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STABLE FUZZY CLASSIFIERS

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Abstract. A possible interpretation, in terms of fuzzy classification models (fuzzy classifiers), of one of the general principles of choosing a scientific theory – a consistency principle – is considered. A concept of a stability measure of unsupervised fuzzy classifiers is introduced. A general scheme of computing the above measure is proposed. A concrete algorithm for implementing the general scheme and examples of its application are given.

Key words: unsupervised fuzzy classifier, self-consistency principle, stability property, admissible modifications of initial sample, relative stability index, absolute stability index.

Introduction. In this paper a problem of the interpretation of one of the general principles of the selection of a scientific theory – the consistency principle – is analyzed. More concretely, we consider this principle in the context of unsupervised fuzzy classification models (which we will also call fuzzy cluster-analysis models or fuzzy automatic classification models).

It has been previously noted (Vatlin, 1992), that the consequence of the primary character of the classification models with respect to other forms of a theoretical knowledge is necessity of the interpretation of the above mentioned principle as a self-consistency principle. However, peculiarity of the unsupervised classification models doesn't allow formulating the self-consistency principle in terms of a self-guessing property, on the analogy of what has been done in the case of supervised fuzzy classifiers (Vatlin, 1992).

The reason of the latter circumstance is stipulated by the absence of some learning information of extrapolation type in problems of (fuzzy) automatic classification. As a result the (fuzzy) cluster-analysis models cannot be compared according to the extent of recognizing by them one part of a learning set after training it on the other part of this set.

Practically the one possibility to evaluate the degree of self-consistency of a (fuzzy) classification model is related with the analysis of the results of using this model under analogous conditions. In problems of (fuzzy) cluster analysis such conditions are given by so-called admissible modifications of the initial sample X, that is, by such samples of experimental observations which are "similar" (in a predefined sense) to the initial sample (Moreau, Jain, 1987; Krasnoproshin, Vatlin, 1989).

It is reasonable to consider, as the most valid classification model, that one which has the maximum stability on all the set of admissible variations of the sample X. Really, random classifications of experimental observation will be, as a rule, unstable since they reflect the results of only one separate experiment. At the same time, if the classification model "catches" real regularities, present in the data being analyzed, it must repeat itself in a majority of experiments. As a result, the stability of such classification (on a set of admissible modifications of the initial sample) will always be higher than that in inadequate classifications.

The samples representing the admissible modifications of X may be formed in different ways. If a required additional information is available these samples may even "go beyond the limits" of the initial set of observations (Forsyth, 1980). However, in situations when it is difficult to obtain such additional information, some conclusions on the validity of one or another classification model of the initial sample can be reached by moving not outside the X set but "inside" it. Really, if some classification reflects real regularities of the experimental data X then this classification must also "work" on (representative) subsets from X. Similarly, the classifications "working" on the (representative) subsets from X, must differ slightly from (restrictions on these subsets) the valid classification of the whole set X. Stability of the valid classification model should be observed.

The stability of classifications, reflecting the objective regularities of the physical world represents one of the fundamental peculiarities of the human way of thinking. So, for example, it is impossible to imagine that the structure of the classification scheme in a sense of the number of classes of the Hubble extragalactic nebulas or Mendeleev Table structure prove to be unstable, dependent on the peculiarities of a concrete experiment.

As a consequence, while revealing the valid classification models, it is important to analyze the stability of classification properties expressed by some heuristic criterion with respect to "internal" admissible modifications of the initial sample X, rather than to estimate the degree of implementing such criterion on this sample.

In order to describe the general scheme of stability analysis of unsupervised fuzzy classification models, basic notions and definitions in Section 1 of the paper are given. In Section 2 an original algorithm for stability analysis of fuzzy classification model realizing the above scheme is considered.

1. Basic concepts and definitions. A general scheme of stability analysis of unsupervised fuzzy classification models. Let X be a protocol of experimental observations of a concrete object Z, such that $X = \{X^m\}$, $m \in M$, $X^m = \{x_1^m, x_2^m, \dots, x_n^m\}$, $x_i^m = (x_{i1}^m, x_{i2}^m, \dots, x_{ip}^m)$, $x_{ij}^m \in \mathbf{R}^1$, $i = \overline{1,n}$, $j = \overline{1,p}$, $m \in M$.

Further, if we deal with a fixed experiment with the above object we shall omit the superscript and write merely $X = \{x_1, x_2, \dots, x_n\}$, $x_i = (x_{i1}, x_{i2}, \dots, x_{ip})$, $i = \overline{1,n}$.

Let us denote by V_{nL} – the set of matrices of dimension $n \times L$ $(n, L \in N,\ l \geqslant 2,\ n > L)$, the elements of which are arbitrary real-valued numbers μ_{ij}. A set

$$V_{fL} = \Big\{ v \in V_{nL} \big| \mu_{ij} \in [0,1], \quad \forall i = \overline{1,n},\ \forall j = \overline{1,L}; \\ \sum_{j=1}^{L} \mu_{ij} = 1,\ \forall i = \overline{1,n}, \quad 0 < \sum_{i=1}^{n} \mu_{ij} < 1, \quad \forall j = \overline{1,L} \Big\} \tag{1}$$

will be called a fuzzy L-partitions space of the sample X.

The values μ_{ij} will be interpreted as a measure of belonging the i-th object of the initial sample X to the j-th fuzzy class F_j, $j = \overline{1, L}$.

The unsupervised fuzzy classifier is such a (finite) family of (admissible) mappings $G = \{g_L\}$, $L \in [L_0, L_1]$ that

$$g_L : X \to V_{fL}, \ L \in [L_0, L_1],$$

where L_0 and L_1 denote lower and upper bounds of a possible variation of classes in X (in the most general case $L_0 = 2$, $L_1 = n - 1$).

Let $\{X, A, U, \widetilde{G}, \Phi, \Psi\}$ be an ordered tuple, in which X is an initial sample; A, $\mathrm{Card} A = D$ is such a set of indices that $\{X^\alpha\}$, $\alpha \in A$ is a parametrical family of admissible modifications of sample X (without loss of generality we assume that $X^0 = X$); $U = \bigcup_{\alpha \in A} V_{fL}^\alpha$, where V_{fL}^α is a set, the elements of which are all possible fuzzy partitions of X^α into L classes, $\alpha \in A$.

A family of mappings $\widetilde{G} = \{g_L^\alpha\}_{L \in [L_0, L_1]}$, $g_L^\alpha : X^\alpha \to V_{fL}^\alpha$ will be called an extension of the unsupervised fuzzy classifier $G = \{g_L\}_{L \in [L_0, L_1]}$, $g_L : X \to V_{fL}$, and the mappings g_L^α – will be called α-cutoffs of the admissible mappings g_L, $\alpha \in A$.

Nonnegative real-valued functions $f_L^\alpha : V_{fL}^\alpha \to \mathbf{R}^1$ constituting set $\Phi = \{f_L^\alpha\}_{\alpha \in A}$, $L \in [L_0, L_1]$ will be defined as a heuristic validation criterion of α-cutoffs g_L^α.

The numbers

$$\beta_L^\alpha = f_L^\alpha \left(g_L^\alpha(X^\alpha) \right), \qquad \alpha \in A, \ L \in [L_0, L_1]$$

will be called relative stability indices of mappings g_L.

The value

$$\gamma_L = \Psi \left(\beta_L^1, \beta_L^2, \ldots, \beta_L^D \right)$$

will be called an absolute stability index for g_L, $L \in [L_0, L_1]$, where Ψ – is a nonnegative real-valued function, determined from the Cartesian product $\mathbf{R}^D$, $\Psi : \mathbf{R}^D \to \mathbf{R}^1$.

The admissible mapping g_L will be considered as a ε-stable (or merely stable) according to the system of heuristic validation criteria $\{f_L^\alpha\}$, $\alpha \in A$, $L \in [L_0, L_1]$ and according to the family of admissible modifications of the initial sample – $\{X^\alpha\}$, $\alpha \in A$, iff.

$$\gamma_L \leqslant \varepsilon.$$

Let X be a fixed initial sample, and $G = \{g_L\}_{L \in [L_0, L_1]}$ is an unsupervised fuzzy classifier such, that $g_L : X \to V_{fL}$. Let us denote by W_ε a set of all stable admissible mappings realized by the classifier G. If $\mathrm{Card}W_\varepsilon = 1$ then the problem of analyzing the stability of the unsupervised fuzzy classification model is solved uniquely. In situations, when $\mathrm{Card}W_\varepsilon > 1$ we have a partial solution of the above problem consisting in the possibility of ordering the admissible stable mappings by the values of their absolute criteria

$$g_{L1} < g_{L2} < g_{L3} \cdots < g_{Lm_\varepsilon} \iff \gamma_{L1} \leqslant \gamma_{L2} \leqslant \cdots \leqslant \gamma_{Lm_\varepsilon}, \quad m_\varepsilon = \mathrm{Card}W_\varepsilon .$$

If $\mathrm{Card}W_\varepsilon = 0$ the problem has no solution. In this case further actions of stability analysis methods of the unsupervised classification models should be directed to the verification and refinement of a priori representation of the initial sample structure and of the character of heuristic validation criteria for α-cutoffs g_L^α.

2. Algorithm for fuzzy classifier analysis. Let, as a family of admissible modifications of the initial sample X, a set of its pseudocopies $S = \{X^1, X^2, \ldots, X^D\}$ be specified. We shall define the structure of the S set so that in any X^α, $\alpha = \overline{1,D}$ a small part of objects from X is missing. The choice of these objects is made in a random manner and it doesn't depend on the way of forming the other elements of the S set.

We'll denote by $v_{fL} = \|\mu_{ij}\|$, $i = \overline{1,n}$, $j = \overline{1,L}$ a matrix of an arbitrary fuzzy partition of X into L classes. Let $\mu_{ij}|X^\alpha$ be restrictions of the functions μ_{ij} on sets X^α, and $v_{\alpha L}$ be matrices of a fuzzy partition corresponding to the above restrictions. We'll also denote by v_L^α the matrix, formed as a result of an arbitrary "immediate" partition of the X into L fuzzy classes, $\alpha = \overline{1,D}$, $L \in [L_0, L_1]$.

On elements of sets $K_L^\alpha = V_{\alpha L} \cup V_L^\alpha$, $\alpha = \overline{1,D}$, $L \in [L_0, L_1]$, where $V_{\alpha L}$ and V_L^α are spaces of all possible matrices $v_{\alpha L}$ and v_L^α respectively, we shall define a function of distance (Mirkin and Tcherny, 1970)

$$\rho(v_{\alpha L}, v_L^\alpha) = \frac{1}{2} \sum_{t,g=1}^{M\alpha} |r_{tg} - s_{tg}|, \tag{2}$$

where by means of r_{tg} and s_{tg} we denote elements – indicators R and S of matrices $v_{L\alpha}$ and v_L^α, $M_\alpha = \text{Card}X^\alpha$, $\alpha = 1, D$, $L \in [L_0, L_1]$. (If $V, W \in K_L^\alpha$ are such that $W, V \in V_{L\alpha}$ or $W, V \in V_L^\alpha$ then relation (1) is valid as before).

Let us choose as heuristic estimate of validation of restrictions $\mu_{ij}|X^\alpha$ the values of the function ρ associated with them, i.e., we put

$$\beta_L^\alpha = f_L^\alpha(g_L^\alpha(X^\alpha)) = \rho(v_{\alpha l}, v_L^\alpha), \qquad \alpha = \overline{1, D},\ L \in [L_), L_1].$$

Taking into account the way of constructing the pseudocopies X^α, $\alpha = \overline{1, D}$, the value of β_L^α may be considered as a value of random variable β_L

$$\beta_L : V_{fL} \to \mathbf{R}^1.$$

Then, at the fixed unsupervised classifier $G = \{g_L\}_{L \in [L_0, L_1]}$, there will be D individual values of the random variable β_L, $L \in [L_0, L_1]$, corresponding to each admissible mapping g_L.

Let us consider as stable only those admissible mappings g_L for which $\gamma = \Psi(\beta_L^1, \beta_L^2, \ldots, \beta_L^D) = \frac{1}{D} \sum_{\alpha=1}^{D} \beta_L^\alpha \leqslant \varepsilon$. As a result, the sequence of operations, implementing the general scheme of stability analysis of the unsupervised fuzzy classification models may be structured as follows.

V – Algorithm

Step 1. By means of admissible mappings $g_L \in G$, $L \in [L_0, L_1]$ classify the initial sample X into L fuzzy classes. Construct the matrices of fuzzy classification
$v_{fL} = ||\mu_{ij}||$, $\quad i = \overline{1, n}$, $j = \overline{1, L}$, $L \in [L_0, L_1]$.

Step 2. Form the matrices $v_{\alpha L}$, corresponding to the restrictions of functions μ_{ij} on subsets X^α, $\alpha = \overline{1, D}$, $L \in [L_0, L_1]$.

Step 3. Classify each of the pseudocopies of the initial sample X into L fuzzy classes with the help of α-cutoffs of admissible mappings $g_L - g_L^\alpha$. Construct matrices of fuzzy partitions v_L^α, $\alpha = \overline{1, D}$, $L \in [L_0, L_1]$.

Step 4. Calculate the values of relative stability indices of admissible mappings g_L,

$\beta_L^\alpha = \rho(v_{\alpha L}, v_L^\alpha)$, $\qquad \alpha = \overline{1, D}$, $L \in [L_0, L_1]$.

Step 5. Calculate the values of the absolute stability indices for g_L, $\gamma_L = \frac{1}{D} \sum_{\alpha=1}^{D} \beta_L^\alpha$.

Step 6. Verify the stability of the admissible mappings g_L by the system of heuristic validation criteria – $\{f_L^\alpha\}$, $L \in [L_0, L_1]$.

$$\gamma_L \leqslant \varepsilon.$$

Step 7. If Card$W_\varepsilon > 1$ or Card$W_\varepsilon = 1$, then form either complete or partial solution of the problem of validity analysis of the unsupervised (fuzzy) classification model $G = \{g_L\}$ $L \in [L_0, L_1]$.

Else, (Card$W_\varepsilon = 0$) go to the revision of the a priori representation of the character of the heuristic criteria of validation $\{f_L^\alpha\}$, $\alpha = \overline{1, D}$, $L \in [L_0, L_1]$ and the structure of the initial sample X.

Conclusion. The study carried out on experimental and model problems confirm the efficiency of the V algorithm. So, for example, while revealing, by means of the family of SEMI-FUZZY algorithms (Selim, Ismail, 1983), the most valid number of classes in the Fisher sample, the admissible mapping with $L^* = 3$ has been identified unconditionally. This corresponds to the true structure of the set being analyzed (Bezdek, 1975). At the same time, the application of conventional algorithms for validation analysis of various classifications of this sample, results in a wrong model with $L^* = 2$ as the most "preferable" (Bezdek, 1975). Thus, the application of the V algorithm allows us to cope with the situations in which the classical algorithm doesn't work.

The V algorithm shows a good ability for revealing the true number of classes in solving problems, associated with the identification of non-spherical and linearly-unseparable classes (Zahn, 1971).

In essence, the proposed algorithm takes on an intermediate position between bootstrap and "geometrical" algorithms, which have already been tested while solving a number of practical problems (Krasnoprshin, Vatlin, 1989; Vatlin, Moroz, 1988). The stable classification models formed within the framework of the V algorithm are a suitable tool for generalizing an experimental information and a means of analyzing and forecasting processes in complex engineering systems.

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