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RESTORATION OF AN ENTIRE EPICORTICAL ELECTROCORTICOGRAM FROM A SPATIALLY SAMPLED ELECTROENCEPHALOGRAM

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Abstract. The restoration of an entire epicortical ECoG-potential pattern from spatially sampled EEG-potentials can be performed according to the general principle of inverse spatial filtering: if a forward spatial filter is an averaging one, then a mutually inverse spatial filter can form a sharpened pattern from a slurred sampled pattern thus restoring more or less a spatial contrast of a primary source pattern. Brain coverings are the averaging forward spatial time-lag free filter in the transformation "eECoG $\longrightarrow$ EEG"; several models have been already developed for calculating the weight coefficients of this filter. Proposed here method to obtain weight coefficients of the mutually inverse spatial time-lag free filter of EEG is based on the generalized inversion of forward filter's weight coefficients matrix. The developed algorithm is verified using 3-dimensional depiction of numerically simulated patterns: the "real" eECoGs are visually compared with the restored eECoGs, that were obtained from the corresponding spatial samples of the "real" EEGs.

Key words: inverse problem of electroencephalography; space filtering of electroencephalogram;

Abbrevations:

EEG – electroencephalogram;
ECoG – electrocorticogram;
eECoG – epicortical ECoG;
FP – forward problem of electroencephalography;
IP – inverse problem of electroencephalography;
FSF – forward spatial time-lag free filter of eECoG;

ISF – inverse spatial time-lag free filter of EEG;
GI – generalized inversion;
ED – equivalent dipole;
MS – multilayer sphere (multisphere);
RD – radial dipole.

1. Introduction. One of the earliest problems in the EEG genesis theory was prompted by the necessity to simulate the potential distribution on the scalp from the given set of intracranial sources; it was called "the forward problem of electroencephalography" (the term "forward" was borrowed from the field theory of classical physics). Later models for FP solving gained an additional significance as an essentially important element of the pressing applied problem named "the inverse problem of electroencephalography" (the term "inverse" is also taken from the classical field theory). Recall that IP solutions are aimed at the estimation of the intracranial electrical activity using the derived EEGs.

The first and most popular up-to-date approach to the IP is the "equivalent dipole" method (Henderson *et al.*, 1975); sophisticated modern algorithms can compose sets of several EDs (Salu *et al.*, 1990).

The principal drawbacks of the ED-method are well known:

1) its ambiguity, because an attempt is made to reconstruct 3-dimensional intracranial source patterns from 2-dimensional EEG-potential patterns;

2) the attempt to replace a lot of spatial sources dispersed all over the brain by a relatively small set of several dipoles, e.g., point sources, is rather often misleading (Nunez, 1997).

Another possible and apparently more perspective approach to the IP consists in a restoration of the potentials on some interesting for an investigator intracranial surface from the sampled EEG. The IP-solving in such a way means an attempt to obtain patterns of potential sharpened in comparison with EEG-potential patterns, i.e., it can result in a restoration of the spatial contrast of intracranial potential patterns, as far as it is known, this contrast is lost more or less in EEG-potential patterns due to a strong slurring effect of brain tissues and brain coverings in the EEG genesis (Pfurtscheller *et al.*, 1975). The possibility to analyze the restored sharpened potential patterns instead of discontrasted EEG-patterns might be very attractive for clinicians and biophysicists.

This second method to solve the IP is less artificial than the ED-method, since no assumptions on intracranial source patterns are necessary a priori. The simplest variant of such an approach to the IP consists in the restoration of an eECoG from an EEG: in this case an attempt is made to estimate the potential on the 2-dimensional epicortical surface using EEG-potentials, sampled also on the 2-dimensional scalp surface. There are no EEG sources between the epicortical and the scalp surface (the potentials of scalp muscles should be filtered as artifacts) therefore there is no principle ambiguity.

The results of eECoG restoration from EEG presented in literature are rather insufficient: first of all, primitive completely inadequate FP-models were used in numerical simulation, and secondly, the IP was solved using not perspective algorithms – decomposition of sampled EEG in the basis of spherical harmonics, or solving a small square system of linear equations (so was modeled the forward transformation "eECoG $\longrightarrow$ EEG") (Schneider, 1972; Henderson *et al.*, 1975; Hjorth, 1975; Chapman *et al.*, 1976; Freeman, 1980); all that is too poor for the IP of EEG.

The sketch of a new algorithm for eECoG restoration on the entire episurface of the cortex from a spatially sampled EEG is presented in this preliminary report. Numerical simulation results show its efficiency in solving the IP in such a way.

2. The algorithm of eECoG restoration from EEG. The resistivity of most tissues (with the possible exception of those which exhibit a considerable degree of anisotropy) is not strongly frequency-dependent in the range extending from d.c. to a few hundred kc/s (Geddes, 1967), i.e., the brain envelopes conduct the currents of intracranial sources almost passively in the classical frequency range of EEG rhythms ($\lesssim 100$ Hz). It means that the momentary values of EEG-potentials are some time-lag free mapping of the momentary values of intracranial sources, and analyzed here forward transformation "eECoG $\longrightarrow$ EEG" can be described by a such integral equation:

$$W(X,Y) = \int\limits_{x}\int\limits_{y} H(X,Y \mid x,y)V(x,y)\,dF(x,y); \qquad (1)$$

here: $W(X,Y)$ is the momentary value of EEG-potential; $V(x,y)$ is the momentary value of eECoG-potential; $h(X,Y \mid x,y)$ is the weight function of forward transformation "eECoG $\rightarrow$ EEG"; $dF(x,y)$ is the differential element of the epicortical surface; (x,y) is any 2-dimensional system of coordinates.

The integral in equation (1), as is the practice of numerical calculation, can be approximated by its integral sum with the necessary accuracy; in this way the weight function $H(.)$ should be normalized by the square of the chosen elementary finite area of the episurface and that allows to reduce (1) to such a system of linear equations:

$$w_i = \sum_{j=1}^{J} h_{ij}v_j; \quad i = 0, 1, \ldots, I; \qquad (2)$$

here: w_i is the absolute (e.g., referred to zero potential of infinity) EEG-potential at the i-th point on the scalp surface; v_j is the absolute eECoG-potential at some point of the j-th elementary finite area of cortex episurface; h_{ij} is the weight coefficient that describes the input of the unit absolute eECoG-potential on the j-th elementary finite area of episurface into the i-th absolute EEG-potential; J is the total number of points of the "grid" that covers the episurface, obviously J is large; $(I+1)$ is the total number of the "electrodes" on the scalp.

Real registrations do not sample absolute potentials; the difference of them at some two points of a measured object is the only physical reality to be sampled. That is why "bipolar derivation", when the difference of two absolute EEG-potentials at some points on the scalp is lead to a amplifier so formatting one elementary EEG-derivation, is the only way to registry an EEG, realizable in a practice of biophysical experiment or clinical testing.

A little bit of comments: the attempts to put the reference electrode (often misleadingly named as an "indifferent electrode") in some special places of the scalp, head or even body and thus to obtain the constant reference potential, uninfluenced by brain electrical activity, resulted in a failure (Stephenson *et al.*, 1951; Lehtonen *et al.*, 1971). The so called 'monopolar derivation' (Cobb, 1958), "averaged electrode derivation" (Goldman, 1950), "source derivation" (Hjorth, 1975), generalized as "differential derivation" (Gutman *et al.*, 1979) are some combinations of bipolar derivations in more or less transparent way (Shimoliunas, 1992).

Therefore, any theory of EEG numerical processing oriented to practical applications should use the bipolar EEG-derivation as the "elementary brick" from the very beginning; in the time-lag free case it is the bipolar EEG-sampling:

$$s_i = w_i - w_0 = \sum_{j=1}^{J}(h_{ij} - h_{0j})v_j = \sum_{j=1}^{J} h'_{ij}v_j; \quad i = 1, ..., I; \quad (3)$$

here: I is the total number of EEG-samplings; obviously $I \ll J$.

To restore the eECoG from the EEG means to solve the system of linear equations (3). The generalized inversion algorithm of linear algebra processes rectangular $(I \neq J)$ systems of linear equations, and, if $J > I$, it obtains a solution with an added arbitrary constant (Moore, 1935; Penrose, 1953). In the case of eECoG estimations from EEG an additive constant potential of calculated eECoG can be cut off as meaningless physically, and the restoration of eECoG can be calculated as such a linear sum of EEG-samplings:

$$\widehat{v}_j = \sum_{i=1}^{I} f_{ji}s_i; \quad j = 1, \ldots, J; \quad (4)$$

here: $\widehat{v}_j$ is the restoration of eECoG at the j-th point of cortex episurface; f_{ji} is the weight coefficient of the input of the i-th EEG-sampling to the j-th restored eECoG-potential.

Such is the sketch of the algorithm used for numerical simulation here. Let us outline some content of this formalism.

From the biophysical point of view, the weight coefficients $\{h_{ij}\}$ "are" brain coverings in the EEG-pattern formation from eECoG-patterns. Between the cortex episurface and sampling EEG electrodes on the scalp's surface there are such anatomical structures: epicortical space (it contains liquor, meninges), three-layer skull, multilayer scalp. In what form and how many of them will be included in $\{h_{ij}\}$ depends on the complexity and achieved adequacy of the FP-solving model; that is the object of a special, rather a complicated biophysical analysis (Frank, 1952; Geisler *et al.*, 1961; Schneider, 1974; Rush *et al.*, 1968,1969; Kavanagh *et al.*,1978; Munck *et al.*, 1993; Berg *et al.*, 1994). It is obvious that first of all a practical value of the conclusions based on the performed numerical simulation depends on the adequacy of the used FP-solving model.

From the mathematical point of view, the set of weight coefficients $\{h_{ij}\}$ "is" the forward spatial time-lag free filter forming the EEG-potential patterns from the eECoG-potential patterns. The biophysical analysis proved that EEG-patterns are badly slurred as compared with the eECoG-patterns, what means the FSF $\{h_{ij}\}$ is an averaging spatial time-lag free filter (Luccki *et al.*, 1962). The calculation of $\{f_{ji}\}$ means the construction of a mutually inverse spatial time-lag free filter aimed at restoring the contrast of the primary eECoG-patterns lost in EEG-patterns.

Obviously the work of any more or less sound mutually inverse spatial filter will result in some restoration of the wanted contrast; the main question is whether the sharpening of a spatially sampled EEG, obtained by GI-algorithm, converting these EEGs in some approximation of the real eECoG-pattern, will be noticeable.

3. Model for numerical simulation. There are three things to be determined before starting the numerical simulation of any ISF-procedure converting EEG into eECoG:

1) FP-model in the framework of which the forward weight coefficients $\{h'_{ij}\}$, $\{i = 1, \dots, I\}$, $\{j = 1, \dots, J\}$ will be calculated;

2) pixels on the scalp at which "electrodes" deriving EEG-potentials will be attached, i.e., the set of points $\{i = 0, 1, \dots, I\}$;

3) the grid covering the cortex episurface, in the nodes of which eECoG will be restored, i.e., the set of points $\{j = 1, \dots, J\}$.

3.1. FP – model. Any FP-model assumed as adequately describing the forward transformation "eECoG → EEG", can be used for ISF of EEGs by the GI-method; obviously, that more precise this FP-model will be that more realistic estimations of eECoG will be obtained. The goal of this paper is to verify in principle the possibility of the ISF-GI algorithm to restore an eECoG on the entire episurface of the cortex from a spatially sampled EEG, therefore the spherical approximation of cerebral cranium, traditionally used in the analytical/numerical EEG modeling, is chosen. This spherical approximation means a choice from several well known multisphere models (see references in the previous section) and, consequently, leads to the work in the spherical system of coordinates (r, O, φ). It is possible to obtain the general simple formula expressing h_{ij} for MS with any number of different conductivity shells (brain coverings) around a homogeneous ball (brain):

$$h_{ij} = 0.9 \sum_{l=1}^{25} (2l+1) c(l) P_l(\cos\gamma); \qquad (5a)$$

$$cos\gamma = cos\theta_i \cdot cos\theta_j + sin\theta_i sin\theta_j cos(\varphi_i - \varphi_j); \qquad (5b)$$

here $P_l(.)$ is the ordinary Legendre polynomial of degree l; the complexity of a chosen MS affects only the coefficients $\{c(l),\ l = 1, 2, \ldots\}$, since only they represent all of those brain coverings, which are included in an MS-model (Shimoliunas, 1988).

Sufficient adequacy of forward transformation "eECoG → EEG" modeling, in the frame of spherical approximation of cerebral cranium, can be achieved by MS-5, i.e., a homogeneous ball-brain surrounded by: a) one homogeneous layer – epicortical liquor and meninges in it (averaged conductivity); b) three layers of different conductivity – skull, consisting of inner/outer plates and diploe; c) one outer homogeneous layer – scalp (averaged conductivity). The unified algorithm to calculate these $\{c(l),\ l = 1, 2, \ldots\}$ is presented in (Shimoliunas, 1988). The values of MS-5 geometrical parameters are settled here according to the biometric data presented in Povitsky *et al.* (1974) and Ary *et al.* (1981) and correspond to used in the most popular MS of Rush *et al.* (1968,1969). The values of necessary conductivities of brain cranium tissues are estimated according to the biometric data compendium collected by Geddes *et al.* (1967).

3.2. The set of points for spatial EEG sampling. The EEG interelectrode distances must be not shorter as approximately 2 cm in real experiments, because "it appears that pixels smaller than 1 cm are unlikely to reflect significant differences in the scalp potential field" (Epstein, 1985). In the process of numerical simulation performed here EEG-sampling points are spaced by 2.5 cm over the spherical "scalp" of MS-5, the outer radius of which is equal to 9.4 cm and differs from the conventional value equal to 9.2 cm given by Rush *et al.* (1968,1969), due to an epicortical split present in MS-5 (its width being equal to 0.2 cm). Thus there are used $I = 79$ EEG-samples to restore an eECoG by ISF-GI algorithm (their spherical coordinates are presented in Table 1).

The amount $I = 79$ seems unreal in comparison with that used in the traditional EEG-deriving system "10–20" (here $I \leqslant 19$; Jasper, 1958), but modern systems of spatial EEG sampling already have $I = 64; 128; 256$ channels (Tucker, 1993).

Table 1. The EEG spatial sampling net

No. of circle	i from	i till	Θ of circle
0	1	1	0
1	2	7	15
2	8	19	30
3	20	36	45
4	37	56	60
5	57	79	75
"Referent electrode": $\theta = 90^0$, $\varphi = 0$; (common for all "active electrodes")			

3.3. The grid of eECoG restoration points. A preparatory numerical simulation showed that the eECoG-potential can be averaged over the area not exceeding approximately 1 cm^2 with the loss of accuracy of EEG-potential calculation not exceeding several percents; for EEG measurements, the noise of which is of the same range, it is quite acceptable.

The interdistance 0.5 cm of eECoGs restoration points on the spherical 'epicortex' with the radius equal to 8 cm (this value is chosen as by Rush *et al.*, 1968, 1969) means averaging of eECoGs over the area of approximately 0.8 cm^2. According to the spherical approximation used here these points are also put on the circles with the common center $\theta = 0$, (here is the point $j = 1$), there are 25 circles total, the interdistance between the neighbor circles is $\theta = 3.6^0$ (i.e., 0.5 cm), the last circle being on $\theta = 90^0$. Thus, there are $J = 1650$ points, uniformly spread over the entire episurface of cortex, at which eECoG-potentials will be restored. Such density is quite sufficient for visual analysis.

4. Results of numerical simulation. RDs are the adequate sources to describe realistic generators of brain activity (Munck, 1988). RDs in the cortex produce sharp "peaks" in the eECoG-potential patterns that are presented by slurred "bells" in the EEG-potential patterns (Kearfott, 1991; Shimoliunas, 1992). That is the reason why the sets of RDs are reasonable test-sources of artificial eECoG/EEG patterns used as verification objects of the sharpening effect of sampled EEGs performed by ISF-GI.

The stages of numerical simulation were such: 1) set of RDs was composed; 2) "real" eECoG and "real" EEG patterns of the set were calculated in the frame of FP-model MS-5; 3) the corresponding spatial EEG sample was calculated according to the above described scheme of "electrodes" on the "scalp"; 4) ISF-GI convertion of "sampled" EEGs into the potentials at the nodes of covering epicortex grid, i.e., the calculation of restored eECoGs, completed the simulation. All these potentials patterns – "real" eECoG, estimated eECoG, "real" EEG patterns – are depicted as 3-dimensional surfaces without any additional numerical processing, because their values are calculated in the nodes of a dense grid. The "sampled" EEG values are linearly interpolated, because their pixels on the "scalp" are not so numerous. The sequence of simulated surfaces presentation is the same in all the analyzed situations: a) "real" eECoG; b) estimated eECoG; c) "real" EEG; d) spatially sampled EEGs; comparing them it is easy to verify the ability of ISF-GI to convert sampled EEGs into an pattern – estimated eECoG; because this estimated eECoG pattern is so similar to the "real" eECoG pattern it can be named "restored" eECoG pattern.

A little bit about the key of reading the presented here figures: the center of any depicted surface corresponds to $\theta = 0$, the peripheral circle corresponds to $\theta = 90^0$; if the potential over the entire surface ("cortex" or "scalp") was zero, a flat disc would be drawn.

First of all it is necessary to see how much the "peak" of 1 RD in the restored eECoG pattern approaches the "peak" of the "real" eECoG pattern after sharpening performed by ISF-GI on the "bell" of "sampled" EEGs; the results of numerical simulation, when one and the same RD was located in two different positions in the cortex, are presented in Fig. 1, 2.

A second point to be verified is the ability of ISF-GI to manage with the sampled EEGs of 2 RDs if they are close enough in the cortex; the discernment of their "peaks" in the restored eECoG pattern, that are distinguishable in the "real" eECoG pattern but are slurred into one "bell" in the sampled EEGs, is presented in Fig. 3, 4.

And for completion the EEG-sampling of the complex set of 5 RDs is processed by ISF-GI and presented in Fig. 5: from the unrecognizable slurred one huge "bell" in the sampled EEGs the 5 "peaks" appear in the restored eECoG pattern, being rather like the original 5 "peaks" in "real" eECoG pattern.

5. Discussion. A comparison of the "peak" of potential in the restored eECoG pattern with the "peak" of potential in the "real" eECoG pattern and the "bell" of potential in the EEG pattern, when 1 RD is placed in the cortex (Fig. 1, 2), shows the effectiveness of the sharpening made by ISF-GI: the sampled EEG "bell" is converted into a "peak" in the restored eECoG pattern that is rather similar to the "peak" in the "real" eECoG pattern. It should be stressed here that the relief of sampled EEGs used by ISF-GI is rather a poor reflection of the 'real' EEG pattern relief – the latter is almost continuous in comparison with the rather moderate set of sampled EEG pixels. The discernibility effect does not depend on the RD location in the cortex. As it could be expected some noise of estimation – additional wavelets on the periphery of the restored "peak", oscillating at high space frequencies, have appeared; that is caused by a deficiency of high space frequency harmonics in the sampled EEG pattern (recall: in the forward transformation "eECoG $\longrightarrow$ EEG" considerable decrement of their amplitudes is conditioned by brain coverings – the potent averaging FSF).

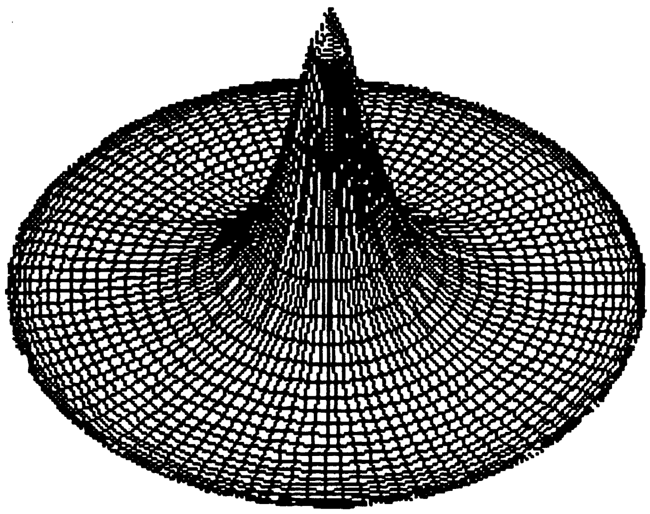
The next point to be analyzed is to estimate the minimal interdistance of two RDs in the cortex, the "peaks" of which will be discerned by ISF-GI in the restored eECoG pattern. The numerical simulation review of these interdistances showed that such a minimal distance is approximately 4 cm (4.2 cm interdistance corresponds to $\gamma = 30^0$ on the spherical "epi-cortex" with the radius 8.0 cm); for so located RDs the overlay of noise wavelets does not noticeably distort the obtained sharpening of the sampled EEG-"bell", i.e., does not prevent the discernment of their "peaks" in the restored eECoG pattern (Fig. 3, 4). As earlier, the restoration of eECoG pattern does not depend on the location of these two "close" RDs in the cortex.

No doubt, often there can be more than two interesting focuses of electrical activity in the cortex, whose "peaks" of potential can stand out against a background eECoG of the current electrical brain activity. Such a situation is simulated using 5 RDs moved apart from one another by estimated above 4 cm (Fig. 5). Their electrical activity is slurred by brain coverings into one big "bell" of potential in the EEG-sample and "real" EEG pattern; but even in this case this deficiency of high space frequencies in the sampled EEGs did not prevent the ISF-GI to restore 5 "peaks" of potential in the eECoG (surely, with some peripheral wavelets added as the noise of the ISF). That is an impressing contrast in comparison with the reflection of these 5 RDs in the EEG, where these 5 sources of 'something interesting/important in the cortex' are reflected completely erroneously as one big focus, thus misleading the interpretation of electrical activity on the surface of the cortex towards only one big source somewhere in the brain.

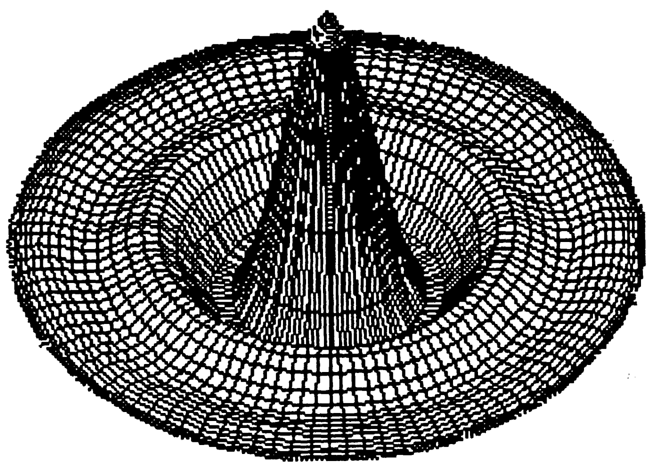
Thus, the restoration of an eECoG on the entire surface of epicortex by ISF-GI of a spatially sampled EEG is possible not only in principle; the obtained numerical simulation results presented here are the sufficient basis for further development of ISF of a spatially sampled EEG performed by GI-algorithm. The main stumbling-block can be the intrinsic feature of any mutually inverse procedure – its instability that increases noises present in a sample to be converted. It means that the overlaying distortions ("wavelets") in the restored eECoG pattern will be caused not only by the deficiency of high space frequency harmonics in the EEG sample, but also by the background eECoG of current brain activity mixed with sampling noises of the devices deriving real EEG. The stability of conversion "EEG $\longrightarrow$ eECoG" by ISF-GI is the foremost object for future theoretical investigations, preceding the practical implementation of eECoGs, restored in this way, in biophysical experiments and clinical tests.

To conclude this preliminary numerical analysis let us emphasize that the spherical approximation used here for FP-solving, i.e., the calculation of $\{h_{ij}\}$ in the frame of the chosen MS, is not the intrinsic feature of the proposed ISF-GI algorithm. Any other model of cerebral cranium, considered to be more realistic (ellipsoidal approximation or even numerical describtion), can serve for practical implementations of GI-conversion "EEG $\longrightarrow$ eECoG" in the future; the question is, if only that will result in a better restoration of an eECoG in the biophysical/clinical analysis of a real EEG.

Without doubt, the attempts to deal with in the restored (in any reasonably way) eECoGs instead of EEGs should be investigated carefully as possibly leading to the development of new perspective not invasive methods of brain electrical activity analysis. The ISF-GI algorithm presented here is among the firsts and may appare in the future as not being the best one.

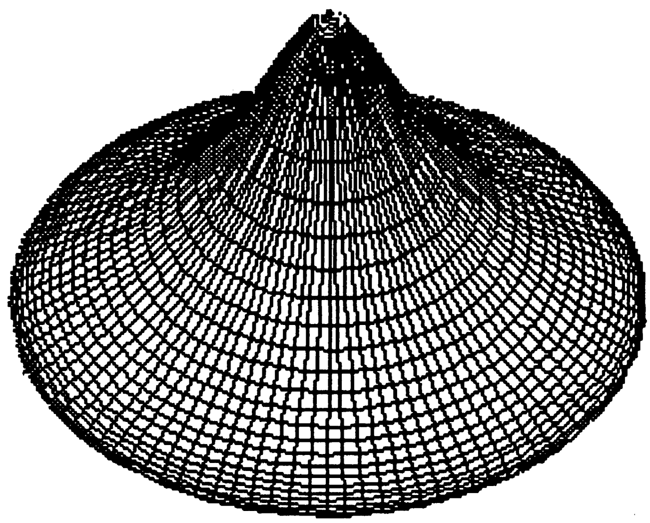


a) "real" eEoG

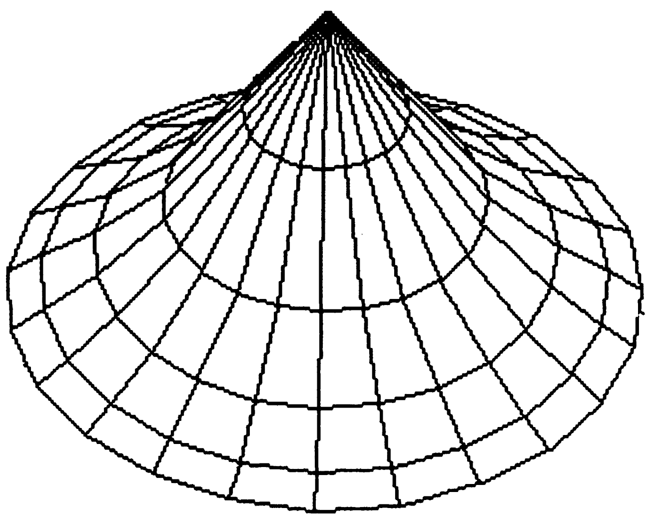


b) restored eECoG

Fig. 1. To be continued.

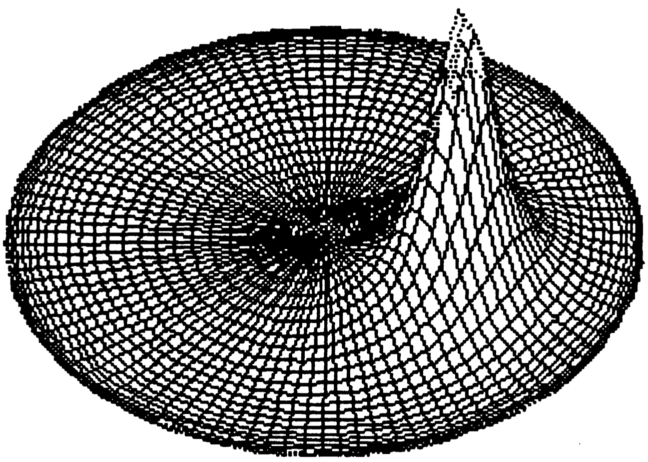


c) "real" EEG

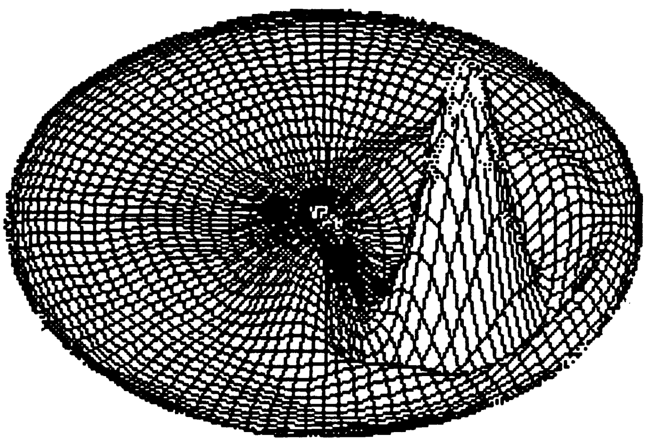


d) sampled EEG

Fig. 1. The patterns of potential when 1 RD is placed in the cortex; its coordinates are: $\theta_1 = 0$, $(\varphi_1 = 0)$.

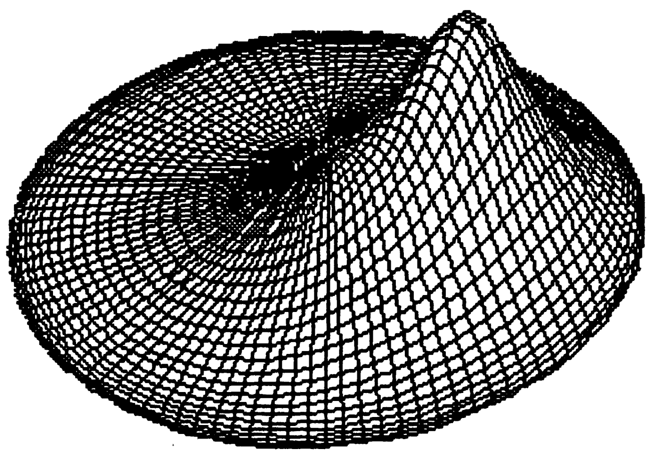


a) "real" eECoG

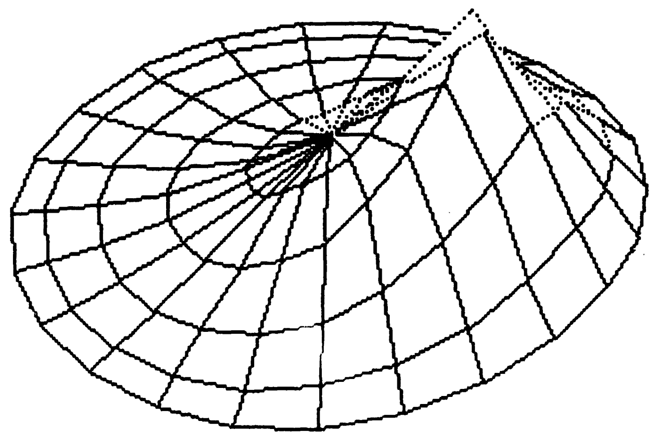


b) restored eECoG

Fig. 2. To be continued.

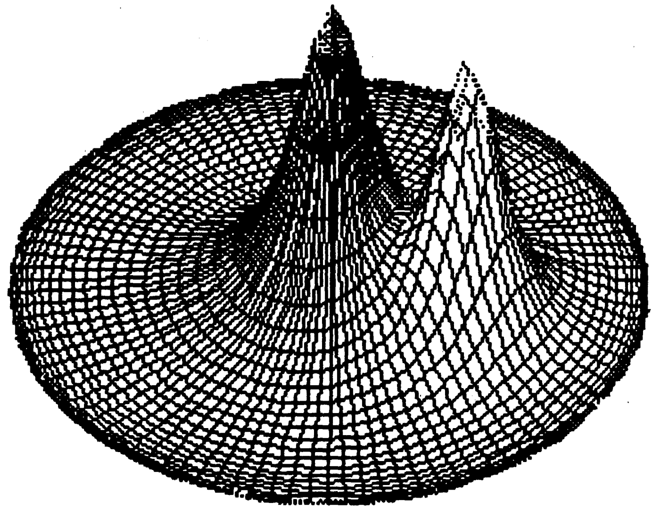


c) "real" EEG

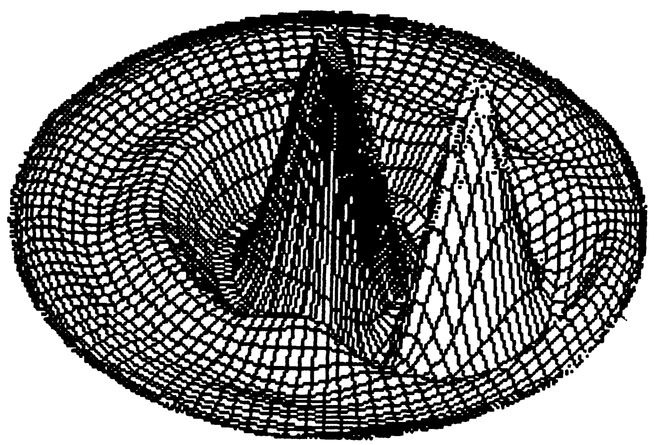


d) sampled EEG

Fig. 2. The patterns of potential when 1 RD is placed in the cortex; its coordinates are: $\theta_1 = 30^0$, $\varphi_1 = 0$.

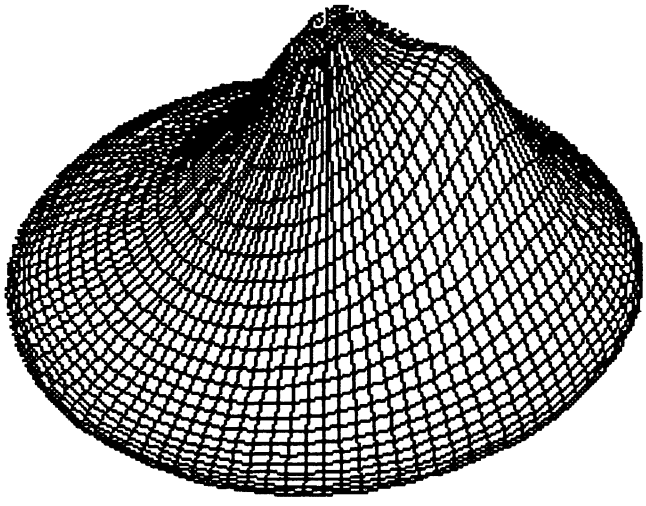


a) "real" eECoG

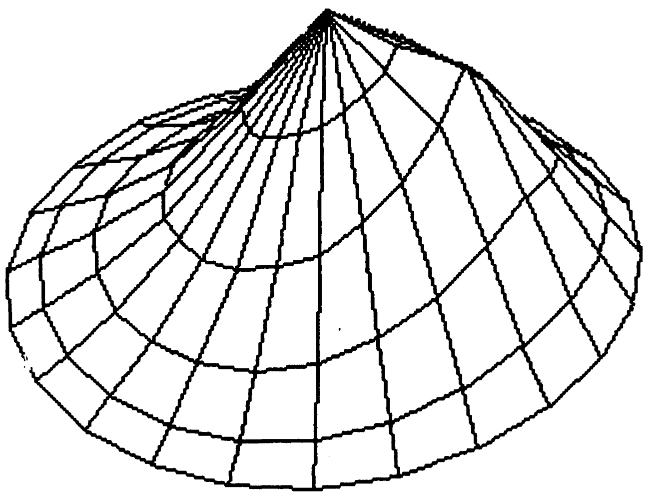


b) restored eECoG

Fig. 3. To be continued.

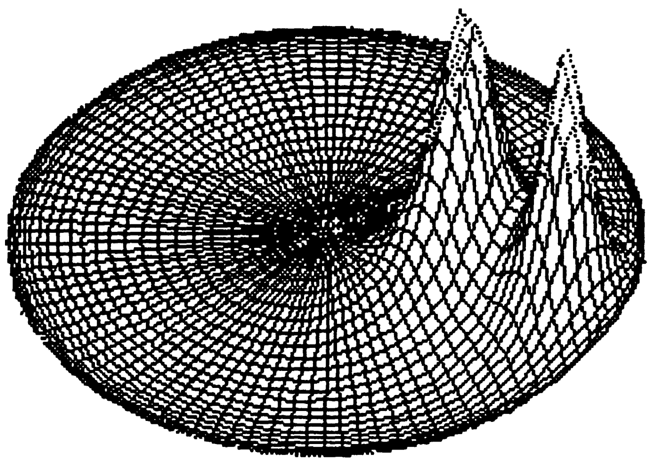


c) "real" EEG

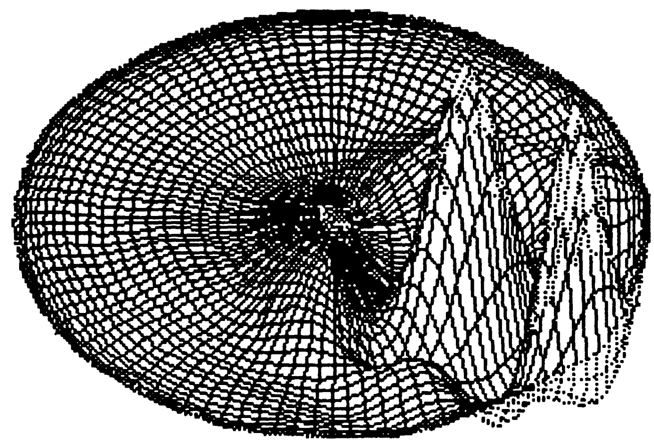


d) sampled EEG

Fig. 3. The patterns of potential when 2 RDs are placed in the cortex. Their coordinates are: 1) $\theta_1 = 0$; and 2) $\theta_2 = 30^0$; $\varphi_2 = 0$.

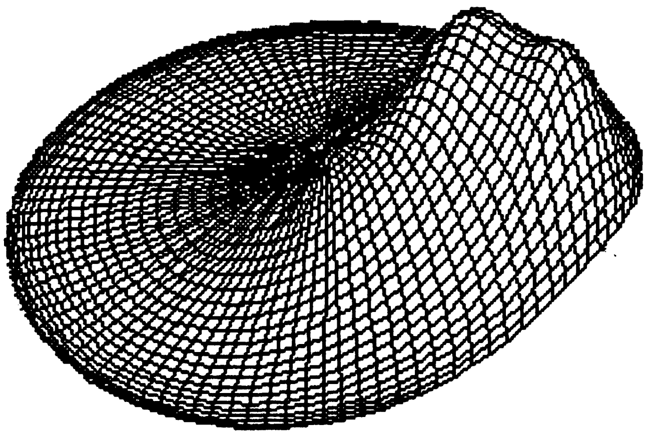


a) "real" eECoG

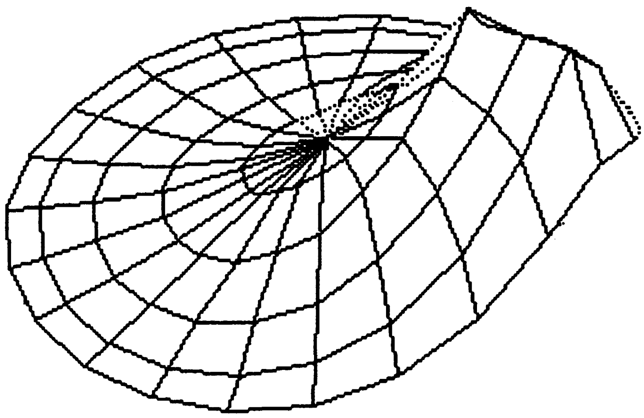


b) restored eECoG

Fig. 4. To be continued.

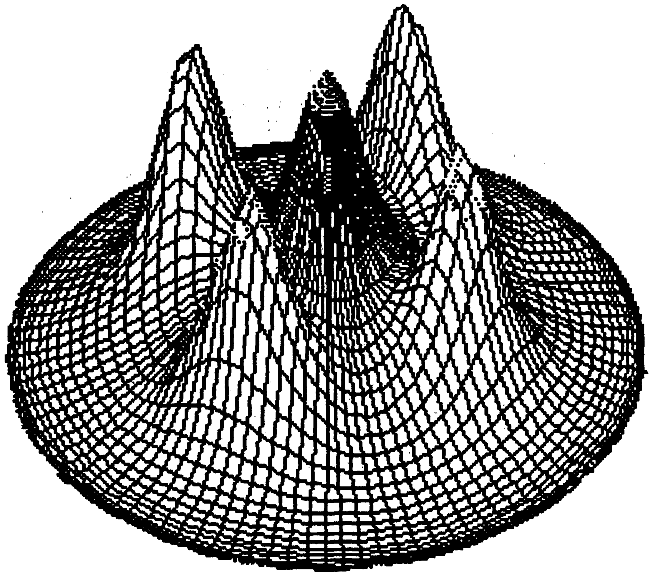


c) "real" EEG

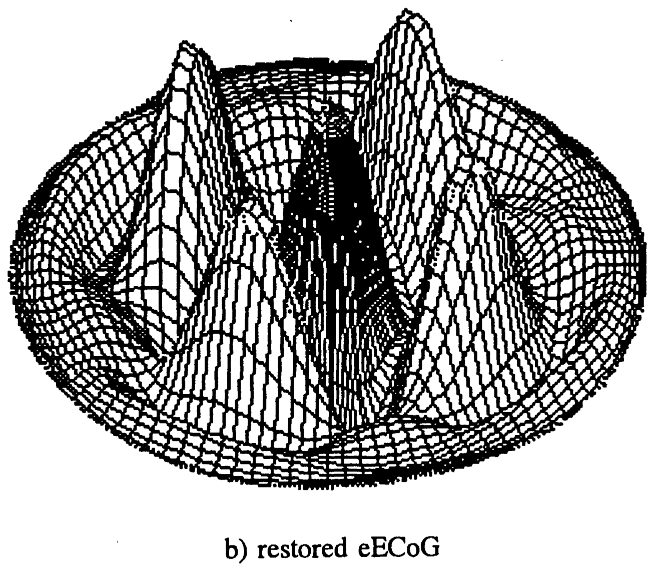


d) sampled EEG

Fig. 4. The patterns of potential when 2 RDs are placed in the cortex. Their coordinates are: 1) $\theta_1 = 30^0$; $\varphi_1 = 0$; and 2) $\theta_2 = 60^0$; $\varphi_2 = 0^0$.

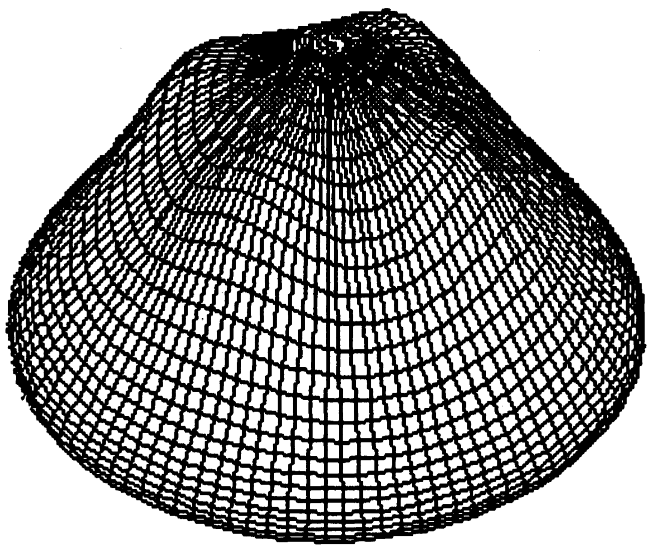


a) "real" eECoG

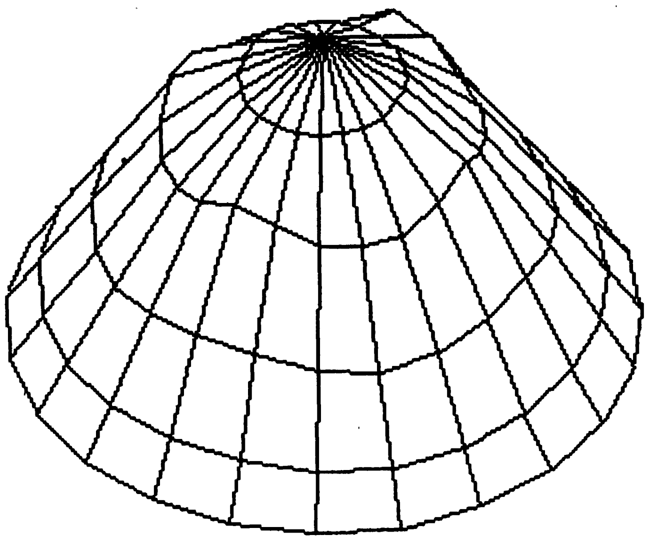


b) restored eECoG

Fig. 5. To be continued.



c) "real" EEG



d) sampled EEG

Fig. 5. The patterns of potential when 5 RDs are placed in the cortex. Their coordinates are: 1) $\theta_1 = 0$; 2) $\theta_2 = 30^0$; $\varphi_2 = 0$; 3) $\theta_3 = 30^2$; $\varphi_3 = 90^0$; 4) $\theta_4 = 30^0$; $\varphi_4 = 180^0$; 5) $\theta_5 = 30^0$; $\varphi_5 = 270^0$.

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