked to a group assessment scenario, where each alternative is evaluated by the experts (specialists) $E_1, E_2, \dots, E_l$ under the PHF framework. The initial results, analysed by the experts, are depicted as the PHF decision matrices, as given below:

$$U_m = [\aleph_m^{rs}]_{n \times q} = \left[\left\langle \bigcup_b \{ \delta_{rsm}^{(b)} (p_{rsm}^{(b)}) \} \right\rangle \right]_{n \times q} \quad (m = 1, 2, \dots, l). \tag{11}$$

The proposed framework includes the following steps.

Step 1: Use PHFAAWA or PHFAAWG operator to get the aggregated PHF matrix (A-PHF-M).

A-PHF-M is $[\aleph^{rs}]_{n\times q} = \left[\left\langle\bigcup_b\{\delta_{rs}^{(b)}(p_{rs}^{(b)})\}\right\rangle\right]_{n\times q}$ where:

$$\aleph^{rs} = PHFAAWA\left(\aleph_1^{rs}, \aleph_2^{rs}, \ldots, \aleph_l^{rs}\right) = \bigoplus_{m=1}^{l}\left(\varpi_m \aleph_m^{rs}\right) \tag{12}$$

OR

$$\aleph^{rs} = PHFAAWG\left(\aleph_1^{rs}, \aleph_2^{rs}, \ldots, \aleph_l^{rs}\right) = \bigotimes_{m=1}^{l}\left(\aleph_m^{rs}\right)^{\varpi_m} \tag{13}$$

(ϖ_m being the weight of E_m).

Step 2: Calculate the "degree of consensus" (DC) for each expert.

Let us consider the following fuzzy matrices.

$$F_m = \left[\Delta_m^{rs}\right]_{n\times q} = \left[\sum_b\left(\delta_{rsm}^{(b)} \times p_{rsm}^{(b)}\right)\right]_{n\times q}, \tag{14}$$

$$F = \left[\Delta^{rs}\right]_{n\times q} = \left[\sum_b\left(\delta_{rs}^{(b)} \times p_{rs}^{(b)}\right)\right]_{n\times q}. \tag{15}$$

The correlation measure (CM) $\Omega_s^{(m)}$ for E_m under H_s can be defined as:

$$\Omega_s^{(m)} = \frac{\sum_{r=1}^{p}\left[\left(\frac{D_{rs}^{(m)}}{D_s^{(m)}} - \frac{1}{p}\sum_{r=1}^{p}\frac{D_{ij}^{(m)}}{D_s^{(m)}}\right) \times \left(\frac{D_{rs}}{D_s} - \frac{1}{p}\sum_{r=1}^{p}\frac{D_{rs}}{D_s}\right)\right]}{\sqrt{\sum_{r=1}^{p}\left(\frac{D_{ij}^{(m)}}{D_s^{(m)}} - \frac{1}{p}\sum_{r=1}^{p}\frac{D_{rs}^{(m)}}{D_s^{(m)}}\right)^2} \times \sqrt{\sum_{r=1}^{p}\left(\frac{D_{rs}}{D_s} - \frac{1}{p}\sum_{r=1}^{p}\frac{D_{rs}}{D_s}\right)^2}}, \tag{16}$$

where

$$\begin{aligned}
&\bar{\zeta}_m^{(rs)} = \left\langle\max_r \Delta_m^{rs}\right\rangle, \quad \underline{\zeta}_m^{(rs)} = \left\langle\min_r \Delta_m^{rs}\right\rangle, \quad \bar{\zeta}^{(rs)} = \left\langle\max_r \Delta^{rs}\right\rangle, \quad \underline{\zeta}^{(rs)} = \left\langle\min_r \Delta^{rs}\right\rangle,\\
&D_{rs}^{(m)} = \text{Distance}\left(\zeta_m^{(rs)}, \bar{\zeta}_m^{(rs)}\right) = \left|\zeta_m^{(rs)} - \bar{\zeta}_m^{(rs)}\right|,\\
&D_s^{(m)} = \text{Distance}\left(\bar{\zeta}_m^{(rs)}, \underline{\zeta}_m^{(rs)}\right) = \left|\bar{\zeta}_m^{(rs)} - \underline{\zeta}_m^{(rs)}\right|,\\
&D_{rs} = \text{Distance}\left(\bar{\zeta}^{(rs)}, \zeta^{(rs)}\right) = \left|\bar{\zeta}^{(rs)} - \zeta^{(rs)}\right|,\\
&Dist_s = \text{Distance}\left(\bar{\zeta}^{(rs)}, \underline{\zeta}^{(rs)}\right) = \left|\bar{\zeta}^{(rs)} - \underline{\zeta}^{(rs)}\right|.
\end{aligned}$$

Next, CD $\vartheta^{(m)}$ of E_m can be defined as:

$$\vartheta^{(m)} = \frac{1}{q}\sum_{s=1}^{q}\Omega_s^{(m)} \quad (m = 1, 2, \ldots, l). \tag{17}$$

Obviously $\vartheta^{(m)} \in [0, 1]$. If ϑ denotes the predefined DC for each expert, then $\vartheta^{(m)} \geqslant \vartheta$ must be fulfilled. For $\vartheta^{(m)} < \vartheta$, then we have to consider other values of θ till $\vartheta^{(m)} \geqslant \vartheta$ is achieved for all E_m.

Step 3: Calculation of criteria weights.

The divergence measure, as given in Eq. (18), exhibits the difference between rth alternative and other alternatives under sth criterion.

$$DIV_{rs} = \frac{1}{n-1} \sum_{z=1}^{n} C\big(\Delta^{rs}, \Delta^{zs}\big). \tag{18}$$

where $C(\Delta^{rs}, \Delta^{zs})$ which corresponds the cross-entropy measure between Δ^{rs} and Δ^{zs} is defined as:

$$\begin{aligned} C\big(\Delta^{rs}, \Delta^{zs}\big) = {} & \Delta^{rs} \times \ln\left(\frac{2\Delta^{rs}}{\Delta^{rs} + \Delta^{zs}}\right) + \Delta^{zs} \times \ln\left(\frac{2\Delta^{zs}}{\Delta^{rs} + \Delta^{zs}}\right) \\ & + \big(1 - \Delta^{rs}\big) \times \ln\left(\frac{1 - \Delta^{rs}}{1 - \frac{1}{2}(\Delta^{rs} + \Delta^{zs})}\right) \\ & + \big(1 - \Delta^{zs}\big) \times \ln\left(\frac{1 - \Delta^{zs}}{1 - \frac{1}{2}(\Delta^{rs} + \Delta^{zs})}\right). \end{aligned} \tag{19}$$

The overall divergence corresponding to sth criterion is calculated as:

$$DIV_s = \frac{1}{n-1} \sum_{r=1}^{n} \sum_{z=1}^{n} C\big(\Delta^{rs}, \Delta^{zs}\big). \tag{20}$$

Hence, upon solving the optimization model, criteria weights can be determined.

$$\begin{aligned} & \mathrm{Max}\chi = \sum_{s=1}^{q} \beta_s \frac{1}{n-1} \sum_{r=1}^{n} \sum_{z=1}^{n} C\big(\Delta^{rs}, \Delta^{zs}\big), \\ & \text{Subject to: } \beta_s \in \Xi, \sum_{s=1}^{q} \beta_s = 1, \quad \beta_s \geqslant 0 \forall s, \end{aligned} \tag{21}$$

where β_s represents the weight of sth criterion and Ξ represents the set of partial information regarding criteria weights.

Step 4: Derive priority order of the alternatives using MULTIMOORA.

Step 4.1: Determine the significance values (SVs) of alternatives by "ratio system" (RS) technique.

Suppose ζ_r^+ and ζ_r^- denote the level of significance of N_r in relation to benefit and cost attributes respectively. They can be calculated by:

$$\zeta_r^+ = \bigoplus_{s \in \mathrm{Benefit}} \big(\beta_s \Delta^{rs}\big), \qquad \zeta_r^- = \bigoplus_{s \in \mathrm{Cost}} \big(\beta_s \Delta^{rs}\big). \tag{22}$$

Determine the SV for N_r utilizing Eq. (23):

$$\zeta_r = \zeta_r^+ - \zeta_r^- \quad (r = 1, 2, \dots, n). \tag{23}$$

Step 4.2: Calculate the overall SVs (OSVs) of alternatives by "ratio point" (RP) model.

The OSVs (ϑ_s) of RP model for the sth criterion (F_s) are obtained using Eq. (24).

$$\vartheta_s = \begin{cases} \max_r \Delta^{rs}, & \text{for } s \in \text{Benefit}, \\ \min_r \Delta^{rs}, & \text{for } s \in \text{Cost}. \end{cases} \tag{24}$$

Use Eq. (25) to calculate the weighted distances (WDs) for all alternatives.

$$\Omega_{rs} = \beta_s \times \left|\Delta^{rs} - \vartheta_s\right|. \tag{25}$$

Determine the maximum WD based on Eq. (26).

$$\eta_r = \max_s \Omega_{rs} \quad (r = 1, 2, \dots, p). \tag{26}$$

Step 4.3: Calculate SVs of the alternatives by FMF model.

Suppose α_r^+ and α_r^+ denote the level of significance of N_r related to benefit and cost attributes respectively. We calculate them as:

$$\alpha_r^+ = \bigotimes_{s \in \text{Benefit}} \left(\Delta^{rs}\right)^{\beta_s}, \qquad \alpha_r^- = \bigotimes_{s \in \text{Cost}} \left(\Delta^{rs}\right)^{\beta_s}. \tag{27}$$

Determine the SV of N_r using Eq. (28).

$$\tau_r = \frac{\alpha_r^+}{\alpha_r^-} \quad (r = 1, 2, \dots, n). \tag{28}$$

Step 4.4: Calculate the final priority values (FPVs) of alternatives.

The FPV of N_r by improved Borda Rule, is calculated as follows:

$$\begin{aligned} B(N_r) = {} & \frac{2}{n(n+1)}\Big[\tilde{\zeta}_r \times \big(n - rank(\tilde{\zeta}_r) + 1\big) - \tilde{\eta}_r \times rank(\tilde{\eta}_r) \\ & + \tilde{\tau}_r \times \big(n - rank(\tilde{\tau}_r) + 1\big)\Big] \quad (r = 1, 2, \dots, n). \end{aligned} \tag{29}$$

Here, $\tilde{\zeta}_r, \tilde{\eta}_r, \tilde{\tau}_r$ are the scores (normalized) of B_r by RS, RP and FMF models, respectively. Alternatives are ranked based on their FPVs.

5. Case Study

5.1. *Problem Description*

One of the main forces for macroeconomic expansion and technological improvement in India is the automobile sector. By 2026, volume-wise, India is predicted to become the third-largest vehicle market in the world (Miglani, 2019). Approximately 22.70 million vehicles, including passenger, commercial, three- and four-wheeler vehicles, were produced in India during the 2020–2021 fiscal year (Luthra *et al.*, 2016). By 2026, the Indian automobile sector is projected to generate \$300 billion in revenue (Mathivathanan *et al.*, 2018). The automobile sector in India generates around 37 million direct and indirect jobs, accounts for 49% of all manufacturing output, and contributes 7.1% of the country's GDP (Borkhade *et al.*, 2022).

In this paper, a well-known Indian producer of commercial vehicles (referred to as Company Z), which has a robust domestic and global market and has been in operation for the previous forty years, is examined. The business holds an ISO 9001 accreditation. As one of the leading car manufacturers in India, Company Z is well-known for its excellent performance in terms of mileage, payload, strength and durability, luxury and comfort, convenience of warranty and servicing, and so on. The corporation has been working to achieve sustainability and agility in their operations. The case company has therefore put outsourcing rules into place and is searching for a trustworthy outsourcing vendor.

To identify and assess agile and sustainability criteria, a committee comprising three experts was formed. The committee chose six agile criteria, namely production flexibility and capability ($F1$), service level and lead time ($F2$), multi-skilled and flexible workforce ($F3$), collaboration with partners ($F4$), customer driven innovation ($F5$), delivery and sourcing flexibility ($F6$), and six sustainability criteria, namely: product price ($F7$), resource consumption ($F8$), green product ($F9$), green manufacturing process ($F10$), workers' occupational health and safety ($F11$), social welfare and community development ($F12$). A brief description about the criteria is presented in Table 2.

The initial evaluations provided by the three experts for the outsourcing vendors Altras (N_1), Bertrandt (N_2), Tata technology (N_3) and EDAG (N_4) are presented in Tables 3, 4, and 5 respectively.

5.2. *Problem Solution*

PHFAAWA operator, as shown in Eq. (13), is employed to derive the aggregated PHF matrix for $\theta = 2$, with expert weights of 0.35, 0.40, and 0.25. The initial PHF assessment matrices are converted to fuzzy matrices based on Eq. (14). The converted aggregated matrix (for $\theta = 2$) is formed employing Eq. (15), as shown in Table 6. Suppose the minimum DC is $\vartheta = 0.80$. DC value for each expert are calculated using Eqs. (16) and (17) for different values of Aczel-Alsina parameter θ have been given in Fig. 1. Since $\vartheta^{(3)} = 0.7930$ for $\theta = 3$, $\vartheta^{(3)} = 0.7857$ for $\theta = 4$, and $\vartheta^{(3)} = 0.7815$ for $\theta = 5$, so these values of θ cannot be considered. We can choose either $\theta = 1$ or $\theta = 2$. Let us take $\theta = 2$. Then the DC are: $\vartheta^{(1)} = 0.9118$, $\vartheta^{(2)} = 0.8186$, and $\vartheta^{(3)} = 0.8056$. Since $\min_m \vartheta^{(m)} = 0.8056$, we are done with the consensus reaching among experts.

Next, based on the Eqs. (18)–(21), we have the following optimization model in linear form:

$$\begin{aligned}\text{Max } Z = {} & 0.1314\beta_1 + 0.3583\beta_2 + 0.3702\beta_3 + 0.1167\beta_4 + 0.1139\beta_5 + 0.2844\beta_6 \\ & + 0.1858\beta_7 + 0.3089\beta_8 + 0.1050\beta_9 + 0.5059\beta_{10} + 0.2100\beta_{11} + 0.3157\beta_{12}\end{aligned}$$

Table 2
Significance of the considered MOV selection criteria.

Category	Criteria	Significance
Agile	Production flexibility and capability (F1)	Ability of the vendor to adapt their production process to meet changing demands, enabling faster product variations or customized orders.
	Service level and lead time (F2)	The time it takes to fulfil an order and deliver the product, crucial for meeting customer expectations and maintaining competitiveness in dynamic markets.
	Multi-skilled and flexible workforce (F3)	The versatility of the workforce in handling various tasks, ensuring smooth transitions in production processes without delays or quality compromise.
	Collaboration with partners (F4)	The extent of cooperation with supply chain partners to improve efficiency, transparency, and responsiveness.
	Customer-driven innovation (F5)	Innovation driven by customer needs, which helps in creating customized or advanced products that meet market demands.
	Delivery and sourcing flexibility (F6)	Ability to adjust delivery schedules and sourcing strategies according to demand fluctuations, reducing risks and delays in the supply chain.
Sustainability	Product price (F7)	The cost at which the product is sold, influencing both affordability and the vendor's ability to maintain competitive pricing in the market.
	Resource consumption (F8)	The efficiency with which the vendor uses resources, such as energy and raw materials, to minimize waste and environmental impact.
	Green product (F9)	A product designed with environmental considerations, using sustainable materials and processes to minimize environmental impact.
	Green manufacturing process (F10)	The adoption of eco-friendly manufacturing processes that reduce environmental harm, such as energy consumption and emissions.
	Workers' occupational health and safety (F11)	Focus on the health, safety, and well-being of workers in the production environment, ensuring a safe working atmosphere and compliance with regulations.
	Social welfare and community development (F12)	Contributions to societal well-being, including community engagement, promoting social equity, and supporting local development.

Table 3
Primary assessments by the 1st expert.

MOV	F_1	F_2	F_3	F_4
N_1	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.8(0.5), 0.9(0.5)\rangle$	$\langle 0.4(0.6), 0.5(0.4)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$
N_2	$\langle 0.8(1)\rangle$	$\langle 0.7(1)\rangle$	$\langle 0.6(1)\rangle$	$\langle 0.8(1)\rangle$
N_3	$\langle 0.5(1)\rangle$	$\langle 0.6(1)\rangle$	$\langle 0.3(1)\rangle$	$\langle 0.4(1)\rangle$
N_4	$\langle 0.1(0.5), 0.3(0.5)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$	$\langle 0.1(0.5), 0.2(0.5)\rangle$	$\langle 0.2(0.5), 0.3(0.5)\rangle$
	F_5	F_6	F_7	F_8
N_1	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.5(0.5), 0.7(0.5)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$
N_2	$\langle 0.6(1)\rangle$	$\langle 0.7(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.8(1)\rangle$
N_3	$\langle 0.3(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.7(1)\rangle$	$\langle 0.5(1)\rangle$
N_4	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.2(0.6), 0.4(0.4)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$	$\langle 0.3(0.4), 0.4(0.6)\rangle$
	F_9	F_{10}	F_{11}	F_{12}
N_1	$\langle 0.3(0.4), 0.6(0.6)\rangle$	$\langle 0.5(0.5), 0.7(0.5)\rangle$	$\langle 0.6(0.5), 0.9(0.5)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$
N_2	$\langle 0.6(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.7(1)\rangle$
N_3	$\langle 0.1(1)\rangle$	$\langle 0.3(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.3(1)\rangle$
N_4	$\langle 0.2(0.5), 0.4(0.5)\rangle$	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.2(0.5), 0.3(0.5)\rangle$

Table 4
Primary assessments by the 2nd expert.

MOV	F_1	F_2	F_3	F_4
N_1	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.5(0.6), 0.8(0.4)\rangle$	$\langle 0.6(0.5), 0.7(0.5)\rangle$
N_2	$\langle 0.7(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.8(1)\rangle$
N_3	$\langle 0.7(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.2(1)\rangle$	$\langle 0.6(1)\rangle$
N_4	$\langle 0.5(0.5), 0.6(0.5)\rangle$	$\langle 0.1(0.5), 0.3(0.5)\rangle$	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$
	F_5	F_6	F_7	F_8
N_1	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.5(0.5), 0.8(0.5)\rangle$	$\langle 0.4(0.5), 0.6(0.5)\rangle$	$\langle 0.5(0.5), 0.7(0.5)\rangle$
N_2	$\langle 0.5(1)\rangle$	$\langle 0.6(1)\rangle$	$\langle 0.8(1)\rangle$	$\langle 0.7(1)\rangle$
N_3	$\langle 0.3(1)\rangle$	$\langle 0.2(1)\rangle$	$\langle 0.6(1)\rangle$	$\langle 0.5(1)\rangle$
N_4	$\langle 0.4(0.5), 0.5(0.5)\rangle$	$\langle 0.1(0.6), 0.3(0.4)\rangle$	$\langle 0.4(0.5), 0.6(0.5)\rangle$	$\langle 0.2(0.4), 0.3(0.6)\rangle$
	F_9	F_{10}	F_{11}	F_{12}
N_1	$\langle 0.5(0.4), 0.6(0.6)\rangle$	$\langle 0.8(0.5), 0.9(0.5)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.6(0.5), 0.7(0.5)\rangle$
N_2	$\langle 0.5(1)\rangle$	$\langle 0.6(1)\rangle$	$\langle 0.4(1)\rangle$	$\langle 0.6(1)\rangle$
N_3	$\langle 0.5(1)\rangle$	$\langle 0.3(1)\rangle$	$\langle 0.7(1)\rangle$	$\langle 0.4(1)\rangle$
N_4	$\langle 0.6(0.5), 0.9(0.5)\rangle$	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.4(0.5), 0.7(0.5)\rangle$	$\langle 0.3(0.5), 0.4(0.5)\rangle$

Table 5
Primary assessments by the 3rd expert.

MOV	F_1	F_2	F_3	F_4
N_1	$\langle 0.7(0.5), 0.8(0.5)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$	$\langle 0.3(0.6), 0.6(0.4)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$
N_2	$\langle 0.6(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.8(1)\rangle$	$\langle 0.5(1)\rangle$
N_3	$\langle 0.2(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.7(1)\rangle$
N_4	$\langle 0.3(0.5), 0.4(0.5)\rangle$	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.5(0.5), 0.7(0.5)\rangle$	$\langle 0.4(0.5), 0.6(0.5)\rangle$
	F_5	F_6	F_7	F_8
N_1	$\langle 0.3(0.5), 0.5(0.5)\rangle$	$\langle 0.5(0.5), 0.6(0.5)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$
N_2	$\langle 0.6(1)\rangle$	$\langle 0.8(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.7(1)\rangle$
N_3	$\langle 0.2(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.6(1)\rangle$	$\langle 0.3(1)\rangle$
N_4	$\langle 0.1(0.5), 0.2(0.5)\rangle$	$\langle 0.3(0.6), 0.5(0.4)\rangle$	$\langle 0.5(0.5), 0.7(0.5)\rangle$	$\langle 0.1(0.4), 0.3(0.6)\rangle$
	F_9	F_{10}	F_{11}	F_{12}
N_1	$\langle 0.3(0.4), 0.5(0.6)\rangle$	$\langle 0.6(0.5), 0.8(0.5)\rangle$	$\langle 0.8(0.5), 0.9(0.5)\rangle$	$\langle 0.4(0.5), 0.5(0.5)\rangle$
N_2	$\langle 0.6(1)\rangle$	$\langle 0.9(1)\rangle$	$\langle 0.7(1)\rangle$	$\langle 0.8(1)\rangle$
N_3	$\langle 0.1(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.5(1)\rangle$	$\langle 0.4(1)\rangle$
N_4	$\langle 0.2(0.5), 0.4(0.5)\rangle$	$\langle 0.4(0.5), 0.5(0.5)\rangle$	$\langle 0.2(0.5), 0.3(0.5)\rangle$	$\langle 0.1(0.5), 0.3(0.5)\rangle$

Subject to: $\beta_1 + \beta_2 + \cdots + \beta_{12} = 1$ and $\beta_1, \beta_2, \ldots, \beta_{12} \geqslant 0$.

Suppose that the partial weights information for the criteria are:

$$\begin{aligned}\Xi = \{&0.03 \leqslant \beta_1 \leqslant 0.07, 0.05 \leqslant \beta_2 \leqslant 0.07, 0.15 \leqslant \beta_3 \leqslant 0.25, 0.05 \leqslant \beta_4 \leqslant 0.08,\\ &0.05 \leqslant \beta_5 \leqslant 0.08, 0.08 \leqslant \beta_6 \leqslant 0.15, 0.10 \leqslant \beta_7 \leqslant 0.15, 0.20 \leqslant \beta_8 \leqslant 0.25,\\ &0.08 \leqslant \beta_9 \leqslant 0.15, 0.15 \leqslant \beta_{10} \leqslant 0.25, 0.03 \leqslant \beta_{11} \leqslant 0.08, 0.02 \leqslant \beta_{12} \leqslant 0.08\}.\end{aligned}$$

This leads to the following solution:
$\beta_1 = 0.03$, $\beta_2 = 0.05$, $\beta_3 = 0.15$, $\beta_4 = 0.05$, $\beta_5 = 0.05$, $\beta_6 = 0.08$, $\beta_7 = 0.10$, $\beta_8 = 0.20$, $\beta_9 = 0.09$, $\beta_{10} = 0.15$, $\beta_{11} = 0.03$, $\beta_{12} = 0.02$ with $\mathrm{Max}Z = 94.7805$.

Table 6
Converted fuzzy matrix (for $\theta = 2$).

MOV	F_1	F_2	F_3	F_4	F_5	F_6
N_1	0.7423	0.7677	0.5530	0.697	0.4158	0.6348
N_2	0.7281	0.8631	0.8262	0.7622	0.5658	0.7051
N_3	0.5865	0.5409	0.3492	0.5888	0.2799	0.4260
N_4	0.4809	0.4354	0.4469	0.4998	0.3956	0.3001
	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
N_1	0.6624	0.6322	0.5200	0.7763	0.7962	0.5841
N_2	0.8717	0.7433	0.5658	0.8467	0.7822	0.7051
N_3	0.6416	0.4654	0.3598	0.3714	0.6061	0.3705
N_4	0.5590	0.3014	0.6269	0.4260	0.4735	0.2748

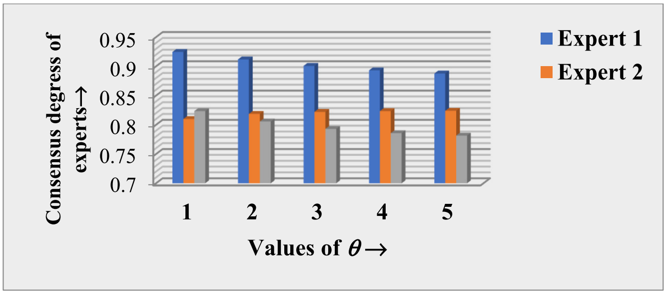

Fig. 1. Consensus degrees of experts for various values of θ.

Table 7
SVs of MOVs by RS approach.

MOV	ζ_r^+	ζ_r^-	ζ_r
N_1	0.448841	0.192678	0.256163
N_2	0.527224	0.235825	0.291398
N_3	0.288221	0.157240	0.130980
N_4	0.312025	0.116175	0.195850

SVs of the alternatives are calculated using RS approach, as described in Eqs. (22) and (23), and are presented in Table 7.

RPs ϑ_s ($s = 1, 2, \dots, 12$) are then computed using Eq. (24), as follows: $\vartheta_1 = 0.742344787$, $\vartheta_2 = 0.863080243$, $\vartheta_3 = 0.826210559$, $\vartheta_4 = 0.762183501$, $\vartheta_5 = 0.565789937$, $\vartheta_6 = 0.705054017$, $\vartheta_7 = 0.558990445$, $\vartheta_8 = 0.301380812$, $\vartheta_9 = 0.626947861$, $\vartheta_{10} = 0.846700199$, $\vartheta_{11} = 0.796187962$, $\vartheta_{12} = 0.705054017$.

Next, the weighted distances are estimated using Eq. (25) and presented in Table 8.

Finally, using Eq. (26) of RP model, the maximum distances of the alternatives are calculated, as given below:

$$\eta_1 = 0.06616, \quad \eta_2 = 0.08837, \quad \eta_3 = 0.07154, \quad \eta_4 = 0.06310.$$

Table 8
Distance from each alternative to the RPs.

Criteria	N_1	N_2	N_3	N_4
F_1	0	0.000426	0.004675	0.007843
F_2	0.004768	0	0.016107	0.021383
F_3	0.040975	0	0.071546	0.056903
F_4	0.003258	0	0.008671	0.013119
F_5	0.007500	0	0.014292	0.008509
F_6	0.005623	0	0.022323	0.032399
F_7	0.010341	0.0312736	0.008264	0
F_8	0.066161	0.088376	0.032801	0
F_9	0.009625	0.005504	0.024046	0
F_{10}	0.010562	0	0.071296	0.063105
F_{11}	0	0.000421	0.005701	0.009680
F_{12}	0.002419	0	0.006692	0.008605

Table 9
Outcomes by FMF model.

MOV	α_r^+	α_r^-	τ_r
N_1	0.723450	0.875551	0.826279
N_2	0.813743	0.929537	0.875428
N_3	0.528133	0.820902	0.643356
N_4	0.559902	0.742272	0.754308

Table 10
Final priority values of the MOV alternatives.

	RS model		RP model		FMF model		$B(N_r)$
MOV	$\tilde{\zeta}_r$	Rank	$\tilde{\eta}_r$	Rank	$\tilde{\tau}_r$	Rank	
N_1	0.5643	2	0.4534	2	0.5298	2	0.2376
N_2	0.642	1	0.6057	4	0.5614	1	0.2390
N_3	0.2886	4	0.4904	3	0.4125	4	−0.077
N_4	0.4315	3	0.4325	1	0.4837	3	0.1398

Next, Eqs. (27)–(28) of FMF model are used to compute the SVs of alternatives, as given in Table 9.

The final priority values of the alternatives are derived using Eq. (29), as presented in Table 10.

Hence, the priority order of the vendors is: $N_2 \succ N_1 \succ N_4 \succ N_3$ where the sign "$\succ$" signifies "superior to". Therefore, the most suitable vendor is N_2 (Bertrandt).

6. Discussions

The entire discussion is classified into two parts: (A) sensitivity analysis of criteria weights, (B) comparison of the proposed approach with the extant methods, and (C) managerial implications.

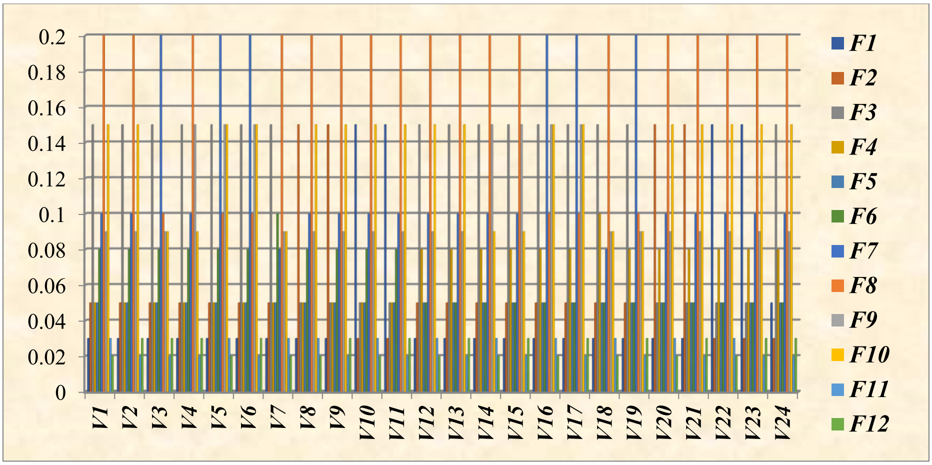

Fig. 2. Various criteria weight sets.

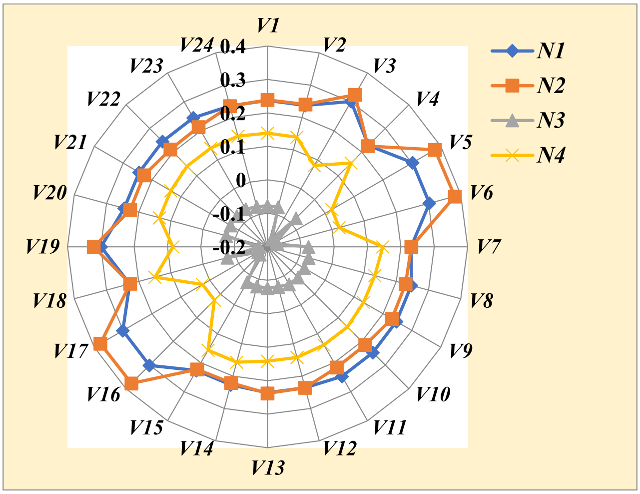

Fig. 3. Priority values of alternatives for various sets of criteria weights.

6.1. *Sensitivity Analysis of Criteria Weights*

In this section, sensitivity analysis is conducted to understand the effect of criteria weights on the ranking order. This is achieved using 24 different weight sets (CWS1, CWS2, CWS3, . . . , CWS24), as depicted in Fig. 2, formed by considering twenty-four arbitrary combinations of the criteria weights. Especially, this is valuable in achieving a broader scope of criteria weights for taking a look at the performance of the created model. The final priority scores of alternatives are shown in Fig. 3. The positioning places of alternatives along with the Spearman's rank correlation coefficient (SRCC) values (Saha *et al.*, 2023) have been calculated for those 24 weight sets and depicted in Fig. 4. The average

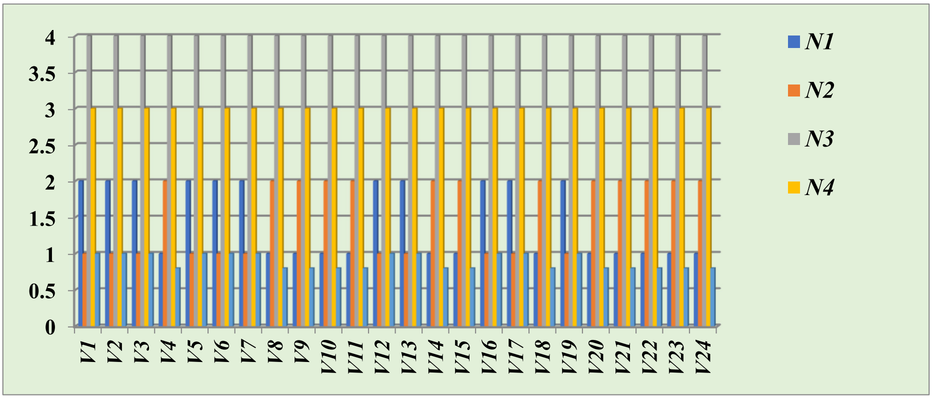

Fig. 4. Ranking positions and SRCC values.

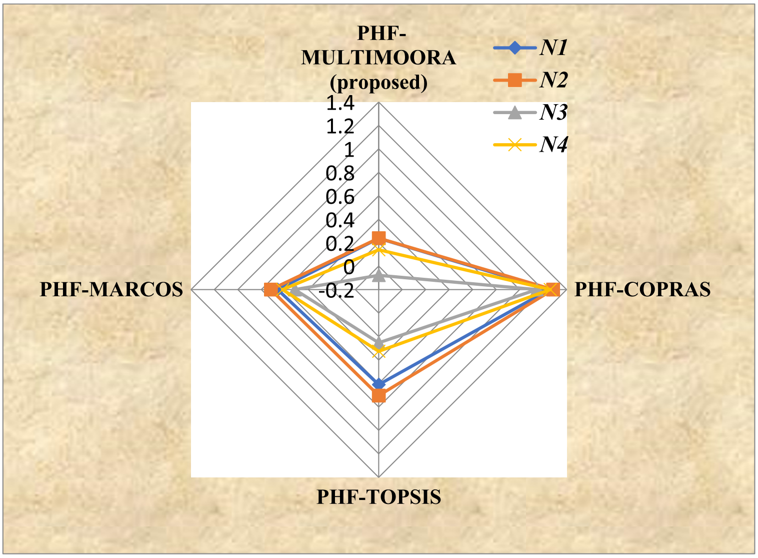

Fig. 5. Comparative analysis.

SRCC value is calculated as '0.8916', indicating a strong correlation (Saha *et al.*, 2023) between the ranking positions. Therefore, the priority order of the alternatives obtained using the proposed framework is reliable.

6.2. *Comparative Study*

This section aims to compare the proposed model with existing ones, specifically PHF-COPRAS (Krishankumar *et al.*, 2022), PHF-MARCOS (Liu *et al.*, 2023b), and PHF-TOPSIS (Qi, 2023). These tools have been applied to the case study considered by us. Outcomes are presented in Fig. 5. According to Fig. 5, the priority order derived by each technique is $N_2 \succ N_1 \succ N_4 \succ N_3$ which matches perfectly the results from our suggested strategy.

The main benefits of the proposed framework are:

1. Existing methods, especially those relying on t-norm and t-conorm operators, often fail to handle bias or disagreement among experts effectively. Aczel-Alsina AOs integrated in the proposed framework include an adjustable parameter, allowing them to better account for expert biases and preferences, which leads to a more robust consensus among the group. This makes the proposed method more effective in situations where expert opinions are uncertain or divergent.
2. The existing methodologies (Krishankumar *et al.*, 2022; Liu *et al.*, 2023b; Qi, 2023) might result in information loss when determining criteria weights due to the absence of optimization models. Consequently, the accuracy of the inputs cannot be thoroughly evaluated by current methods (Krishankumar *et al.*, 2022; Liu *et al.*, 2023b; Qi, 2023). In contrast, the proposed framework uses a cross-entropy based model to determine the criteria weights, which enhances the accuracy and relevance of the weights assigned to each criterion. This model helps to reduce uncertainty in the weight assignment process, which is often a challenge in other methods that rely on fixed or subjective weight distributions.
3. The proposed framework integrates a consensus-building procedure for decision-makers, whereas existing PHFS methods such as PHF-COPRAS (Krishankumar *et al.*, 2022), PHF-MARCOS (Liu *et al.*, 2023a), and PHF-TOPSIS (Qi, 2023) lack the capability to adjust the consensus level among experts.
4. PHF-MULTIMOORA is more efficient compared to PHF-COPRAS (Krishankumar *et al.*, 2022), PHF-MARCOS (Liu *et al.*, 2023b), and PHF-TOPSIS (Qi, 2023) as "ratio system", "reference point" and "full multiplicative form" are included in that. Moreover, PHF-MULTIMOORA approach is simple, highly robust, and has less computation time in comparison to PHF-COPRAS (Krishankumar *et al.*, 2022), PHF-MARCOS Liu *et al.* (2023b), and PHF-TOPSIS (Qi, 2023).

6.3. *Adaptability and Versatility of Proposed Framework Across Industries*

The integration of PHF sets allows the framework to handle uncertainty and vagueness in expert opinions, making it suitable for a variety of decision-making contexts across industries. PHF sets enable the incorporation of multiple conflicting and uncertain evaluations, which is crucial in sectors like healthcare, where decisions often rely on subjective judgments about treatments, resource allocation, and medical technologies. Aczel-Alsina aggregation operations, with their adjustable parameters, provide the flexibility to modify the aggregation process based on the specific requirements of different industries, allowing for fine-tuning of the consensus mechanism to match the decision-making environment. For example, in the context of sustainable supplier selection, the framework can be employed to aggregate diverse criteria such as environmental impact, cost-effectiveness, and compliance with sustainability standards. By adjusting the parameters of the aggregation operations, different weightings can be applied to these criteria depending on the sustainability goals of the industry. Similarly, in financial risk assessment, MULTIMOORA method, in combination with PHF aggregation, can be used to rank investment portfolios based on criteria like return potential, risk exposure, and liquidity, offering a comprehensive evaluation framework under uncertainty. The adopted cross-entropy-based criteria weighting model ensures that the weight distribution reflects the relative importance of each criterion, which is particularly useful in industries like transportation, where fuel efficiency, safety, and cost criteria may vary in significance based on the region or operational conditions. The versatility of the framework is further enhanced by its multi-objective optimization approach, which can be applied to a wide range of problems like urban infrastructure development or public policy formulation. These technical aspects make the framework highly adaptable, capable of addressing the specific needs of different industries while maintaining its reliability and accuracy in aggregating expert judgments.

7. Conclusions

The shortfalls in existing methods for managing uncertainty in decision-making are addressed using PHF sets to systematically handle uncertainty. Traditional aggregation operations for PHF sets often lack adaptability, which has been improved by introducing PHF Aczel-Alsina weighted averaging and geometric operations, providing greater flexibility in aggregating uncertain information. A consensus-building approach is proposed to identify the best alternative within a PHF environment, effectively mitigating the influence of biased expert perspectives. The proposed framework also incorporates a cross-entropy based model to determine criteria weights, ensuring that the weighting process accurately reflects the real-world importance of each criterion. MULTIMOORA method is then applied to establish the priority orders of alternatives, offering a comprehensive approach to decision-making. This combination of techniques improves both the reliability and objectivity of the decision process. A detailed case study on MOV selection demonstrates the practical applicability of the framework, showing how it can be used to make informed and balanced decisions in real-world scenarios. Sensitivity analysis and comparative studies further validate the effectiveness of the proposed framework in managing the uncertainties in PHF-based decision-making. These analyses confirm that the framework performs well in adjusting to different levels of uncertainty, thus ensuring robust and consistent results.

The framework may be extended in future research by incorporating additional criteria. New factors can be added or existing ones modified depending on specific industry or business requirements to better reflect the evolving needs of decision-making environments. The integration of dynamic or time-varying uncertainty models within PHF sets can also be explored to handle uncertainties that evolve over time. The proposed framework can be further extended to other MCGDM problems across various sectors like supply chain management, energy planning, or healthcare, where uncertainty and hesitation play significant roles in decision outcomes. The adaptability of PHF Aczel-Alsina operations in these diverse contexts should be investigated in future studies. The framework can also be expanded to include machine learning or artificial intelligence-based techniques for dynamic consensus-building, enabling the system to learn from previous decision outcomes and improve the aggregation and consensus processes over time, refining the decision-making process in environments with large amounts of data and complex decision criteria.

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