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Two-Stage EDAS Decision Approach with Probabilistic Hesitant Fuzzy Information

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Abstract. This paper develops a two-stage decision approach with probabilistic hesitant fuzzy data. Research challenges in earlier models are: (i) the calculation of occurrence probability; (ii) imputation of missing elements; (iii) consideration of attitude and hesitation of experts during weight calculation; (iv) capturing of interdependencies among experts during aggregation; and (v) ranking of alternatives with resemblance to human cognition. Driven by these challenges, a new group decision-making model is proposed with integrate methods for data curation and decision-making. The usefulness and superiority of the model is realized via an illustrative example of a logistic service provider selection.

Key words: case-based approach, EDAS, entropy measure, group decision-making, Regret theory, Maclaurin symmetric mean.

1. Introduction

Hesitant fuzzy set is a popular type of traditional fuzzy set that allows multiple membership grades for a particular entity (Torra, 2010). Motivated by the flexibility of HFS,

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numerous scholars adopt HFS for group decision-making (GDM) (Rodríguez *et al.*, 2014). However, Xu and Zhou (2016) rightly pointed out that HFS does not consider the occurrence probability of each element and thus, probabilistic hesitant fuzzy set (PHFS) is put forward to overcome the issue. PHFS is an intuitive generalization to HFS that allows multiple membership grades and associates the occurrence likelihood of each element, which provides potential information for rational GDM. Driven by the flexibility of PHFS, many researchers have proposed decision models with PHFS and readers can refer to the next section for clarity.

1.1. *Research Challenges/Gaps and Motivation*

From the literature review presented in the next section alongwith the summarized view of characteristics of the extant PHFS models, following research challenges can be identified:

- Direct elicitation of probability alongwith membership grade is difficult from experts' point of view.
- Due to unavoidable hesitation in the decision-making process, missing elements are possible. Extant decision models with PHFS do not consider missing elements, and methodical imputation of the same is ignored.
- Extant models with PHFS do not consider expert weight calculation, which causes human intervention and subjective bias in decision-making.
- Further, attitudes of experts are ignored during the weight calculation of criteria in state-of-the-art PHFS-based decision models. This is vital information in criteria weight estimation as the initial opinion of each criterion is obtained from the experts.
- Experts possess some interdependencies, which are not adequately captured during aggregation of preferences. Besides, the operators do not use experts' weights that are calculated methodically.
- Finally, ranking of alternatives must consider the nature of criteria and produce results close to human driven decision-making or cognition.

Based on these identified challenges, following research questions emanate:

- How to determine the confidence associated with more than one membership grade?
- How to address the issue of missing or unavailable preferences in the decision matrices?
- How to reduce subjectivity, bias, and human intervention in the weight assessment process?
- How to capture the interdependencies among experts during preference aggregation?
- How to rank alternatives by considering decision process close to human style decision-making?

The challenges identified by the authors are supported by the summarized view of characteristics of extant PHFS models provided in the next section. Besides, these challenges bring out the crucial research questions, which infer that there is an urge for a novel integrated framework under PHFS context for promoting rational decision-making. Motivated by the research challenges/questions pointed out above, in this article, authors put forward integrated decision approaches with PHFS data to rationally make decisions with less human intervention and subjectivity. To achieve the objective, following research contributions are put forward.

1.2. *Novel Contributions of the Research*

The presented challenges motivated the authors to make the following research contributions:

- A mathematical model is proposed by using a distance measure to compute the occurrence probability of each factor.
- A case-based approach is proposed to impute the missing elements under PHFS context.
- Weights of factors and experts are decided rationally by proposing an attitude-based entropy measure and regret/rejoice factor, respectively. In general, the literature survey reveals that the attitudes of decision-makers are not taken during criteria weight estimation into consideration, and the weights of experts are directly obtained. For instance, the results of Kao (2010) and Koksalmis and Kabak (2019) indicate the need for a rational criteria significance determination method and a rational expert weight calculation method, respectively.
- Moreover, Maclaurin symmetric mean (MSM) is extended to PHFS for the aggregation of preferences with weights of experts acquired methodically from the regret/rejoice factor.
- Lastly, the EDAS technique is extended to PHFS context for rational alternatives' ranking. EDAS takes into account the nature of factors and yields results close to human-like decisions with resemblance to human cognition.

Before proceeding further, it is important to discuss the reason behind proposing such approaches as contributions in this article. As discussed above, determination of occurrence probability (confidence) for each membership grade is not easy and hence, a mathematical model is formulated to determine the confidence values methodically, which not only reduces the overhead, but also mitigates bias and subjectivity. Further, missing entries and non-availability of preferences are natural in practical decision problems owing to the pressure and hesitation. To address the issue, a case based approach is developed, which not only imputes missing values, but also holds the properties of PHFS during data imputation. Further, weight values of decision-makers and criteria are methodically decided by considering hesitation and attitudes of experts for rational determination. Also, interdependencies among experts are captured during preference aggregation alongwith consideration of methodically determined weights with generalized function that can aggregate preferences and represent other functions as special cases based on parameter values. Finally, a ranking approach is presented with the intent of considering criteria type and providing ranks close to human cognition.

Thus, Section 2 offers the essential idea required for framing the theoretical ground. Section 3 ensures the core contributions of the current work. Section 4 demonstrates the applicability of the decision model developed. A comparative investigation is carried out in Section 5 to address the pros and cons of the model. Finally, concluding remarks with future research directions are described in the last section.

2. Literature Review

Li and Wang (2017) introduced a new family of correlation measures under PHFS and used them for commodity selection. Gao *et al.* (2017) prepared a dynamic reference point approach with PHFS for rapid decision-making. Ding *et al.* (2017) established a new mathematical model under PHFS context with partial weight information for evaluating VR projects. Zhou and Xu (2017a) prepared a novel value at risk model with PHFS for tail ended decision-making and employed the model in stock evaluation in China. Hao *et al.* (2017) introduced a new version of PHFS known as the probabilistic dual hesitant fuzzy set (PDHFS) and presented aggregation operators for Arctic geopolitical risk assessment. Li and Wang (2018a) prepared a new prioritized aggregation operator under PHFS for faculty selection and investment option selection. Jiang and Ma (2018) proposed weighted arithmetic/geometric operators for evaluating a public company. Zhou and Xu (2017c) developed new methods for the probability calculation of probabilistic hesitant fuzzy preference relations (PHFPRs) and an integrated decision model for research candidate selection. Zhou and Xu (2017b) prepared a GDM model by analysing the group consistency of PHFPRs, extending the idea for stock evaluation to the growth enterprise market.

Zhang *et al.* (2017) proposed novel operational laws and integration concepts with a detailed discussion on the core properties under PHFS context and utilized them for safety evaluation in the automotive industry. Tian *et al.* (2018) prepared a prospect theory-based consensus model for assessment of sequential investment in venture capitals. Li and Wang (2017) extended outranking methods by using the possibility degree under a PHFS environment for the rational selection of research candidates. Bashir *et al.* (2018) presented a decision model with PHFPRs by determining/repairing consistency and employed the same for commodity assessment for investment. Song *et al.* (2018) put forward a novel comparison formula for the possibility degree under PHFS and presented a ranking method for hospital selection in China. Zhang *et al.* (2018) extended the TODIM method under PHFS for venture capital project evaluation. Li and Wang (2018b) proposed a new mathematical model with PHFPRs by addressing missing values in these relations and solved the new energy project selection. Su *et al.* (2019) introduced entropy measures for PHFS and proposed a decision model to evaluate the Belt & Road case study. Garg and Kaur (2020) put forward fresh correlation transactions under PDHFS and performed the same for personnel evaluation. Liang *et al.* (2020) developed a combined approach with score function and prospect theory under PHFS for evaluation of cars. Li *et al.* (2019a) extended the best-worst method with dominance degree under PHFS context and used the same for apt selection of an investment company. He and Xu (2019) extended the reference ideal solution concept with distance measures to evaluate water conservation projects using PHFS information. Wu *et al.* (2019) extended the grey model with PHFS and introduced a novel distance-based TOPSIS method for emergency decision-making. Gong and Chen (2019) developed a new objective programming model with a variant of PDHFS and applied the model for venture capital evaluation.

Song *et al.* (2019) introduced correlation transactions under PHFS and performed the same for cluster analysis. Li *et al.* (2020a) made a novel extension to the ORESTE method with Euclidean distance and used the same for research topic selection. Guo *et al.* (2020)

developed a combined decision support tool with Choquet integral and TODIM as per PHFS for the appropriate selection of a CO_2 storage site. Farhadinia and Herrera-Viedma (2020) ameliorated the PHFS concept and used the same approach for defining new operations and aggregation operators, which were used for safety evaluation in the automotive industry. Garg and Kaur (2020) modified the Maclaurin symmetric mean operator for PDHFS for quantifying gesture information of patients with brain hemorrhage. Farhadinia *et al.* (2020) introduced correlation and distance measures under PHFS for strategy evaluation. Li *et al.* (2020b) developed a group consensus reaching approach with PHFS for apt selection of candidates. Jin *et al.* (2020) prepared a consistency check/repair model along with data envelopment approach for logistic assessment. Farhadinia and Xu (2021) proposed new comparison schemes with multiplication and exponential formulae under PHFS and demonstrated their applicability in apt hospital selection in China. Şahin and Altun (2020) extended the MACBAC approach to a neutrosophic variant of PHFS and selected suitable investment company. Liu *et al.* (2020) suggested a combined framework with regret theory and put forward new mathematical models for water filling and maximum entropy concepts under PHFS context to assess venture capitalists.

Liao *et al.* (2022a, 2022b) extended ranking approaches such as TODIM and EDAS with cumulative prospect theory and entropy scheme under PHF environment for rational decisions. Furthermore, Ning *et al.* (2022) presented CODAS with new distance/emtropy measure for assessing credit risk of enterprises under the variant of PHFS, Jin *et al.* (2022) integrated PHFS with rough sets and developed fuzziness based weight approach and PROMETHEE II for decision-making, Xu *et al.* (2022) combined fuzzy entropy, hesitancy entropy, and PHFSs, Garg *et al.* (2022) extended WASPAS to PHFS field and used it for transport application, Divsalar *et al.* (2022) put forward Choquet integral with TODIM approach for PHFS and utilized the same for the decision process, Liu *et al.* (2022b) extended cumulative residual entropy under PHFS area, Hu and Pang (2022) discussed the nexus between similarity and entropy along with the required properties and features in PHFS environment, Liu *et al.* (2022a) proposed a BWM-ITARA based methodology under PHFS environment, Liu and Guo (2022), and Wang *et al.* (2022) developed distance measures along with the essential properties and theoretical foundation for decision-making. Moreover, Chen *et al.* (2022) developed a GRA-TOPSIS approach under qROF-PHFSs for effectiveness analysis, Zhou *et al.* (2022) discussed quantitative element probability and qualitative element preference under PHFS and utilized the idea for MCDM, Fang (2023) revisited the concept of uncertainty measures for PHFS and discussed properties and theoretical aspects for better handling uncertainty, Qi (2023) introduced a PHF TOPSIS model for assessing the quality of public charging stations, and Xia *et al.* (2023) developed normal wiggly PHFS and presented fundamental aspects of the set and described its usefulness in war application. Very recently, Jiang *et al.* (2024), Ashraf *et al.* (2023), Jang *et al.* (2023), Chen *et al.* (2023), Yang *et al.* (2023), and Zhao *et al.* (2023) applied PHFS-based MCDM meothods to solve challenging real-life problems.

Research focus in the recent times is growing with respect to PHFS models for decision-making. Su *et al.* (2024) present new entropy functions under PHFS and used the functions for Belt and Road application. In Su *et al.* (2025), a Fogg behavioural aggregation with CRITIC method is proposed under PHFS for knowledge sharing partner

Table 1
Review on extant PHFS decision models.

Source	Methodical DM weights	Hesitation of experts	Imputation of missing values	Interdependencies during data fusion	Probability calculation	Attitude of experts
Wang and Li (2017)	No	No	No	No	No	No
Zhou and Xu (2017a)	No	No	No	No	No	No
Jiang and Ma (2018)	No	No	No	No	No	No
Zhou and Xu (2017c)	No	No	No	No	Yes	No
Tian *et al.* (2018)	No	No	No	No	No	No
Bashir *et al.* (2018)	No	No	No	No	No	No
Song *et al.* (2018)	No	No	No	No	No	No
Zhang *et al.* (2018)	No	No	No	No	No	No
Li and Wang (2018b)	No	No	Yes	No	No	No
Wu *et al.* (2019)	No	No	No	No	No	No
Li *et al.* (2019a)	No	Yes	No	No	No	No
Su *et al.* (2019)	No	Yes	No	No	No	No
Li *et al.* (2020a)	No	Yes	No	No	No	No
Guo *et al.* (2020)	No	No	No	Yes	No	No
Liang *et al.* (2020)	No	No	No	No	No	No
Jin *et al.* (2020)	No	Yes	No	No	No	No
Liu *et al.* (2020)	No	Yes	No	No	Yes	No
Liao *et al.* (2022a)	No	Yes	No	No	No	No
Liao *et al.* (2022b)	No	Yes	No	No	No	No
Liu and Guo (2022)	No	No	No	No	No	No
Chen *et al.* (2022)	No	No	No	No	No	No
Wang *et al.* (2022)	No	Yes	No	No	No	No
Divsalar *et al.* (2022)	No	Yes	No	Yes	No	No
Fang (2023)	No	Yes	No	No	No	No
Zhao *et al.* (2023)	No	Yes	No	Yes	No	No
Jiang *et al.* (2024)	No	Yes	No	Yes	Yes	No

selection. Bi-directional trust model is proposed with PHFS integrated with cloud model and new correlation function and social network concepts, which is tested in an emergency decision situation (Jiang *et al.*, 2024). Service quality of the public charging system is evaluated via PHF-TOPSIS model (Qi, 2023).

As a result, Table 1 presents the review of feature discussion of extant PHFS models and from the table, it is clear that there is an urge for the proposed framework to intuitively address the research gaps and promote rational decision-making. Table 1 supports the identified research gaps and as a result it can be observed that the challenges mentioned in Section 1 are in line to the briefing of extant PHFS models in Table 1. Motivated by the gaps, novel contributions are detailed in the next section, with the view of circumventing the gaps.

3. New Decision Model under PHFS

3.1. *Preliminaries*

This section offers the main idea of HFS and PHFS that forms the foundation for the research work.

Definition 1 (Torra, 2010). Z is a set that is fixed; an HFS on Z is a function h that produces a subset in the unit interval and is scientifically written as:

$$\overline{Z} = \{z, h_{\overline{Z}}(z) \mid z \in Z\}, \tag{1}$$

where $h_{\overline{Z}}(z)$ is a subset in the unit interval, which represents the membership grade of an element $z \in Z$.

Definition 2 (Xu and Zhou, 2016). Z is a set that is fixed; an PHFS on Z is a pair that is given by:

$$H_p = \{z, h_{H_p}(\gamma_i|p_i) \mid \mathrm{z} \in \mathrm{Z}\}, \tag{2}$$

where $h_{H_p}(\gamma_i|p_i)$ denotes the membership grade along with the occurrence probability of an element z on the set H_p, $0 \leqslant \gamma_i \leqslant 1$, $0 \leqslant p_i \leqslant 1$ and $\sum_i p_i \leqslant 1$.

Remark 1. The sum of occurrence probability among instances is less than or equal to 1 because of the idea of partial ignorance. Via normalization, the sum could be equal to 1. Let $h_{H_p}(\gamma_i|p_i) = h_i = (\gamma_i^l|p_i^l)$ be a probabilistic hesitant fuzzy element (PHFE) with $l = 1, 2, \ldots, \#h_i$ and the set of PHFEs builds a PHFS.

The PHFS has the following advantages that motivated authors to consider the set in this study: (i) it can accept multiple preferences for a particular instance owing to the hesitant fuzzy nature, which is not possible in other fuzzy variants; (ii) further, each element can be associated with an occurrence probability, which can be viewed as the confidence associated with that particular element. Such features are missing in other fuzzy forms. It must be noted that interval variant of PHFS also has merits and authors plan its utilization for the future.

Definition 3 (Xu and Zhou, 2016). Consider three PHFEs, h, h_1 and h_2, as highlighted before. Some operations are as follows:

$$h_1 \oplus h_2 = \bigcup_{a=1,2,\ldots\#h_1, b=1,2,\ldots,\#h_2} \{\gamma_a + \gamma_b - \gamma_a\gamma_b | p_a p_b\}, \tag{3}$$

$$h_1 \otimes h_2 = \bigcup_{a=1,2,\ldots\#h_1, b=1,2,\ldots,\#h_2} \{\gamma_a\gamma_b | p_a p_b\}, \tag{4}$$

$$h^c = \bigcup_{a=1,2,\ldots\#h} \{(1-\gamma_a) \mid p_a\}, \tag{5}$$

$$h^\lambda = \bigcup_{a=1,2,\ldots\#h} \{(\gamma_a)^\lambda \mid p_a\}\lambda \geqslant 0, \tag{6}$$

$$\lambda h = \bigcup_{a=1,2,\ldots\#h} \{1-(1-\gamma_a)^\lambda \mid p_a\}\lambda \geqslant 0. \tag{7}$$

The operations discussed in equations (3)–(7) form the theoretical base of PHFS.

3.2. *Imputing Missing Values*

This section discusses the systematic procedure for imputation of missing values in preference matrices. Due to implicit confusion, pressure, and hesitation, experts are unable to provide complete preference matrices. Pieces of literature on PHFS reveal that previous approaches do not consider missing values, and to effectively handle the problem, the case-based methodology is introduced in this section. General imputation methods like random fill, binning, and arithmetic residue (Han *et al.*, 2012) are not suitable for imputation of values in preference matrices due to the interdependencies among experts and criteria along with the varying nature of criteria, which must be considered during imputation. Besides, the proposed case-based approach considers the personal choice of experts on each alternative, which is ignored by the general/common methods.

To circumvent the challenges, the case-based approach is put forward.

Case 1: Out of t preference matrices, an entry (i, j) is missing from a single preference matrix, which may be imputed in the following manner:

$$h_{ij} = \left(\prod_{kk=1}^{t^{av}} (\gamma_{ij}^{l})^{dw_{kk}} \,\middle|\, \prod_{kk=1}^{t^{av}} (p_{ij}^{l})^{dw_{kk}} \right), \tag{8}$$

where t^{av} are number of experts who provide their values to a particular (i, j) entry; and $dw_{kk} = \frac{1}{t^{av}}$ and $\sum_{kk} dw_{kk} = 1$.

Case 2: Out of t preference matrices, a particular entry (i, j) is missing in all t matrices, for which we adopt equation (9)) to impute the values:

$$h_{ij} = \left(\prod_{ii=1}^{m^{av}} (\gamma_{ij}^{l})^{dw_{ii}} \,\middle|\, \prod_{ii=1}^{m^{av}} (p_{ij}^{l})^{dw_{ii}} \right), \tag{9}$$

where m^{av} is the number of alternatives that have values; and dw_{ii} is the weight of the iith alternative that is in the unit interval and $\sum_{ii} dw_{ii} = 1$.

Each expert provides his/her personal choice for each alternative in the form of PHFE. Except for the alternative that has the missing value, PHFEs of all other alternatives are considered. They are converted to a single value using $\sum_{l}(\gamma_{ij}^{l}.p_{ij}^{l})$, which are then normalized to obtain the weights in the unit interval and $\sum_{l}(\gamma_{ij}^{l}.p_{ij}^{l}) = 1$.

Case 3: In a preference matrix, a particular entry (i, j) is available, and others are missing. It can be imputed by repeating the available entry row-wise if the entire row is missing and/or column-wise if the entire column is missing.

Case 4: In all t preference matrices, a particular column is missing and can be imputed in the following manner. First, the characteristic of the criterion (column) is clarified. Provided that it is a non-cost type, then the mean of other non-cost type criteria (columns) in that row is calculated. If the entire row is missing, it is a special case of Case 2, and hence, it can be imputed by using equation (9).

3.3. *Probability Calculation Method*

This section offers a new probability calculation approach that utilizes the available information for formulating a mathematical model that could be solved to obtain the occurrence probability for each element. Previous studies on PHFS have shown that occurrence probabilities are directly given by the experts, causing difficulties and overheads. The study by Zhou and Xu (2017c) provides a method for the probability calculation in preference relations, which motivated the authors to develop an approach for the occurrence probability calculation in decision matrices.

Occurrence probability, as discussed earlier, is seen as a confidence value and determination of the probability supports the usage of PHFS, unlike the direct assignment of probability value by an expert during rating. Elicitation of elements is comfortable, while assigning a confidence value is ordeal for experts and hence, we propose a procedure to calculate. A mathematical model is developed with the help of HFEs and can be solved via the optimization toolbox of MATLAB® to extract occurrence probability values for each element.

Model 1:

$$MinZ = \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{k}\left(\left|\gamma_{ij}^{k} - \gamma_{ij}^{k+}\right| - \left|\gamma_{ij}^{k} - \gamma_{ij}^{k-}\right|\right).$$

Subject to:

$$p_{ij}^{k} \in [0,1], \quad \gamma_{ij}^{k} \in [0,1], \quad \gamma_{ij}^{k+} \in [0,1], \quad \gamma_{ij}^{k-} \in [0,1],$$
$$\sum_{k} p_{ij}^{k} \leqslant 1, \quad \forall k = 1, 2, \dots, h.$$

In Model 1, $\gamma_{ij}^{k+} = \max_{j \in benefit}(\gamma_{ij}^{k})$ or $\min_{j \in cost}(\gamma_{ij}^{k})$ and $\gamma_{ij}^{k-} = \max_{j \in cost}(\gamma_{ij}^{k})$ or $\min_{j \in benefit}(\gamma_{ij}^{k})$. When Model 1 is solved using an optimization toolbox, the occurrence probability for each HFE is determined, and the PHFS property is held (as per Definition 2). We formulate the optimization problem using distance from ideal and anti-ideal solutions. It is a minimization problem where distance of datapoint to ideal solution must be minimum and distance of datapoint to anti-ideal solution must be maximum. Some typical advantages of the proposed mathematical model are: (i) it is easy and straightforward; and (ii) takes the nature of criteria into account for determining the occurrence probability.

3.4. *Regret/Rejoice Factor for Experts' Weights*

This sub-section focuses on a new framework for deciding the weights of experts. As mentioned earlier, Koksalmis and Kabak (2019) pointed out the need for a methodical computation of decision makers' weights to mitigate inaccuracies. Driven by such claim, in this section, a regret/rejoice factor-based weight calculation approach is introduced. The steps are depicted below.

Step 1: t decision matrices are obtained with order $m \times n$, where m demonstrates the number of alternatives, and n demonstrates the number of factors. PHFS information is adopted as the input.

Step 2: Transform the t matrices into single-valued matrices by applying equation (10):

$$sh_{ij} = \sum_{l=1}^{\#h} \left(\gamma_{ij}^{l} . p_{ij}^{l}\right), \tag{10}$$

where $\#h$ represents no of instances in an IVPHFE.

Step 3: Calculate the utility value from the regret theory formulation using equation (11), which is adapted from Gong *et al.* (2019):

$$UT_l = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(vf(sh_{ij}) + RT\left(vf(sh_{ij})\right) - vf^{pos}(sh_{ij})\right), \tag{11}$$

where $vf(.)$ is the von Neuman Morgestern utility function with a power operation denoted as $(.)^a$, and a ranges from 0 to 1. $RT(\tau)$ is the regret theory function that is given by $1 - e^{-(\zeta\tau)}$, where $\zeta \geqslant 0$ is the risk aversion factor. $vf^{pos}(sh_{ij})$ is maximum von Neuman utility for benefit criteria/attribute type and $vf^{pos}(sh_{ij})$ is minimum von Neuman utility for cost criteria/attribute type. From Eq. (11), the regret/rejoice of selection over no selection is considered.

Step 4: Standardize the utility values from previous step to obtain the weight values of the experts, which subsequently forms a $1 \times t$ vector. Equation (12) is employed to get the weight vector:

$$dw_l = \frac{UT_l}{\sum_l UT_l}, \tag{12}$$

where dw_l is the weight of the lth expert.

3.5. *Attitude-Based Entropy Measure*

Herein, we introduce a novel framework for the criteria weight computation by presenting an attitude-based entropy measure. Generally, weights of criteria are decided either with partially known information or fully unknown information. If the weights of criteria are determined with partially known, decision-makers share their opinions of each criterion as inequality constraints, and in the latter context, such information is not existing. It should be noted that the former context adds an overhead to the expert and may not be possible in several practical situations.

To mitigate this issue, the latter context was further developed with popular approaches, such as AHP (Peng and Liu, 2017), BWM (Mi and Liao, 2019), and information/divergence measures (Mishra *et al.*, 2020) for ratio analysis, among others. However, these methods do not consider the attitude of experts and are unable to capture the hesitation during preference elucidation. Driven by these challenges, herein, a new attitude-based entropy measure is introduced for factors weight determination under PHFS context. Steps for weight determination are given below:

Step 1: Form an expert opinion matrix ($t \times n$) with PHFEs as preference information for each criterion, where t represents the number of experts, and n represents the number of factors.

Step 2: Equation (7) must be applied to all PHFEs of the matrix to obtain a weighted opinion of experts. Attitude values of decision-makers that are gathered as weights from the previous sub-section are used as the scalar value, and a weighted matrix is obtained based on equation (7).

Step 3: Apply a deviation measure to all elements of the matrix (from Step 2) using equation (13):

$$D_{lj} = |sh_{lj} - \overline{sh_j}|, \tag{13}$$

where $sh_{lj} = \sum_{k=1}^{\#h}(\gamma_{lj}^k . p_{lj}^k)$; and $\overline{sh_j}$ is the mean for the jth factor.

Step 4: The information entropy measure is applied to form a vector of entropy values that is of order $1 \times n$. Equation (14) is applied for determining the entropy values:

$$EY_j = \sum_l \left(-\frac{1}{n}\left(\frac{D_{lj}}{\sum_l D_{lj}} ln\left(\frac{D_{lj}}{\sum_l D_{lj}}\right)\right)\right), \tag{14}$$

where $\frac{D_{lj}}{\sum_l D_{lj}}$ is the normalized deviation, EY_j is the entropy value of the jth criterion or factor.

Step 5: Equation (15) is utilized to normalize the entropy values to form a weight vector of order $1 \times n$, which provides the weights of criteria:

$$cw_j = \frac{EY_j}{\sum_j EY_j}, \tag{15}$$

where cw_j is the weight of the jth factor.

It should be mentioned that the weight of each criterion is in the unit interval.

3.6. *Maclaurin Operator for Aggregating PHFEs*

This section focuses on presenting a new aggregation operator under PHFS context for preference aggregation. Existing aggregation operators under PHFS (Jiang and Ma, 2018), Li and Wang (2018a) do not capture interdependencies among experts effectively, and to circumvent this issue, the Maclaurin operator is extended to PHFS for aggregation.

The Maclaurin symmetric mean (MSM) (Maclaurin, 1729) operator is a generalized operator that is capable of representing other arithmetic/geometric operators by parameter adjustments. The operator has the ability to capture interdependencies among decision-makers by adopting weight values and risk appetite values of experts during the formulation. Bearing in mind the literature survey above, it is clear that experts' weights are not calculated systematically, which inspired inaccuracies, as argued by Koksalmis and Kabak (2019).

To overcome the issue, in the present sub-section, a weight MSM operator is introduced to aggregate PHFEs by acquiring weights methodically from Section 3.3. We define the operator below and discuss key properties.

Definition 4. PHFEs are aggregated through the PH-WMSM operator ($\beta^n \rightarrow \beta$) and is given by,

$$\begin{aligned}
&\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1,h_2,\ldots,h_t)\\
&\quad=\left(\left(1-\left(\prod_{l=1}^{t}\left(1-\prod_{ll=1}^{v}\gamma_{ij}^{\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}},\\
&\qquad\left(\left(1-\left(\prod_{l=1}^{t}\left(1-\prod_{ll=1}^{v}p_{ij}^{\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}},
\end{aligned}\tag{16}$$

where $v = \lceil \frac{t}{2} \rceil$, $\lambda_1, \lambda_2, \ldots, \lambda_v$ are risk appetite elements that could receive possible values from the set $\{1, 2, \ldots, t\}$; and dw_l is the weight of the lth expert.

Property 1: Idempotent

h_i for all $i = 1, 2, \ldots, t$ be a set of PHFEs. If $h_i = h$ for all $i = 1, 2, \ldots, t$, then $\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) = h$.

Proof.

$$\begin{aligned}
&\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1,h_2,\ldots,h_t)\\
&\quad=\left(\left(1-\left(\prod_{l=1}^{t}\left(1-\prod_{ll=1}^{v}\gamma_{ij}^{\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}},\\
&\qquad\left(\left(1-\left(\prod_{l=1}^{t}\left(1-\prod_{ll=1}^{v}p_{ij}^{\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}.
\end{aligned}$$

By expanding the terms:

$$\begin{aligned}
&\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1,h_2,\ldots,h_t)\\
&\quad=\left(\left(1-\left(\left(1-\prod_{ll=1}^{v}\gamma_{ij}^{\lambda_{ll}}\right)^{dw_1+dw_2+\cdots+dw_t}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}},\\
&\qquad\left(\left(1-\left(\left(1-\prod_{ll=1}^{v}p_{ij}^{\lambda_{ll}}\right)^{dw_1+dw_2+\cdots+dw_t}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}.
\end{aligned}$$

Since the sum of experts' weights equal unity, we get:

$$= \left(\left(1 - \left(\left(1 - \prod_{ll=1}^{v} \gamma_{ij}^{\lambda_{ll}}\right)\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}, \left(\left(1 - \left(\left(1 - \prod_{ll=1}^{v} p_{ij}^{\lambda_{ll}}\right)\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}$$
$$= (\gamma_{ij} \mid p_{ij}) = h \qquad \square$$

Property 2: Bounded

For all $\lambda_1, \lambda_2, \ldots, \lambda_v$; $h^- \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) \leqslant h^+$, where $h^- = \min\left(\sum_k \gamma_{ij}^k . p_{ij}^k\right)$; and $h^+ = \max\left(\sum_k \gamma_{ij}^k . p_{ij}^k\right)$ for all $i = 1, 2, \ldots, t$.

Proof. Suppose that h be the aggregated PHFE. Based on monotonicity and idempotent properties. Then,

$$\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\left(h^-, h^-, \ldots, h^-\right) \leqslant h \quad \text{and}$$
$$\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\left(h^+, h^+, \ldots, h^+\right) \geqslant h.$$

Through combining the inequalities, one gets

$$\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\left(h^-, h^-, \ldots, h^-\right) \leqslant h \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\left(h^+, h^+, \ldots, h^+\right).$$

Thus, $h^- \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) \leqslant h^+$. $\square$

Property 3: Commutative

For any permutation h_i^{**},

$$\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) = \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\left(h_1^{**}, h_2^{**}, \ldots, h_t^{**}\right).$$

Proof.

$$\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\left(h_1^{**}, h_2^{**}, \ldots, h_v^{**}\right)$$
$$= \left(\left(1 - \left(\prod_{l=1}^{t}\left(1 - \prod_{ll=1}^{v} \gamma_{ij}^{**\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}},$$
$$\left(\left(1 - \left(\prod_{l=1}^{t}\left(1 - \prod_{ll=1}^{v} p_{ij}^{**\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}$$
$$= \left(\left(1 - \left(\prod_{l=1}^{t}\left(1 - \prod_{ll=1}^{v} \gamma_{ij}^{\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}},$$
$$\left(\left(1 - \left(\prod_{l=1}^{t}\left(1 - \prod_{ll=1}^{v} p_{ij}^{\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}$$
$$= \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t). \qquad \square$$

Property 4: Monotonicity

If there exist a set of PHFEs h_i^* for all $i = 1, 2, \ldots, t$, such that $h_i \leqslant h_i^*$ for all $i = 1, 2, \ldots, e$, then

$$\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) \\ \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}\big(h_1^*, h_2^*, \ldots, h_t^*\big).$$

Proof. Let

$$\gamma_{ij}^* = \left(\left(1 - \left(\prod_{l=1}^{t}\left(1 - \prod_{ll=1}^{v} \gamma_{ij}^{*\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}$$

and

$$p_{ij}^* = \left(\left(1 - \left(\prod_{l=1}^{t}\left(1 - \prod_{ll=1}^{v} p_{ij}^{*\lambda_{ll}}\right)^{dw_l}\right)\right)\right)^{\frac{1}{\sum_{ll}\lambda_{ll}}}.$$

Also, $\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) = h$ and $\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1^*, h_2^*, \ldots, h_t^*) = h^*$. Based on the score and deviation functions adapted from Xu and Zhou (2016), we can infer that $s(h_i) \leqslant s(h_i^*)$, and if $s(h_i) = s(h_i^*)$, then $d(h_i) \geqslant d(h_i^*)$ for all $i = 1, 2, \ldots, e$ as $h_i \leqslant h_i^*$. Now, $s(h) \leqslant s(h^*)$ and when $s(h) = s(h^*)$, $d(h) \geqslant d(h^*)$. Thereby, $h \leqslant h^*$, and so $\text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1^*, h_2^*, \ldots, h_t^*)$. □

Theorem 1. *Aggregation of PHFEs utilizing the PH-WMSM operator yields a PHFE.*

Proof. It must be noted that Definition 2 provides the characteristics of PHFS. From the definition, it is clear that the element γ_{ij}^k and the associated occurrence probability p_{ij}^k are in the unit interval with the sum of occurrence probability less than or equal to 1. From the bounded property, it is evident that the result of the aggregation operator is within the lower and upper bounds, that is, $h^- \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) \leqslant h^+$. By extending the idea, we get $h^- = 0 \leqslant \text{PH-WMSM}^{(v,\lambda_1,\lambda_2,\ldots,\lambda_v)}(h_1, h_2, \ldots, h_t) \leqslant h^+ = 1$, which indicates that $0 \leqslant (\gamma_{ij}^k\{p_{ij}^k) \leqslant 1$. Further, the input for the PH-WMSM operator are PHFEs that satisfy the inequality $\sum_k p_{ij}^k \leqslant 1$ for all $k = 1, 2, \ldots, \#h_i$. By using the bounded property, it is inferred that the occurrence probability of the aggregated value also follows the inequality $\sum_k p_{ij}^k \leqslant 1$, and hence, the aggregated value from PH-WMSM operator is a PHFE. □

3.7. *Ranking Method with PHFEs*

This sub-section introduces a fresh extension to the EDAS technique under the PHFS environment for rational ranking of alternatives. EDAS (Keshavarz Ghorabaee *et al.*, 2015) is an attractive ranking method that follows the distance measure between possible choices and average values. Because of the simplicity and flexibility of the technique, numerous researchers adopt EDAS for MCDM. For instance, Keshavarz Ghorabaee *et al.* (2016) gave a fuzzy extension to EDAS for supplier selection. Kahraman *et al.* (2017) extended EDAS to IFS and applied the disposal technique selection method for solid wastes. Feng *et al.* (2018) proposed a new model by extending EDAS to hesitant fuzzy linguistic information and focused on the project selection problem. Peng and Liu (2017) developed a new model via a neutrosophic soft EDAS methodology using a novel similarity measure to select an optimal investment for software projects. Ecer (2018) developed an integrated framework with fuzzy AHP and EDAS for third party logistic provider selection. Karaşan and Kahraman (2018) made a suitable selection of sustainable goals for the United Nations by proposing the EDAS approach with interval-valued neutrosophic fuzzy information. Mi and Liao (2019) proposed an integrated approach under HFS by using BWM and EDAS methods for choosing insurance products in commercial sectors. Zhang *et al.* (2019) developed a framework with picture 2-tuple linguistic information by extending the geometric operator and EDAS to evaluate green suppliers. Li *et al.* (2019b) suggested a decision model with a power operator and EDAS for linguistic neutrosophic information using the property management company selection model. Recently, Mishra *et al.* (2020) employed the IFS-based EDAS approach for HCWT assessment. Liang *et al.* (2020) also presented an IFS-based EDAS for the proper selection of energy-efficient projects for green building construction. Aldalou and Perçin (2020) prepared a fuzzy decision model by integrating an entropy measure and EDAS for assessing financial indicators of food and drink firms in Istanbul. Yanmaz *et al.* (2020) introduced a new approach with interval-valued Pythagorean EDAS for car selection. Further, Ecer *et al.* performed the intuitionistic fuzzy EDAS model for evaluating cryptocurrencies, Lei *et al.* (2022) introduced the PDHL-EDAS approach, Mishra *et al.* (2022) developed a Fermatean fuzzy EDAS methodology, Batool *et al.* (2022) offered a Pythagorean probabilistic hesitant fuzzy EDAS approach, Hashemkhani Zolfani *et al.* (2021) utilized the EDAS method for international market selection, and Menekse and Camgoz Akdag (2023) introduced interval-valued spherical fuzzy EDAS. Torkayesh *et al.* (2023) prepared a detailed review on EDAS approach and discussed the applicability and possible extesions in the decision-making field. Motivated by the flexibility of EDAS, in this sub-section, a new extension of EDAS with PHFEs and the steps for ranking are presented as follows.

Step 1: Aggregated matrix of order $m \times n$ is gathered from the previous sub-section as input for the PHFS-based EDAS method. Also, the criteria weight vector of order $1 \times n$ obtained from the previous sub-section is used as the input.

Step 2: Determine the weighted aggregated matrix by using equation (17) that is of order $m \times n$:

$$hw_{ij} = \left(1 - \left(1 - \gamma_{ij}^{k}\right)^{cw_j} \,\middle|\, 1 - \left(1 - p_{ij}^{k}\right)^{cw_j}\right), \tag{17}$$

where cw_j is the weight of the jth criterion obtained from the previous section.

Step 3: Determine the average value of preferences for each alternative over different criteria using equation (18):

$$\overline{hw_i} = \left(\frac{\sum_{j=1}^{n} \gamma_{ij}^{k(a)}}{n} \middle| \frac{\sum_{j=1}^{n} p_{ij}^{k(a)}}{n} \right), \tag{18}$$

where $\overline{hw_i}$ is the average PHFE for the ith alternative.

It must be noted that hw_{ij} is a PHFE in the form $(\gamma_{ij}^{k(a)}|p_{ij}^{k(a)})$.

Step 4: Determine the positive and negative distances from the average (PDA, NDA) for each alternative that yields a vector of order $1 \times m$:

$$PDA_i = d(hw_{ij}, \overline{hw_i}), \tag{19}$$

$$NDA_i = d(\overline{hw_i}, hw_{ij}), \tag{20}$$

where $d(a1, a2)$ is the distance between two PHFEs, $a1$ and $a2$.

Although equations (19) and (20) seem similar, they vary in terms of the nature of criteria. That is, in equation (19), PDA_i is determined by taking the complement of the preferences in the cost type, and in equation (20), NDA_i is calculated by taking the complement of the preferences in the benefit type. By this way, the ranking method effectively considers the nature of criteria in their formulation. Both equations (19) and (20) form a vector of order $1 \times m$ that is used in the next step for ranking alternatives. Equation (5) presents the complement operation.

$$d(a1, a2) = \sqrt{\sum_{k=1}^{\#h} \left((\gamma_{ij}^{k}.p_{ij}^{k})_{a1} - (\gamma_{ij}^{k}.p_{ij}^{k})_{a2} \right)^2}.$$

Here, $a1$ and $a2$ are any two PHFEs.

Step 5: Estimate the rank values of each alternative with Eq. (21), in which values from Step 4 are utilized to form the rank values:

$$RV_i = \left(\frac{PDA_i - \min_i(PDA_i)}{\max_i(PDA_i) - \min_i(PDA_i)} \right) + \left(\frac{NDA_i - \min_i(NDA_i)}{\max_i(NDA_i) - \min_i(NDA_i)} \right), \tag{21}$$

where RV_i is the rank value of the ith alternative; $\min_i(*)$ is the minimum operator; and $\max_i(*)$ is the maximum operator.

Arrange the rank values in descending order to find the ranking/prioritization order of the alternatives.

Before presenting the case study and to clearly recognise the practicality of the introduced methodology, it is essential to explain the working mechanism of the proposed decision model with PHFS information. Fig. 1 provides the working model that begins with collecting HFEs from experts for each alternative over each criterion. The team of experts are decided by the top officials. These experts finalize the alternatives and criteria for the process of decision-making. The model begins with the imputation of HFEs, then occurrence probability values are calculated. Sections 3.1 and 3.2 describe achieving the task. Weights of experts and criteria are determined methodically in order to mitigate

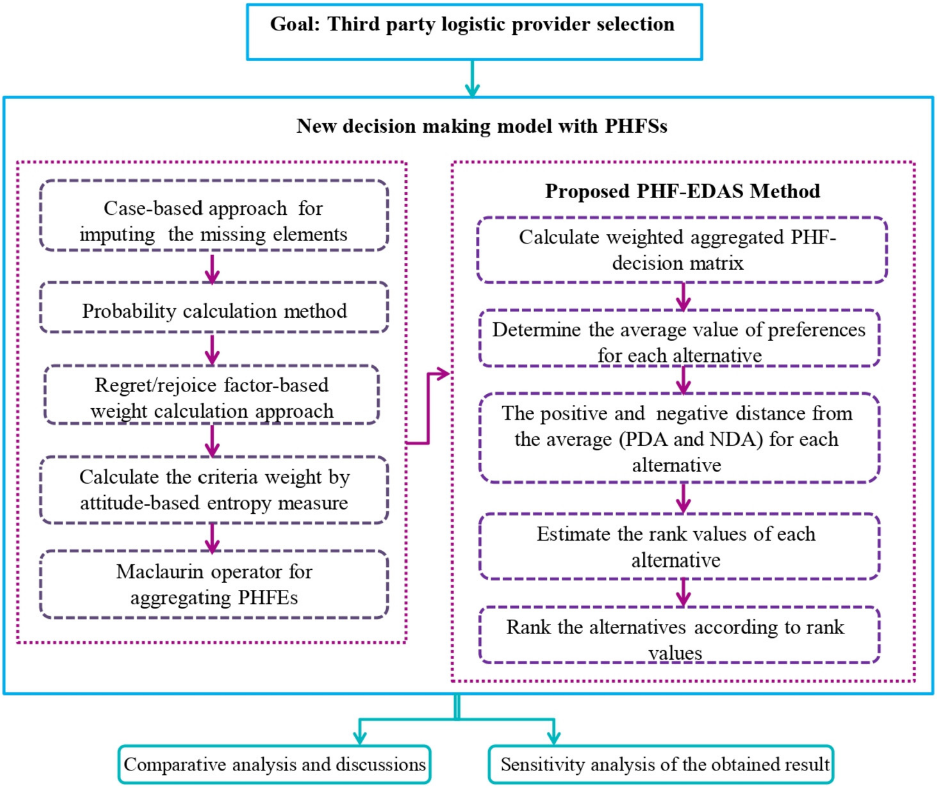

Fig. 1. Workflow of proposed PHFS-based decision model for MCDM.

human bias and inaccuracies in decision-making. Sections 3.3 and 3.4 are used for this purpose. Based on the decision matrices and experts' weights, aggregation is performed to obtain a single aggregated matrix (refer to Section 3.5). Finally, the alternatives are ranked using the aggregated matrix and criteria weight vector. The detailed procedure for ranking is provided in Section 3.6.

The two stage model is desirable because extant decision models do not consider the preprocessing module in the decision process, which is addressed in this study. Imputing missing data and determination of probability is provided as preprocessing module that improves the input to the decision approach where weights of experts and criteria are determined along with data fusion and ranking. Many studies in decision-making considers data to be complete, which may not be practical in real life due to diverse reasons and as a result, a preprocessing procedure is required.

4. Case Study of Logistics Provider Assessment

This section provides a numerical example for demonstrating the applicability of the suggested methodology. A case study of logistic provider evaluation is presented for a food and beverage company in Chennai, India. The company X2Y (denoted as such to keep the company name anonymous) provides tasty snacks, including cakes, cookies, chats, and veg/cheese rolls, and beverages to their customers around Chennai. The company specializes in serving parties and official meetings. X2Y actively follows green standards and adheres to the ISO 14000 and 14001. Raw materials that are needed for the preparation of snacks and beverages are procured from the suppliers who have active green practices and rigid quality control measures. Common raw materials utilized by X2Y are milk with low-fat cream, butter, chocolate, dry fruits, cheese, and organically farmed vegetables. The company spends almost 63% of their money, time, and effort in choosing the best raw materials and suppliers to serve their customers the optimal products. Since taste and health are the primary focus of X2Y, the top officials allocate concrete quality check measures before the raw material is put to the production line.

The company prepares a detailed audit report every quarter, and the top officials identified that there is a significant portion of expenditure on the transportation of raw materials. Further, there is emphasis on sustainability in diverse business practices and since the company focuses on sustainable work, sustainable transport is also an area of focus and idea on urban sustainable transport is obtained from (Moslem, 2024). Idea related to factor selection for evaluating logistic providers can be obtained from (Ulutaş *et al.*, 2024). With the aim to cut costs and manage complex issues related to transportation, the company plans to outsource the transport facility from third-party logistic providers (TLPs). Officials of X2Y have identified many TLPs within the city and decided to utilize a decision model for rational selection of TLP. The officials constituted an expert panel with three experts/DMs, viz., Finance & Audit personnel et_1, Logistic manager et_2, and Senior HR personnel et_3. These experts are allocated 20 days to choose potential TLPs for the selection process. Ten TLPs were initially shortlisted based on phone calls, emails, and on-line presentations. Based on a Delphi approach, five TLPs were finalized for the selection process, which are rated based on seven criteria pertaining to economics, environmental, and social aspects. While 2 of these criteria are cost type criteria, there are five benefits type criteria. Experts adopted a literature review, brainstorming, and voting mechanism to finalize the seven criteria for rating TLPs.

The five TLPs were termed tlp_1, tlp_2, tlp_3, tlp_4, and tlp_5, which are rated based on seven criteria, viz., on-time delivery ct_1, service quality ct_2, adoption of green practices ct_3, customer relationship/harmony ct_4, pollution control strategy ct_5, total cost ct_6, and damage to raw materials during shipment ct_7. By adopting a comprehensive peer discussion with the five TLPs concerning the seven criteria, the experts rated the TLPs with PHFEs. Steps for ranking the TLPs are shown below.

Step 1: Create three preference matrices of order 5×7 by using HFEs. Five TLPs are rated based on seven criteria. Missing values are imputed rationally via the procedure introduced in Section 3.1. Probability values are calculated for the preference matrices by using Section 3.2

$-xx-$ in Table 2 depicts the missing values that are imputed in a rational manner by using the procedure presented in Section 3.2. Imputation is done by adopting the suitable case. Case 1 is applied to impute the entry (tlp_3, ct_2) of et_1 and is given by $\begin{pmatrix} 0.4975|p_{17}^{1} \\ 0.5745|p_{17}^{2} \end{pmatrix}$.

Table 2
Preference information from three experts – PHFEs.

TLPs	Criteria for evaluating TLPs						
	ct_1	ct_2	ct_3	ct_4	ct_5	ct_6	ct_7
				et_1			
tlp_1	$\begin{pmatrix}0.4\vert p_{11}^1\\0.5\vert p_{11}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{12}^1\\0.4\vert p_{12}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{13}^1\\0.55\vert p_{13}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{14}^1\\0.4\vert p_{14}^2\end{pmatrix}$	$-xx-$	$\begin{pmatrix}0.45\vert p_{16}^1\\0.35\vert p_{16}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{17}^1\\0.65\vert p_{17}^2\end{pmatrix}$
tlp_2	$\begin{pmatrix}0.4\vert p_{21}^1\\0.3\vert p_{21}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{22}^1\\0.5\vert p_{22}^2\end{pmatrix}$	$\begin{pmatrix}065\vert p_{23}^1\\0.45\vert p_{23}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{24}^1\\0.55\vert p_{24}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{25}^1\\0.6\vert p_{25}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{26}^1\\0.4\vert p_{26}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{17}^1\\0.7\vert p_{17}^2\end{pmatrix}$
tlp_3	$\begin{pmatrix}0.55\vert p_{31}^1\\0.6\vert p_{31}^2\end{pmatrix}$	$-xx-$	$\begin{pmatrix}0.5\vert p_{33}^1\\0.7\vert p_{33}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{34}^1\\0.65\vert p_{34}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{35}^1\\0.4\vert p_{35}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{36}^1\\0.35\vert p_{36}^2\end{pmatrix}$	$\begin{pmatrix}0.75\vert p_{17}^1\\0.55\vert p_{17}^2\end{pmatrix}$
tlp_4	$\begin{pmatrix}0.6\vert p_{41}^1\\0.7\vert p_{41}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{42}^1\\0.36\vert p_{42}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{43}^1\\0.7\vert p_{43}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{44}^1\\0.45\vert p_{44}^2\end{pmatrix}$	$\begin{pmatrix}0.4\vert p_{45}^1\\0.55\vert p_{45}^2\end{pmatrix}$	$\begin{pmatrix}0.4\vert p_{46}^1\\0.45\vert p_{46}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{17}^1\\0.6\vert p_{17}^2\end{pmatrix}$
tlp_5	$\begin{pmatrix}0.45\vert p_{51}^1\\0.55\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{52}^1\\0.65\vert p_{52}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{53}^1\\0.45\vert p_{53}^2\end{pmatrix}$	$\begin{pmatrix}0.4\vert p_{54}^1\\0.5\vert p_{54}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{55}^1\\0.4\vert p_{55}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{56}^1\\0.5\vert p_{56}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{17}^1\\0.7\vert p_{17}^2\end{pmatrix}$
				et_2			
tlp_1	$\begin{pmatrix}0.55\vert p_{11}^1\\0.6\vert p_{11}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{12}^1\\0.6\vert p_{12}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{13}^1\\0.5\vert p_{13}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{14}^1\\0.6\vert p_{14}^2\end{pmatrix}$	$-xx-$	$\begin{pmatrix}0.55\vert p_{51}^1\\0.5\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{51}^1\\0.7\vert p_{51}^2\end{pmatrix}$
tlp_2	$\begin{pmatrix}0.55\vert p_{21}^1\\0.45\vert p_{21}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{22}^1\\0.55\vert p_{22}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{23}^1\\0.45\vert p_{23}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{24}^1\\0.4\vert p_{24}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{51}^1\\0.6\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{51}^1\\0.6\vert p_{51}^2\end{pmatrix}$	$-xx-$
tlp_3	$\begin{pmatrix}0.5\vert p_{31}^1\\0.55\vert p_{31}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{32}^1\\0.55\vert p_{32}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{33}^1\\0.7\vert p_{33}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{34}^1\\0.5\vert p_{34}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{51}^1\\0.7\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{51}^1\\0.6\vert p_{51}^2\end{pmatrix}$	$-xx-$
tlp_4	$\begin{pmatrix}0.6\vert p_{41}^1\\0.7\vert p_{41}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{42}^1\\0.6\vert p_{42}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{43}^2\\0.35\vert p_{43}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{44}^1\\0.7\vert p_{44}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{51}^1\\0.65\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{51}^1\\0.55\vert p_{51}^2\end{pmatrix}$	$-xx-$
tlp_5	$\begin{pmatrix}0.7\vert p_{51}^1\\0.65\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{52}^1\\0.4\vert p_{52}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{53}^1\\0.5\vert p_{53}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{54}^1\\0.65\vert p_{54}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{51}^1\\0.5\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{51}^1\\0.6\vert p_{51}^2\end{pmatrix}$	$-xx-$
				et_3			
tlp_1	$\begin{pmatrix}0.6\vert p_{11}^1\\0.4\vert p_{11}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{12}^1\\0.65\vert p_{12}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{13}^1\\0.35\vert p_{13}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{14}^1\\0.55\vert p_{14}^2\end{pmatrix}$	$-xx-$	$\begin{pmatrix}0.55\vert p_{16}^1\\0.6\vert p_{16}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{17}^1\\0.5\vert p_{17}^2\end{pmatrix}$
tlp_2	$\begin{pmatrix}0.55\vert p_{21}^1\\0.45\vert p_{21}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{22}^1\\0.65\vert p_{22}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{23}^1\\0.45\vert p_{23}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{24}^1\\0.6\vert p_{24}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{25}^1\\0.6\vert p_{25}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{26}^1\\0.55\vert p_{26}^2\end{pmatrix}$	$\begin{pmatrix}0.35\vert p_{27}^1\\0.4\vert p_{27}^2\end{pmatrix}$
tlp_3	$\begin{pmatrix}0.7\vert p_{31}^1\\0.6\vert p_{31}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{32}^1\\0.6\vert p_{32}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{33}^1\\0.65\vert p_{33}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{34}^1\\0.5\vert p_{34}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{35}^1\\0.55\vert p_{35}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{36}^1\\0.7\vert p_{36}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{37}^1\\0.55\vert p_{37}^2\end{pmatrix}$
tlp_4	$\begin{pmatrix}0.7\vert p_{41}^1\\0.6\vert p_{41}^2\end{pmatrix}$	$\begin{pmatrix}0.45\vert p_{42}^1\\0.4\vert p_{42}^2\end{pmatrix}$	$\begin{pmatrix}0.7\vert p_{43}^1\\0.6\vert p_{43}^2\end{pmatrix}$	$\begin{pmatrix}0.55\vert p_{44}^1\\0.45\vert p_{44}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{45}^1\\0.7\vert p_{45}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{46}^1\\0.5\vert p_{46}^2\end{pmatrix}$	$\begin{pmatrix}0.65\vert p_{47}^1\\0.7\vert p_{47}^2\end{pmatrix}$
tlp_5	$\begin{pmatrix}0.55\vert p_{51}^1\\0.65\vert p_{51}^2\end{pmatrix}$	$\begin{pmatrix}0.4\vert p_{52}^1\\0.5\vert p_{52}^2\end{pmatrix}$	$\begin{pmatrix}0.75\vert p_{53}^1\\0.55\vert p_{53}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{54}^1\\0.55\vert p_{54}^2\end{pmatrix}$	$\begin{pmatrix}0.75\vert p_{55}^1\\0.45\vert p_{55}^2\end{pmatrix}$	$\begin{pmatrix}0.5\vert p_{56}^1\\0.7\vert p_{56}^2\end{pmatrix}$	$\begin{pmatrix}0.6\vert p_{57}^1\\0.7\vert p_{57}^2\end{pmatrix}$

Case 2 is applied to impute the entry (tlp_1, ct_5) of all experts, and the values are given by $\begin{pmatrix}0.4949|p_{17}^1\\0.4794|p_{17}^2\end{pmatrix}$, $\begin{pmatrix}0.522|p_{17}^1\\0.6078|p_{17}^2\end{pmatrix}$, and $\begin{pmatrix}0.6427|p_{17}^1\\0.5678|p_{17}^2\end{pmatrix}$, respectively. Case 3 is applied to impute values in ct_7 of et_2 and is given by $\begin{pmatrix} 0.5|p_{17}^{1} \\ 0.7|p_{17}^{2} \end{pmatrix}$.

$$et_1 = \begin{pmatrix} (0.2,0.3) & (0.65,0.35) & (0.2,0.1) & (0.6,0.3) & (0.7,0.1) & (0.75,0.25) & (0.5,0.15) \\ (0.3,0.45) & (0.4,0.2) & (0.5,0.45) & (0.4,0.25) & (0.3,0.15) & (0.5,0.2) & (0.6,0.35) \\ (0.5,0.45) & (0.4,0.15) & (0.5,0.1) & (0.6,0.22) & (0.7,0.18) & (0.4,0.27) & (0.5,0.45) \\ (0.3,0.2) & (0.8,0.15) & (0.7,0.3) & (0.75,0.25) & (0.25,0.2) & (0.65,0.22) & (0.35,0.32) \\ (0.45,0.24) & (0.7,0.3) & (0.65,0.2) & (0.32,0.15) & (0.22,0.25) & (0.28,0.32) & (0.76,0.15) \end{pmatrix},$$

$$et_2 = \begin{pmatrix} (0.3,0.3) & (0.1,0.35) & (0.8,0.1) & (0.1,0.3) & (0.7,0.1) & (0.75,0.25) & (0.1,0.15) \\ (0.3,0.45) & (0.4,0.2) & (0.1,0.45) & (0.75,0.25) & (0.85,0.15) & (0.8,0.2) & (0.1,0.35) \\ (0.1,0.45) & (0.4,0.15) & (0.9,0.1) & (0.1,0.22) & (0.1,0.18) & (0.4,0.27) & (0.5,0.45) \\ (0.3,0.2) & (0.8,0.15) & (0.1,0.3) & (0.75,0.25) & (0.25,0.2) & (0.1,0.22) & (0.35,0.32) \\ (0.76,0.24) & (0.1,0.3) & (0.65,0.2) & (0.85,0.15) & (0.75,0.25) & (0.28,0.32) & (0.1,0.15) \end{pmatrix},$$

$$et_3 = \begin{pmatrix} (0.3,0.3) & (0.65,0.35) & (0.2,0.1) & (0.6,0.3) & (0.7,0.1) & (0.75,0.25) & (0.1,0.15) \\ (0.3,0.45) & (0.4,0.2) & (0.1,0.45) & (0.75,0.25) & (0.85,0.15) & (0.5,0.2) & (0.6,0.35) \\ (0.5,0.45) & (0.85,0.15) & (0.9,0.1) & (0.6,0.22) & (0.7,0.18) & (0.73,0.27) & (0.55,0.45) \\ (0.8,0.2) & (0.1,0.15) & (0.7,0.3) & (0.1,0.25) & (0.25,0.2) & (0.1,0.22) & (0.35,0.32) \\ (0.45,0.24) & (0.1,0.3) & (0.65,0.2) & (0.32,0.15) & (0.75,0.25) & (0.679,0.32) & (0.1,0.15) \end{pmatrix}$$

Table 2 demonstrates the HFEs from each decision-maker that form the preference matrices. By using the procedure proposed in Section 3.2 and by solving Model 1 utilizing the optimization toolbox of MATLAB®, the occurrence probability of each HFE in each matrix is obtained and is shown above. Clearly, the probability values satisfy the constraints defined for PHFS.

Step 2: Form another matrix of order 3×7 for criteria weight calculation through PHFEs. Section 3.4 is considered to obtain the criteria weight vector of order 1×7.

$$EY_{lj} = \begin{pmatrix} 0.3155 & 0.1623 & 0.3215 & 0.3196 & 0.3083 & 0.3207 & 0.3343 \\ 0.1618 & 0.3297 & 0.3155 & 0.3107 & 0.3155 & 0.0711 & 0.3155 \\ 0.3296 & 0.3155 & 0.0802 & 0.3155 & 0.3214 & 0.3155 & 0.2614 \end{pmatrix}$$

Table 3 depicts the weight calculation matrix for criteria with PHFEs. Equation (14) is applied to calculate the entropy values for each criterion. Further, the divergence vectors were determined to be 0.193, 0.192, 0.283, 0.054, 0.055, 0.293, and 0.089, respec-

Table 3
Weight calculation matrix for criteria – Experts vs. Criteria.

TLPs	Criteria for evaluating TLPs						
	ct_1	ct_2	ct_3	ct_4	ct_5	ct_6	ct_7
et_1	$\begin{pmatrix} 0.5\|0.5 \\ 0.4\|0.3 \end{pmatrix}$	$\begin{pmatrix} 0.5\|0.35 \\ 0.65\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.45\|0.35 \\ 0.5\|0.5 \end{pmatrix}$	$\begin{pmatrix} 0.5\|0.45 \\ 0.6\|0.5 \end{pmatrix}$	$\begin{pmatrix} 0.6\|0.5 \\ 0.4\|0.3 \end{pmatrix}$	$\begin{pmatrix} 0.55\|0.45 \\ 0.5\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.6\|0.45 \\ 0.7\|0.5 \end{pmatrix}$
et_2	$\begin{pmatrix} 0.6\|0.4 \\ 0.5\|0.45 \end{pmatrix}$	$\begin{pmatrix} 0.45\|0.4 \\ 0.55\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.7\|0.45 \\ 0.6\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.7\|0.5 \\ 0.5\|0.35 \end{pmatrix}$	$\begin{pmatrix} 0.55\|0.4 \\ 0.45\|0.45 \end{pmatrix}$	$\begin{pmatrix} 0.5\|0.5 \\ 0.7\|0.3 \end{pmatrix}$	$\begin{pmatrix} 0.65\|0.4 \\ 0.55\|0.5 \end{pmatrix}$
et_3	$\begin{pmatrix} 0.65\|0.45 \\ 0.7\|0.35 \end{pmatrix}$	$\begin{pmatrix} 0.5\|0.5 \\ 0.65\|0.35 \end{pmatrix}$	$\begin{pmatrix} 0.65\|0.5 \\ 0.45\|0.35 \end{pmatrix}$	$\begin{pmatrix} 0.45\|0.45 \\ 0.5\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.65\|0.35 \\ 0.5\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.65\|0.45 \\ 0.6\|0.4 \end{pmatrix}$	$\begin{pmatrix} 0.5\|0.3 \\ 0.7\|0.55 \end{pmatrix}$

tively. Criteria weights were calculated as 0.1666, 0.1662, 0.2440, 0.0468, 0.0473, 0.2525, 0.0766, respectively, using equation (15).

Step 3: Experts' weights are calculated with the help of preference matrices from Step 1 and Section 3.5 to achieve a vector of order 1×3. Later, the three matrices are aggregated using Section 3.6 to obtain a matrix of order 5×7.

$$RT_{ij}^{et_1} = \begin{pmatrix} 0.2132 & 0.2972 & 0.1787 & 0.2928 & 0.2549 & 0.2782 & 0.2630 \\ 0.2231 & 0.2529 & 0.3045 & 0.2455 & 0.2172 & 0.2576 & 0.3222 \\ 0.3087 & 0.2343 & 0.2464 & 0.2909 & 0.2958 & 0.2376 & 0.3260 \\ 0.2464 & 0.2963 & 0.3276 & 0.3127 & 0.2048 & 0.2589 & 0.2122 \\ 0.2015 & 0.2768 & 0.2775 & 0.1638 & 0.1528 & 0.1782 & 0.2762 \end{pmatrix},$$

$$RT_{ij}^{et_2} = \begin{pmatrix} 0.2545 & 0.2250 & 0.3144 & 0.2152 & 0.2785 & 0.3069 & 0.1787 \\ 0.2615 & 0.2682 & 0.2241 & 0.3183 & 0.3116 & 0.2928 & 0.2378 \\ 0.2387 & 0.2260 & 0.3328 & 0.1787 & 0.1892 & 0.2773 & 0.3133 \\ 0.2464 & 0.3233 & 0.1760 & 0.3343 & 0.2182 & 0.1916 & 0.2708 \\ 0.3395 & 0.1863 & 0.2953 & 0.3227 & 0.3156 & 0.2676 & 0.1787 \end{pmatrix},$$

$$RT_{ij}^{et_3} = \begin{pmatrix} 0.2396 & 0.3178 & 0.1734 & 0.3178 & 0.2995 & 0.3127 & 0.1590 \\ 0.2615 & 0.2622 & 0.2222 & 0.3369 & 0.3389 & 0.2809 & 0.2561 \\ 0.3254 & 0.3116 & 0.3409 & 0.2978 & 0.3108 & 0.3108 & 0.2966 \\ 0.3379 & 0.1496 & 0.3359 & 0.1850 & 0.2269 & 0.1863 & 0.2854 \\ 0.2721 & 0.1958 & 0.3206 & 0.2182 & 0.3369 & 0.3130 & 0.1838 \end{pmatrix}.$$

By applying equation (11), utility values can be obtained for each expert. The regret/rejoice factors shown above are fed as values into equation (11) for obtaining the weights of the experts. These factors are obtained from PHFEs (Table 2 for HFE and occurrence probability matrices). The utility values were determined as 6.881, 6.333, and 7.806 based on equation (11) and Table 4. Equation (12) yields the experts' weights as 0.3274, 0.3013, and 0.3713, respectively, which are considered as attitude values for the criteria weight calculation.

Table 5 shows the aggregated PHFEs that are obtained by aggregating preferences from Table 2 and experts' weight vector. The operator proposed in Section 3.5 is used for aggregation.

Step 4: The weight vector and aggregated matrix from Steps 2 and 3 are used to rank alternatives considering the procedure introduced in Section 3.7.

Table 6 provides the EDAS parameter values, which are vectors of order 1×5. Equations (19)–(21) are used for obtaining the values in Table 6. By applying equations (19)–(20), two vectors of 1×5 are obtained denoted by PDA_i and NDA_i, respectively. By using the distance measure presented in the procedure, the distance values are determined that are finally normalized to obtain the rank values of logistic providers. This is done via equation (21), which also yields a 1×5 vector and from the rank values the ranking order is determined as $tlp_1 \succ tlp_4 \succ tlp_2 \succ tlp_5 \succ tlp_3$.

Table 4
Utility values for each expert.

TLPs	Criteria for evaluating TLPs						
	ct_1	ct_2	ct_3	ct_4	ct_5	ct_6	ct_7
				UT_1			
tlp_1	−0.0454	0.2972	−0.2213	0.2356	0.1421	0.5377	0.3963
tlp_2	−0.0101	0.1307	0.2371	0.0560	0.0057	0.4610	0.6230
tlp_3	0.3087	0.0630	0.0183	0.2287	0.2958	0.3879	0.6380
tlp_4	0.0738	0.2938	0.3276	0.3127	−0.0384	0.4656	0.2122
tlp_5	−0.0868	0.2195	0.1337	−0.2284	−0.2170	0.1782	0.4458
				UT_2			
tlp_1	0.0124	−0.0461	0.2601	−0.1139	0.1730	0.6146	0.1787
tlp_2	0.0382	0.1117	−0.0778	0.2709	0.2999	0.5601	0.3872
tlp_3	−0.0453	−0.0427	0.3328	−0.2416	−0.1495	0.5015	0.6712
tlp_4	−0.0174	0.3233	−0.2460	0.3343	−0.0476	0.1916	0.5088
tlp_5	0.3395	−0.1824	0.1860	0.2882	0.3155	0.4651	0.1787
				UT_3			
tlp_1	−0.0373	0.3178	−0.2795	0.2611	0.1836	0.6504	0.1590
tlp_2	0.0431	0.1057	−0.109	0.3369	0.3389	0.5281	0.5013
tlp_3	0.2882	0.2934	0.3409	0.1833	0.2274	0.6428	0.6537
tlp_4	0.3379	−0.2912	0.3207	−0.2273	−0.0859	0.1863	0.6109
tlp_5	0.0827	−0.1331	0.2599	−0.1109	0.3308	0.6517	0.2436

Table 5
Aggregated information – PHFEs using proposed operator.

TLPs	Criteria for evaluating TLPs			
	ct_1	ct_2	ct_3	ct_4
		et_{123}		
tlp_1	$\begin{pmatrix} 0.548 \mid 0.2826 \\ 0.5245 \mid 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.5374 \mid 0.6137 \\ 0.5935 \mid 0.35 \end{pmatrix}$	$\begin{pmatrix} 0.5807 \mid 0.6659 \\ 0.4942 \mid 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.5807 \mid 0.6659 \\ 0.4942 \mid 0.1 \end{pmatrix}$
tlp_2	$\begin{pmatrix} 0.5208 \mid 0.3 \\ 0.4240 \mid 0.45 \end{pmatrix}$	$\begin{pmatrix} 0.6397 \mid 0.4 \\ 0.5890 \mid 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5832 \mid 0.4160 \\ 0.45 \mid 0.45 \end{pmatrix}$	$\begin{pmatrix} 0.5807 \mid 0.6659 \\ 0.4942 \mid 0.1 \end{pmatrix}$
tlp_3	$\begin{pmatrix} 0.6231 \mid 0.4713 \\ 0.5872 \mid 0.45 \end{pmatrix}$	$\begin{pmatrix} 0.5108 \mid 0.7390 \\ 0.5784 \mid 0.15 \end{pmatrix}$	$\begin{pmatrix} 0.6454 \mid 0.8589 \\ 0.6838 \mid 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.5807 \mid 0.6659 \\ 0.4942 \mid 0.1 \end{pmatrix}$
tlp_4	$\begin{pmatrix} 0.6473 \mid 0.6888 \\ 0.6719 \mid 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5726 \mid 0.7474 \\ 0.5042 \mid 0.15 \end{pmatrix}$	$\begin{pmatrix} 0.6300 \mid 0.6616 \\ 0.6218 \mid 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.5807 \mid 0.6659 \\ 0.4942 \mid 0.1 \end{pmatrix}$
tlp_5	$\begin{pmatrix} 0.6058 \mid 0.6383 \\ 0.6256 \mid 0.24 \end{pmatrix}$	$\begin{pmatrix} 0.5284 \mid 0.5859 \\ 0.5646 \mid 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.6886 \mid 0.65 \\ 0.5105 \mid 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5807 \mid 0.6659 \\ 0.4942 \mid 0.1 \end{pmatrix}$
	ct_5	ct_6	ct_7	
		et_{123}		
tlp_1	$\begin{pmatrix} 0.5781 \mid 0.7 \\ 0.5624 \mid 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.5266 \mid 0.75 \\ 0.5323 \mid 0.25 \end{pmatrix}$	$\begin{pmatrix} 0.5067 \mid 0.4160 \\ 0.6372 \mid 0.15 \end{pmatrix}$	
tlp_2	$\begin{pmatrix} 0.6225 \mid 0.8049 \\ 0.6 \mid 0.15 \end{pmatrix}$	$\begin{pmatrix} 0.5847 \mid 0.6780 \\ 0.5434 \mid 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5250 \mid 0.5660 \\ 0.6526 \mid 0.35 \end{pmatrix}$	
tlp_3	$\begin{pmatrix} 0.6035 \mid 0.6616 \\ 0.6030 \mid 0.18 \end{pmatrix}$	$\begin{pmatrix} 0.5691 \mid 0.6280 \\ 0.6247 \mid 0.27 \end{pmatrix}$	$\begin{pmatrix} 0.6419 \mid 0.5214 \\ 0.6178 \mid 0.45 \end{pmatrix}$	
tlp_4	$\begin{pmatrix} 0.4613 \mid 0.25 \\ 0.6509 \mid 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.5658 \mid 0.5427 \\ 0.5064 \mid 0.22 \end{pmatrix}$	$\begin{pmatrix} 0.6219 \mid 0.35 \\ 0.6753 \mid 0.2993 \end{pmatrix}$	
tlp_5	$\begin{pmatrix} 0.6669 \mid 0.7053 \\ 0.4571 \mid 0.25 \end{pmatrix}$	$\begin{pmatrix} 0.6095 \mid 0.5800 \\ 0.6320 \mid 0.32 \end{pmatrix}$	$\begin{pmatrix} 0.5610 \mid 0.6392 \\ 0.7 \mid 0.15 \end{pmatrix}$	

Table 6
Parameter values of PHFS-based EDAS method.

TLPs	EDAS parameter values		
	PDA_i	NDA_i	RV_i
tlp_1	0.5274	1	1.5274
tlp_2	0.3364	0.8938	1.2303
tlp_3	0	0	0
tlp_4	1	0.3100	1.3100
tlp_5	0.0696	0.8748	0.9444

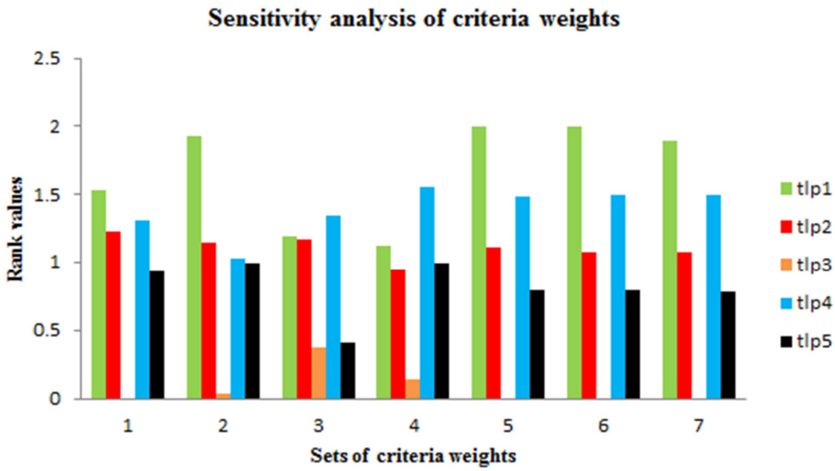

Fig. 2. Sensitivity analysis of criteria weights (Seven sets of criteria weights with single left shift operation).

Step 5: Finally, sensitivity check is conducted over criteria weights to decide the effect of weight alteration on rankings.

Given Fig. 2, it is inferred that there is close competition among TLPs, tlp_1, tlp_2, and tlp_4, with a final ranking of: $tlp_1 \succcurlyeq tlp_4 \succcurlyeq tlp_2 \succ tlp_5 \succ tlp_3$. It is clear that tlp_1 is the most preferred and tlp_3 is the least preferred from the set of five TLPs.

5. Comparative Study with Extant Approaches under PHFS

This section describes the comparative analysis of the proposed approach with the existing methodologies under PHFS context. Analytical factors were considered from both theoretical and numerical perspectives and were extracted from respective literature and intuition. In order to consider homogeneity in comparison, state-of-the-art models that utilize PHFEs are considered. Existing approaches considered for investigation are Jiang and Ma's approach (Jiang and Ma, 2018), Farhadinia *et al.*'s approach (Farhadinia *et al.*, 2020), Li and Wang's approach (Li and Wang, 2018a), Zhou & Xu's approach (Zhou and Xu, 2017a), Divsalar *et al.*'s approach (Divsalar *et al.*, 2022), and *Wang et al.*'s approach

Table 7
Investigation on different PHFS-based decision models.

Features	PHFS-based decision models						
	Proposed	Jiang and Ma (2018)	Farhadinia *et al.* (2020)	Li and Wang (2018a)	Zhou and Xu (2017c)	Divsalar *et al.* (2022)	Wang *et al.* (2022)
Data	PHFEs	PHFEs	PHFEs	PHFEs	PHFEs	PHFEs	PHFEs
Occurrence probability	Calculated – methodically	Not calculated	Not calculated	Not calculated	Not calculated	Not calculated	Not calculated
Missing values	Considered in the study	Not considered in the study					
Imputation	Done methodically	N/A	N/A	N/A	N/A	N/A	N/A
Attitude of of experts	Considered	Not considered	Not considered	Not considered	Not considered	Not considered	Not considered
Experts' weights	Calculated methodically – fully unknown information	Not calculated	N/A	Calculated methodi-cally – partial in-formation is needed	Not calculated	Not calculated	Not calculated
Experts' hesitation	Captured by the model	Not captured by these models				Not captured	Captured
Interdependency among experts	Captured during preference aggregation	Not captured by these models				Captured	Not captured
Nature of criteria	Considered during ranking	Not considered by these models					

Note: N/A is not applicable.

(Wang *et al.*, 2022). Table 7 investigates these models with the proposed model to clearly understand the superiority.

Some interesting innovations/superiorities of the proposed model are presented below.

- The occurrence probability regarding each HFE is calculated systematically by proposing a mathematical model, which is lacking in the existing models.
- Preference matrices with missing values are taken into consideration in the introduced framework, and unlike the state-of-the-art models, these are imputed rationally by proposing a case-based approach.
- Driven by the arguments of Kao (2010) and Koksalmis and Kabak (2019), weights of criteria and experts are determined systematically. During criteria weight calculations, the attitudes of experts are considered along with the hesitation of experts. Further, experts' weights are calculated by considering the regret/rejoice factor.
- Unlike existing models, the preferences are aggregated by capturing the interdependencies among decision-makers and also through methodical weights of experts.
- Methods proposed for the weight calculation are useful when the weight information is fully unknown. Further, existing models do not consider the attitudes of experts during criteria weight calculations.

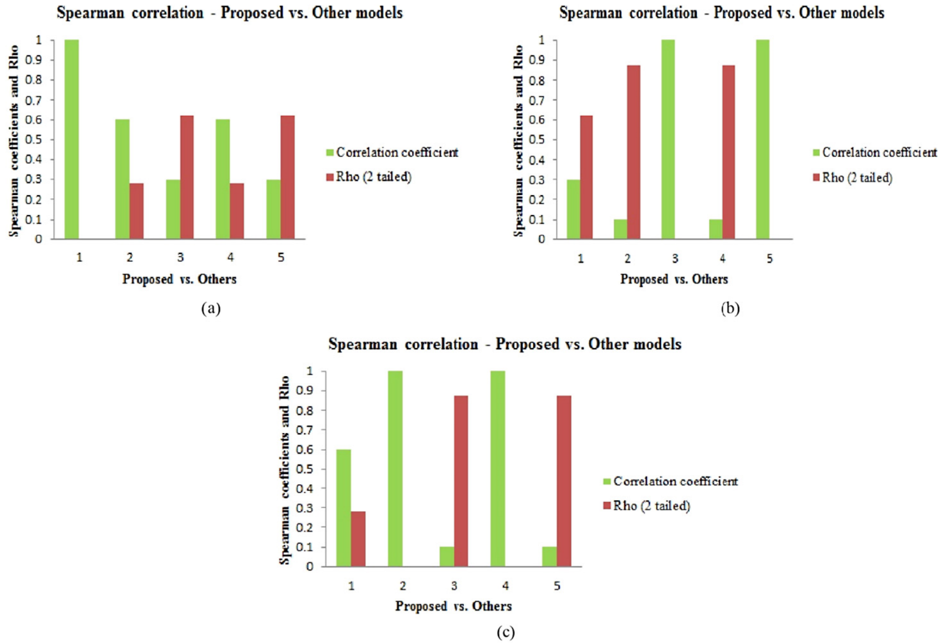

Fig. 3. Spearman correlation coefficients – Proposed model vs. Other models (left top); Models (Ecer, 2018) vs. Other models (right top); and Models (Hu and Pang, 2022; Jang *et al.*, 2023) vs. Others models (mid bottom).

- Finally, alternatives are ranked by properly considering the nature of criteria. Also, the positive and negative distances from the average are considered in the formulation to mimic real-time human ranking processes.

To further appreciate the superiorities of the proposed model, consistency of results from proposed method with the other models is considered. An aggregated matrix along with the criteria weight vector are fed as inputs to the PHFS-based decision models for determining the alternatives' rankings. The rankings produced by the proposed model follow the order: $tlp_1 \succ tlp_4 \succ tlp_2 \succ tlp_5 \succ tlp_3$. Comparatively, the ranking orders deduced by Jiang and Ma's model (Jiang and Ma, 2018) is $tlp_1 \succ tlp_2 \succ tlp_4 \succ tlp_5 \succ tlp_3$; by Farhadinia *et al.* (2020) is $tlp_4 \succ tlp_2 \succ tlp_1 \succ tlp_5 \succ tlp_3$; by Li and Wang (2018a) is $tlp_1 \succ tlp_2 \succ tlp_4 \succ tlp_5 \succ tlp_3$; and by Zhou and Xu (2017c) is $tlp_4 \succ tlp_2 \succ tlp_1 \succ tlp_5 \succ tlp_3$. By applying Spearman correlation on the proposed model versus the other models, we obtain coefficient values of $((1, 0); (0.6, 0.284); (0.3, 0.624); (0.6, 0.284); (0.3, 0.624))$. The first value represents the coefficient, and the second value represents the rho at 2-tailed. Fig. 3 displays the comparisons of the coefficient values along with rho between all models, from which it is obvious that the introduced framework is *fairly consistent* with the available methodologies as results indicate that there are some models that yield ranks that are different from the proposed model.

6. Conclusion

This work proposes a novel decision approach with PHFEs and unknown weight information. In the model, HFEs are first obtained from experts, then the occurrence probability of each HFE is calculated using the mathematical model. Because of hesitation or pressure, decision-makers may not be comfortable to provide all values, which could cause missing entries in the preference matrices that are rationally imputed from a case-based approach. Weights of decision-makers and factors are methodically computed to avoid biases and inaccuracies during decision-making process. Later, preferences are aggregated by properly capturing the interdependencies among experts, and ranking is achieved through the PHFS-based EDAS approach. From the sensitivity analysis (Fig. 2) of criteria weights, the close competition among alternatives is realized. Spearman correlation (Fig. 3) clearly presents the consistency of the introduced approach when compared with other approaches. Table 7 provides comparison of different PHFS-based decision models that clarifies the value addition of the proposed work. Some merits of the framework are: (i) two-stage framework provides module for data curation that includes missing value imputation and confidence determination, which are lacking in extant models; (ii) decision parameters are calculated and so subjectivity is reduced; (iii) experts' interdependencies and hesitation are considered in the decision process. Some weaknesses of the model are that: (i) risk appetite values are not calculated methodically during aggregation; and (ii) the consistency of preference matrices are not systematically checked or repaired.

Managers have provided some implications of the introduced model, such as: (i) it is a simple and ready-to-use tool for business decision-making; (ii) human intervention is mitigated effectively to reduce subjective biases and inaccuracies; (iii) the decision system acts in a bi-directional fashion by providing valuable insights to both customers and service providers for growth and development; (iv) the two-stage construct of the framework facilitates effective input feed to the decision module by performing necessary preprocessing; (v) key decision parameters are calculated and hence, biases are controlled; (vi) the incorporation of the framework for business decisions such as logistic provider selection saves time and offers a methodical tool for selection that is backed by mathematical support and scientific procedure; and (vii) experts need to be trained with PHFS context for efficient preference elicitation and decision-making.

For future work, plans are made to address the weaknesses mentioned above and to propose a new decision model with novel preprocessing steps for data curation under different fuzzy forms including PHFS context. Also, new models can be developed with variants of PHFS such as interval forms. Furthermore, some special fuzzy variants such as parsimonious spherical fuzzy, decomposed fuzzy, quasi q-rung form, and alike can be explored for solving problems within urban transportation context. Besides, machine learning paradigms can be integrated with decision-making methods for solving complex business problems.

Author Contribution

Raghunathan Krishankumar: conceptualization, data curation, formal analysis, methodology, writing – original draft.

Arunodaya R. Mishra: conceptualization, data curation, methodology, writing – original draft.
Pratibha Rani: conceptualization, data curation, formal analysis, methodology, writing – original draft.
Fatih Ecer: supervision, validation, visualization, writing – original draft.
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K.S. Ravichandran: supervision, validation, software, writing – original draft.
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Declarations

Conflict of interest. The authors have no relevant financial or nonfinancial interests to disclose. The authors declare that they have no conflict of interest.

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A.H. Gandomiis a professor of data science and an ARC DECRA fellow at the Faculty of Engineering & Information Technology, University of Technology Sydney. He is also affiliated with Obuda University, Budapest, as a Distinguished Professor. Prior to joining UTS, prof. Gandomi was an assistant professor at the Stevens Institute of Technology and a distinguished research fellow at BEACON Center, Michigan State University. Prof. Gandomi has published over three hundred journal papers and 12 books, which have collectively been cited 60,000 times. He has been named as one of the most influential scientific minds and received the Highly Cited Researcher award (top 1% publications and 0.1% researchers) from Web of Science for six years. In a recent study at Stanford University, released by Elsevier, prof. Amir H. Gandomi is ranked 24th most impactful researcher in the AI and Image Processing subfield in 2023! He also ranked 18th in GP bibliography among more than 17,000 researchers. He has received multiple prestigious awards for his research excellence and impact, such as the 2024 IEEE TCSC Award for Excellence in Scalable Computing (MCR), the 2023 Achenbach Medal, and the 2022 Walter L. Huber Prize, the highest-level mid-career research award in all areas of civil engineering. He has served as associate editor, editor, and guest editor in several prestigious journals, such as AE of IEEE Networks and IEEE IoTJ. Prof. Gandomi is active in delivering keynotes and invited talks. His research interests are global optimisation and (big) data analytics using machine learning and evolutionary computations in particular.