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STABILIZABILITY OF BIBO INTEGRO-DIFFERENTIAL SYSTEMS WITH TWO DISTRIBUTED DELAYS

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Abstract. Control laws' design strategies are developed to stabilize a class of BIBO integro-differential systems with two distributed delays by using an extended system.

Key words: integro-differential systems, distributed delays, stabilizability.

1. Introduction and problem statement. Mathematical models with delays are utilized to deal with a wide class of physical systems. For large delay-differential systems, an approximate method of solution has been developed by Alastruey and González de Mendívil (1994). The stabilizability conditions for systems with general delays in state were extended by Pandolfi (1975) and also by Bhat and Koivo (1976). In a work due to Olbrot (1978) open-loop stabilizability problems for systems with control and state delays were defined. Conditions for the delay-independent stabilization of linear systems were given by Amemiya *et al.* (1986), being the upper bound or the lower bound of the decay rates assignable, and Akazawa *et al.* (1987), by using in the proof matrices with some of their elements being arbitrary. In addition, Fiagbedzi and Pearson (1986, 1990) introduced techniques for the feedback and output feedback stabilization of delay systems by using a generalization of the transformation method. Furthermore, Mori et al. (1983) developed a way to stabilize linear systems with delayed state.

The stability of a linear delay-differential system with a point delay in its state has been studied in different works (Mori *et al.*, 1992; Hmamed, 1985, 1986 a-b; Mori, 1986; Bourlès, 1987 and Mori and Kokame, 1989). In particular, the problem of stability for a scalar differential system with two point delays in its state has been considered by several authors (see, for instance, Jury and Mansour, 1982). However, the exact delay-dependent algebraic stability conditions of such a system were lately provided by Schoen and Geering (1993); such conditions were obtained by using an instability criterion together with the D-decomposition method. In a recent paper (Alastruey *et. al.*, 1994), several criteria in order to design stabilizing control laws for integro-differential systems with two distributed delays in their state were introduced by using an associated extended system under the form of a linear-differential system with two point delays in its state. The method utilized to investigate the stabilizability of a system subject to two bounded known distributed delays was applicable to diagonally dominant systems and was based on testing scalar constraints for the various matrices defining the system. In this paper we particularize the above-mentioned results for the case of a class of BIBO (bi-input bi-output) integro-differential systems with two distributed delays in their state.

The paper is organized as follows: Section 2 introduces an important result on the stability for a system with two point delays in its state that will be used in the sequel. Section 3 shows the main stabilizability result for a class of BIBO systems with two distributed delays in their state. Finally, conclusions end the paper.

2. Stability for a BIBO system with two point delays. The following results will be useful in the next section in order to deduce stabilizability conditions for a BIBO integro-differencial system with two distributed delays in its state given in terms of algebraic relations.

Theorem 2.1 (Alastruey *et. al.*, 1994). *Consider the following linear MIMO system with two point delays in its vector-state*

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h) + A_2 x(t-2h) \tag{1}$$

with initial condition $\varphi_x : [-2h, 0] \to R^n$; $\varphi_x(0) = x(0)$ *being absolutely continuous with possible bounded discontinuities on a subset of* $[-2h, 0]$ *of* *zero measure, and where A_0, A_1 and A_2 are real constant $n \times n$-matrices and $h > 0$. Consider the following $n \times n$-matrices*

$$A_k = A_{kd} + \overline{A}_{kd}, \quad k = 0,1,2, \tag{2}$$

$$D = \text{diag}\left[-\left(|a_{ii}^{2d}| - \frac{\pi}{2h}\right)\right], \tag{3}$$

$$E = -\left(A_{0d} + A_{1d} + A_{2d}\right), \tag{4}$$

$$F = \text{diag}\left[\frac{y_i \cos(y_i h)}{\sin(y_i h)}\right], \tag{5}$$

$$G = \text{diag}\left[\frac{y_i}{sin(y_i h)} + 2a_{ii}^{2d}\cos(y_i h)\right], \tag{6}$$

where A_0, A_1, A_2 are decomposed according to (2) where $A_{kd} =$ diag $(a_{ii}^{kd}$, $(k = 0,1,2)$ are diagonal matrices, and $\overline{A}_{kd} = (a_{ij}^k,\ i \neq j$, otherwise $0) = (\bar{a}_{ij}^k) = A_k = A_{kd}$ are matrices with zero entries in their main diagonal; a_{ii}^{2d} are the elements of the diagonal matrix A_{2d} and $y_i \in [0, \frac{\pi}{h}]$, $(i = 1,\ldots,n)$ are real numbers. Then the time-delay system (1) with D positive is asymptotically stable if the following four conditions hold for some set of values $y_i \in [0, \frac{\pi}{h}]$, $(i = 1,\ldots,n)$

(i) E is positive (7)

(ii) $A_{0d} = F + A_{2d}$, (8)

(iii) $A_{1d} + G$ is positive, (9)

(iv)

$$\sum_{k=0}^{2}\left(\sum_{\substack{i,j=1\\ i\neq j}}^{n} |a_{ij}^k|^2\right)^{1/2}$$
$$< \text{Max}\left\{\left|\left(\sum_{i=1}^{n}|a_{ii}^0|^2\right)^{1/2} - \left(\sum_{i=1}^{n}|a_{ii}^1|^2\right)^{1/2} - \left(\sum_{i=1}^{n}|a_{ii}^2|^2\right)^{1/2}\right|;\right.$$
$$\left|\left(\sum_{i=1}^{n}|a_{ii}^1|^2\right)^{1/2} - \left(\sum_{i=1}^{n}|a_{ii}^0|^2\right)^{1/2} - \left(\sum_{i=1}^{n}|a_{ii}^2|^2\right)^{1/2}\right|;$$
$$\left.\left|\left(\sum_{i=1}^{n}|a_{ii}^2|^2\right)^{1/2} - \left(\sum_{i=1}^{n}|a_{ii}^0|^2\right)^{1/2} - \left(\sum_{i=1}^{n}|a_{ii}^1|^2\right)^{1/2}\right|\right\}. \tag{10}$$

COROLLARY 2.1. Consider the following linear BIBO system with two point delays in its vector-state

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h) + A_2 x(t-2h) \tag{11}$$

with initial condition $\varphi_x : [-2h, 0] \to R^n$ $\varphi_x(0) = x(0)$ being absolutely continuous with possible bounded discontinuities on a subset of $[-2h, 0]$ of zero measure, and where A_0, A_1 and A_2 are real constant $n \times n$-matrices and $h > 0$. Consider the following 2×2-matrices

$$A_k = A_{kd} + \overline{A}_{kd}, \quad k = 0, 1, 2, \tag{12}$$

$$D = \text{diag}\left[-\left(|a_{ii}^{2d}| - \frac{\pi}{2h}\right)\right], \tag{13}$$

$$E = -(A_{0d} + A_{1d} + A_{2d}), \tag{14}$$

$$F = \text{diag}\left[\frac{y_i \cos(y_i h)}{\sin(y_i h)}\right], \tag{15}$$

$$G = \text{diag}\left[\frac{y_i}{\sin(y_i h)} + 2a_{ii}^{2d}\cos(y_i h)\right], \tag{16}$$

where A_0, A_1, A_2 are decomposed according to (12) where $A_{kd} = \text{diag}(a_{ii}^{kd}$, $(k = 0, 1, 2)$ are diagonal matrices, and $\overline{A}_{kd} = (a_{ij}^k,\ i \neq j$, otherwise $0) = (\bar{a}_{ij}^k) = A_k = A_{kd}$ are matrices with zero entries in their main diagonal; a_{ii}^{2d} are the elements of the diagonal matrix A_{2d} and $y_i \in [0, \frac{\pi}{h}]$, $(i = 1, \ldots, n)$ are real numbers. Then the time-delay system (11) with D positive is asymptotically stable if the conditions (7)–(9) for matrices (12)–(16) hold for some set of values $y_i \in [0, \frac{\pi}{h}]$, $(i = 1, 2)$ as well as the following condition:

$$\sum_{k=0}^{2}\left(\sum_{\substack{i,j=1\\ i\neq j}}^{2} |a_{ij}^k|^2\right)^{1/2}$$

$$< \text{Max}\left\{\left|\left(\sum_{i=1}^{2}|a_{ii}^0|^2\right)^{1/2} - \left(\sum_{i=1}^{2}|a_{ii}^1|^2\right)^{1/2} - \left(\sum_{i=1}^{2}|a_{ii}^2|^2\right)^{1/2}\right|;\right.$$

$$\left|\left(\sum_{i=1}^{2}|a_{ii}^1|^2\right)^{1/2} - \left(\sum_{i=1}^{2}|a_{ii}^0|^2\right)^{1/2} - \left(\sum_{i=1}^{2}|a_{ii}^2|^2\right)^{1/2}\right|;$$

$$\left.\left|\left(\sum_{i=1}^{2}|a_{ii}^2|^2\right)^{1/2} - \left(\sum_{i=1}^{2}|a_{ii}^0|^2\right)^{1/2} - \left(\sum_{i=1}^{2}|a_{ii}^1|^2\right)^{1/2}\right|\right\}. \tag{17}$$

Proof. The proof follows immediatly by applying Theorem 2.1 to system (11).

3. Main stabilizability result for a BIBO system with two distributed delays. In this section conditions for a control law to stabilize a BIBO integro-differential system with two distributed delays in its state will be discussed by using an associated extended system. A main result is introduced.

Consider the following BIBO integro-differential system with two distributed delays in its state, and containing a control law with two point delays and a control law with two distributed delays.

$$\begin{aligned}\dot{x}(t) =& A_0 x(t) + \int_0^t A_1 x(t'-h)\,dt' \int_0^t A_2 x(t'-2h)\,dt' \\ &+ B_0^{11} u_1(t) + B_1^{11} u_1(t-h) + B_2^{11} u_1(t-2h) \\ &+ \int_0^t \left[B_0^{22} u_2(t') + B_1^{22} u_2(t'-h) + B_2^{22} u_2(t'-2h)\right] dt' \end{aligned} \tag{18}$$

with $x(t) = g(t)$ for all $t < 0$, where $A_0, A_1, A_2, B_0^{11}, B_1^{11}, B_2^{11}, B_0^{22}, B_1^{22}, B_2^{22} \in W \subset R^{2\times 2}$, $u_1(t), u_2(t) = 0\ \forall t < 0$ and $h \in R^+$.

Theorem 3.1 (Asymptotic stabilizability). *Consider two control laws $u_1(t), u_2(t)$ being continuous and differentiable on $[0,\infty)$ defined by the delay-differential equations*

$$\begin{aligned}\begin{bmatrix}\dot{u}_1(t)\\ \dot{u}_2(t)\end{bmatrix} =& \begin{bmatrix}D_0^{11} & \bar{0}\\ \bar{0} & D_0^{22}\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\end{bmatrix} + \begin{bmatrix}E_0^{11} & \bar{0}\\ \bar{0} & E_0^{22}\end{bmatrix}\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix} \\ &+ \begin{bmatrix}D_1^{11} & \bar{0}\\ \bar{0} & D_1^{22}\end{bmatrix}\begin{bmatrix}x(t-h)\\ \dot{x}(t-h)\end{bmatrix} + \begin{bmatrix}E_1^{11} & \bar{0}\\ \bar{0} & E_1^{22}\end{bmatrix}\begin{bmatrix}u_1(t-h)\\ u_2(t-h)\end{bmatrix} \\ &+ \begin{bmatrix}D_2^{11} & \bar{0}\\ \bar{0} & D_2^{22}\end{bmatrix}\begin{bmatrix}x(t-2h)\\ \dot{x}(t-2h)\end{bmatrix} + \begin{bmatrix}E_2^{11} & \bar{0}\\ \bar{0} & E_2^{22}\end{bmatrix}\begin{bmatrix}u_1(t-2h)\\ u_2(t-2h)\end{bmatrix},\end{aligned} \tag{19}$$

and subject to initializations $u_i(t) = \varphi_{iu}(t)$ on $[-2h, 0]$ with $\varphi_{iu}(t)$ being absolutely continuous with perhaps bounded discontinuities on some subsets $I_i \subset [-2h, 0]$ of zero measures; $i = 1, 2$.

Define the following 4×4-matrices

$$\widetilde{A}_0 \equiv \begin{bmatrix} \overline{0} & I_n \\ \overline{0} & A_0 \end{bmatrix}; \quad \widetilde{A}_1 \equiv \begin{bmatrix} \overline{0} & \overline{0} \\ A_1 & \overline{0} \end{bmatrix}; \quad \widetilde{A}_2 \equiv \begin{bmatrix} \overline{0} & \overline{0} \\ A_2 & \overline{0} \end{bmatrix}; \tag{20}$$

$$\widetilde{B}_0 \equiv \begin{bmatrix} B_0^{11} & \overline{0} \\ \overline{0} & B_0^{22} \end{bmatrix}; \; \widetilde{B}_1 \equiv \begin{bmatrix} B_1^{11} & \overline{0} \\ \overline{0} & B_1^{22} \end{bmatrix}; \; \widetilde{B}_2 \equiv \begin{bmatrix} B_2^{11} & \overline{0} \\ \overline{0} & B_2^{22} \end{bmatrix}; \tag{21}$$

$$\widetilde{D}_0 \equiv \begin{bmatrix} D_0^{11} & \overline{0} \\ \overline{0} & D_0^{22} \end{bmatrix}; \; \widetilde{D}_1 \equiv \begin{bmatrix} D_1^{11} & \overline{0} \\ \overline{0} & D_1^{22} \end{bmatrix}; \; \widetilde{D}_2 \equiv \begin{bmatrix} D_2^{11} & \overline{0} \\ \overline{0} & D_2^{22} \end{bmatrix}; \tag{22}$$

$$\widetilde{E}_0 \equiv \begin{bmatrix} E_0^{11} & \overline{0} \\ \overline{0} & E_0^{22} \end{bmatrix}; \; \widetilde{E}_1 \equiv \begin{bmatrix} E_1^{11} & \overline{0} \\ \overline{0} & E_1^{22} \end{bmatrix}; \; \widetilde{E}_2 \equiv \begin{bmatrix} E_2^{11} & \overline{0} \\ \overline{0} & E_2^{22} \end{bmatrix}. \tag{23}$$

Define the following 8×8*-matrices*

$$\widehat{A}_0 \equiv \begin{bmatrix} \widetilde{A}_0 & \widetilde{B}_0 \\ \widetilde{D}_0 & \widetilde{E}_0 \end{bmatrix}; \quad \widehat{A}_1 \equiv \begin{bmatrix} \widetilde{A}_1 & \widetilde{B}_1 \\ \widetilde{D}_1 & \widetilde{E}_1 \end{bmatrix}; \quad \widehat{A}_2 \equiv \begin{bmatrix} \widetilde{A}_2 & \widetilde{B}_2 \\ \widetilde{D}_2 & \widetilde{E}_2 \end{bmatrix}; \tag{24}$$

$$\widehat{D} \equiv \operatorname{diag}\left[-\left(|\widehat{a}_{ij}^{2d}| - \frac{\pi}{h}\right)\right] \quad \textit{(see Eq. 29 below)}, \tag{25}$$

where $\widehat{a}_{ij}^{2d}$ *are the elements of* $\widehat{A}_{2d}$

$$\widehat{E} \equiv -\left(\widehat{A}_{0d} + \widehat{A}_{1d} + \widehat{A}_{2d}\right) \; \textit{(see Eq. 29 below)}; \tag{26}$$

$$\widehat{F} \equiv \operatorname{diag}\left[\frac{y_i \cos(y_i h)}{\sin(y_i h)}\right]; \tag{27}$$

$$\widehat{G} \equiv \operatorname{diag}\left[\frac{y_i}{\sin(y_i h)} + 2\widehat{a}_{ii}^{2d}\cos(y_i h)\right] \quad \textit{(see Eq. 29 below)}, \tag{28}$$

where $y_i \in [0, \frac{\pi}{h}]$, $(i = 1, \ldots, 8)$, *are real numbers.*

Decompose

$$\widehat{A}_k = \widehat{A}_{kd} + \overline{\widehat{A}}_{kd} \quad (k = 0, 1, 2), \tag{29}$$

where $\widehat{A}_{kd}$ *and* $\overline{\widehat{A}}_{kd}$ *are the diagonal and nondiagonal submatrices defined similarly as for* (1).

Control laws (19) *provide asymptotic stability for system (18) if the following five conditions hold for some set of values* $y_i \in [0, \frac{\pi}{h}]$, $(i = 1, \ldots, 8)$

(i) $\widehat{D}$ *is positive;* (30)
(ii) $\widehat{E}$ *is positive;* (31)
(iii) $A_{0d} = \widehat{F} + \widehat{A}_{2d}$*;* (32)
(iv) $\widehat{A}_{1d} + \widehat{G}$ *is positive;* (33)
(v)

$$\sum_{k=0}^{2}\left(\sum_{\substack{i,j=1\\ i\neq j}}^{8} |\widehat{a}_{ij}^{k}|^2\right)^{1/2}$$
$$< \mathrm{Max}\left\{\left|\left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{0}|^2\right)^{1/2} - \left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{1}|^2\right)^{1/2} - \left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{2}|^2\right)^{1/2}\right|;\right.$$
$$\left|\left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{1}|^2\right)^{1/2} - \left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{0}|^2\right)^{1/2} - \left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{2}|^2\right)^{1/2}\right|;$$
$$\left.\left|\left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{2}|^2\right)^{1/2} - \left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{0}|^2\right)^{1/2} - \left(\sum_{i=1}^{8}|\widehat{a}_{ii}^{1}|^2\right)^{1/2}\right|\right\}. \quad (34)$$

Proof. Consider the free part of system (18)

$$\dot{x}(t) = A_0 x(t) + \int_0^t A_1 x(t'-h)\, dt' \int_0^t A_2 x(t'-2h)\, dt' \quad (35)$$

with $x(t) = g(t)$ for all $t < 0$. By differentiating system (35) one gets

$$\ddot{x}(t) = A_0\dot{x}(t) + A_1 x(t-h) + A_2 x(t-2h) \quad (36)$$

with $\dot{x}(t) = \dot{g}(t)\ \forall t < 0$. Define the 4-vector $\widetilde{x}(t) \equiv \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$. From equations (35) and (36) one gets

$$\dot{\widetilde{x}}(t) = \begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \begin{bmatrix} \overline{0} & I_n \\ \overline{0} & A_0 \end{bmatrix}\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} \overline{0} & \overline{0} \\ A_1 & \overline{0} \end{bmatrix}\begin{bmatrix} x(t-h) \\ \dot{x}(t-h) \end{bmatrix}$$
$$+ \begin{bmatrix} \overline{0} & \overline{0} \\ A_2 & \overline{0} \end{bmatrix}\begin{bmatrix} x(t-2h) \\ \dot{x}(t-2h) \end{bmatrix}. \quad (37)$$

By using definition in equation (20), Eq. 37 can be rewritten as follows

$$\dot{\tilde{x}}(t) = \tilde{A}_0\tilde{x}(t) + \tilde{A}_1\tilde{x}(t-h) + \tilde{A}_2\tilde{x}(t-2h). \tag{38}$$

System (38) is called "associated extended system" of the free system (35). By applying the proposed control laws to system (38) one gets

$$\begin{aligned}\dot{\tilde{x}}(t) =& \tilde{A}_0\tilde{x}(t) + \tilde{A}_1\tilde{x}(t-h) + \tilde{A}_2\tilde{x}(t-2h)\\ &+ \tilde{B}_0\tilde{u}(t) + \tilde{B}_1\tilde{u}(t-h) + \tilde{B}_2\tilde{u}(t-2h),\end{aligned} \tag{39}$$

where

$$\tilde{u}(t) \equiv \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}. \tag{40}$$

Observe that Eqs. (19) and (39) can be rewritten is one single delay-differential equation as follows

$$\begin{aligned}\begin{bmatrix} \dot{\tilde{x}}(t) \\ \dot{\tilde{u}}(t) \end{bmatrix} =& \begin{bmatrix} \tilde{A}_0 & \tilde{B}_0 \\ \tilde{D}_0 & \tilde{E}_0 \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ \tilde{u}(t) \end{bmatrix} + \begin{bmatrix} \tilde{A}_1 & \tilde{B}_1 \\ \tilde{D}_1 & \tilde{E}_1 \end{bmatrix} \begin{bmatrix} \tilde{x}(t-h) \\ \tilde{u}(t-h) \end{bmatrix}\\ &+ \begin{bmatrix} \tilde{A}_2 & \tilde{B}_2 0 \\ \tilde{D}_2 & \tilde{E}_2 \end{bmatrix} \begin{bmatrix} \tilde{x}(t-2h) \\ \tilde{u}(t-2h) \end{bmatrix}.\end{aligned} \tag{41}$$

Define

$$z(t) \equiv \left[\tilde{x}^T(t) \vdots \tilde{u}^T(t)\right]^T, \tag{42}$$

where $z(t)$ is an 8-vector and $\widehat{A}_0, \widehat{A}_1, \widehat{A}_2$ are 8×8 matrices defined in (24). Then, equation (41) can be rewritten as

$$\dot{z}(t) = \tilde{A}_0 z(t) + \tilde{A}_1 z(t-h) + \tilde{A}_2 z(t-2h). \tag{43}$$

If, by hypothesis, conditions (30)–(33) hold, then by Corollary 2.1 system (43) is asymptotically stable, i.e., system (38) is asymptotically stabilizable by control law (40). Observe that the controlled associated extended system (39) becomes

$$\begin{aligned}\dot{x}(t) =& A_0x(t) + \int_0^t A_1x(t'-h)\,dt' \int_0^t A_2x(t'-2h)\,dt'\\ &+ B_0^{11}u_1(t) + B_1^{11}u_1(t-h) + B_2^{11}u_1(t-2h)\end{aligned} \tag{44}$$

with initial conditions $x(\tau) = \varphi_{1x}(\tau);\ \tau \in [2h, 0);\ x(0) = \varphi_{1x}(0);$

$$\begin{aligned}\ddot{x}(t) =& A_0\dot{x}(t) + A_1x(t-h) + A_2x(t-2h) \\ &+ B_0^{22}u_2(t) + B_1^{22}u_2(t-h) + B_2^{22}u_2(t-2h). \end{aligned} \tag{45}$$

The integration of Eq. 45 yields

$$\begin{aligned}\dot{x}(t) =& A_0x(t) + \int_0^t A_1x(t'-h)\,dt' \int_0^t A_2x(t'-2h)\,dt' + \int_0^t B_0^{22}u_2(t')\,dt' \\ &+ \int_0^t B_1^{22}u_2(t'-h)\,dt' + \int_0^t B_2^{22}u_2(t'-2h)\,dt' \end{aligned} \tag{46}$$

with initial conditions $x(\tau) = \varphi_{2x}(\tau);\ \tau \in [2h, 0);\ x(0) = \varphi_{2x}(0)$.

Finally, the application of the superposition principle to the solution (44) and (46) yields

$$\begin{aligned}\dot{x}(t) =& A_0x(t) + \int_0^t A_1x(t'-h)\,dt' \int_0^t A_2x(t'-2h)\,dt' \\ &+ B_0^{11}u_1(t) + B_1^{11}u_1(t-h) + B_2^{11}u_2(t-2h) \\ &+ \int_0^t \left[B_0^{22}u_2(t') + B_1^{22}u_2(t'-h) + B_2^{22}u_2(t'-2h)\right] dt' \end{aligned} \tag{47}$$

with initial conditions $x(\tau) = \varphi_{1x}(\tau) + \varphi_{2x}(\tau)$ and the controlled system (47) – which coincides with (18) – is asymptotically stable, since the stability property is independent from the particular initial values $\varphi_{ix}(\tau)$ provided that for any vector function norm, $||\varphi_{ix}||_{[-2h,0]<\infty}$ $(i = 1, 2)$.

Theorem 3.1 provides a way to evaluate the asymptotic stabilizability of a BIBO delay-differential system with two distributed delays in its state. The evaluation is made on the context of a set of algebraic relations, which are very suitable for computer applications.

4. Conclusions. This paper provides sufficient conditions for testing asymptotic stabilizability of a class of BIBO delay-differential systems with distributed delays in its state.

Conditions are given in terms of algebraic relations, useful for computer applications. The control is given in terms of two independent control laws defined by using dynamic differential equations which contain point delays. The control laws are independent in the sense that each one of them are applied independently to state equation (i.e., they are not interconnected).

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SISTEMŲ SU VĖLINIMU VALDYMAS

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Straipsnyje pateikiama dinaminių sistemų su vėlinimu apžvalga. Aptariami šias sistemas aprašančių skirtuminių lygčių sprendimo metodai, nagrinėjami jų ribojimai ir galimybės. Pateikiamas gana bendro pavidalo tiesinės sitemos su vėlinimu lygties skleidimo Teiloro eilute metodas, nagrinėjamas sistemų su vėlinimu stabilumas. Taip pat pateikiama tarpusavyje surištų sistemų su vėlinimu interpretacija, apžvelgiami darbai, nagrinėjantys netiesines sistemas su vėlinimu. Apžvalgoje nurodomos pagrindinės lygčių sprendimo problemos remiantis paskutinių dviejų dešimtmečių žinomais darbais šioje srityje.