CODAS Method for Multiple Attribute Group Decision Making Under 2-Tuple Linguistic Neutrosophic Environment

Ping WANG\(^1\), Jie WANG\(^2\), Guiwu WEI\(^2\), Jiang WU\(^3\), Cun WEI\(^3\), Yu WEI\(^4\),∗

\(^1\)Institute of Technology, Sichuan Normal University, Chengdu 610101, PR China
\(^2\)School of Business, Sichuan Normal University, Chengdu 610101, PR China
\(^3\)School of Statistics, Southwestern University of Finance and Economics, Chengdu 611130, PR China
\(^4\)School of Finance, Yunnan University of Finance and Economics, Kunming 650221, PR China
e-mail: weiyusy@126.com

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Abstract. In this paper, we present the 2-tuple linguistic neutrosophic CODAS model based on the traditional fuzzy CODAS (combinative distance-based assessment) model and some fundamental theories of 2-tuple linguistic neutrosophic information. Firstly, we briefly review the definition of 2-tuple linguistic neutrosophic sets (2TLNSs) and introduce the score function, the accuracy function, operation laws and some aggregation operators of 2TLNNs. Then, the calculation steps of traditional fuzzy CODAS model are briefly presented. Furthermore, by combining the traditional fuzzy CODAS model with 2TLNNs information, the 2-tuple linguistic neutrosophic CODAS model is established and the computing steps for multiple attribute group decision making (MAGDM) are simply depicted. Our presented model is more accurate and effective for considering the combinative form of two distance measurements, including fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED). Finally, a numerical example for safety assessment of construction project has been given to illustrate this new model and some comparisons between 2TLNNs CODAS model and two 2TLNNs aggregation operators are also made to further illustrate the advantages of the new method.

Key words: multiple attribute group decision making (MAGDM) problems, 2-tuple linguistic neutrosophic sets (2TLNSs), CODAS model, 2-tuple linguistic neutrosophic number weighted average (2TLNNWA) operators, 2-tuple linguistic neutrosophic number weighted geometric (2TLNNWG) operators, 2TLNNs CODAS model, construction project.

1. Introduction

Due to the indeterminacy of DM’s and the decision-making issues, we cannot always give accurate evaluation values of alternatives to select the best project in real MADM problems (Wang et al., 2019a; Wang, 2019; Wu et al., 2019a, 2019b). To conquer this disad-
vantage, fuzzy set theory which was defined by Zadeh (1965) originally used the membership function to describe the estimation results rather than exact real-numbers. Atanassov (1986, 1989) presented another measurement index which named non-membership function as a complement. Smarandache (1999, 2003) introduced the neutrosophic set (NS). Then, Wang et al. (2010) introduced the definition and some operational rules of single-valued neutrosophic sets (SVNSs), where the evaluation information is depicted by truth membership degree, indeterminacy membership degree and falsity membership degree. Obviously, the SVNSs and their extensions can describe the fuzzy and uncertainty degree of a decision maker and can be more suitable for actual multiple attribute decision making problems (Wang et al., 2019c, 2019e, 2019h, 2019j; Wei et al., 2019e, 2019f).

However, the SVNSs can only represent quantitative decision making information and fail to depict the qualitative decision making information. As we all know, the 2-tuple linguistic set (2TLS) (Herrera and Martinez, 2001) can eliminate this limitation. In order to consider both qualitative and quantitative decision making information, Wang et al. (2018a) defined the 2-tuple linguistic neutrosophic sets (2TLNSs), where the truth membership function, indeterminacy membership function and falsity membership function are presented by 2TLNNs. Thus, the 2TLNNs are considered a useful tool to deal with practical MADM applications. In real decision making problems, finding a good way to denote evaluation information is only one aspect (Deng and Gao, 2019; Gao et al., 2019; Li and Lu, 2019; Lu and Wei, 2019). It is also important to know how to deal with this information. To date, The CODAS (combinative distance-based assessment) method, which was originally defined by Keshavarz Ghorabaee et al. (2016), use the combinative form of two distance measurements, including Euclidean and Taxicab distances, which present accurate values to compute the assessment results of alternatives. However, this model cannot be applied in fuzzy environment. To overcome this disadvantage, Keshavarz Ghorabaee et al. (2017) extended the CODAS method to fuzzy environment and proposed a fuzzy CODAS model which used fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED) rather than the crisp distances. Thus, we can easily find that CODAS method can handle fuzzy decision making problems effectively.

Motivated by the 2-tuple linguistic neutrosophic sets (2TLNSs) and the traditional CODAS model, the research question and goal of this paper is to build an extended CODAS model to deal with the 2-tuple linguistic neutrosophic decision making problems. The main novelty and contribution of this paper is the proposition of the 2TLN CODAS model. On the one hand, the 2-tuple linguistic neutrosophic number can express the qualitative and quantitative fuzzy decision making information, on the other hand, the CODAS model has important merits mentioned above. Thus, we can derive accuracy assessment results for construction project by utilizing the 2TLN CODAS model. In order to elaborate the process of putting forward the 2TLN CODAS model, this article is structured in the following way: some related work about 2TLNSs and the CODAS method are given in Section 2. The definition, the score function, the accuracy function, operation rules and some aggregation operators of 2TLNNs are briefly introduced in Section 3. The computing steps of traditional fuzzy CODAS model are briefly presented in Section 4. By combining the fuzzy traditional CODAS model with 2TLNNs information, the 2-tuple linguistic neutrosophic CODAS model is established and the computing steps for MAGDM
problems are simply depicted in Section 5. A numerical example for safety assessment of construction project is given to illustrate this new model and some comparisons between 2-tuple linguistic neutrosophic CODAS model and two 2TLNNs aggregation operators are also made to further illustrate advantages of the new method in Section 6. Section 7 gives some conclusions of our work.

2. Related Work

Previously, a lot of decision-making models such as the VIKOR method (He et al., 2019b; Opricovic and Tzeng, 2004; Wang et al., 2018b), the ELECTRE method (Rashid et al., 2018), the TOPSIS method (Chen, 2000; Lu et al., 2019b), the PROMETHEE method (Balali et al., 2014), the MABAC method (Pamucar and Cirovic, 2015), the EDAS method (Keshavarz Ghorabaee et al., 2015; Wang et al., 2019i; Zhang et al., 2019) and the TODIM method (Gomes and Lima, 1979) have been studied extensively by numerous researchers. Compared with the existing literature, the CODAS model has the advantage of taking the combinative form of ED and HD into account with respect to the intangibility of decision maker (DM) and the uncertainty of decision-making environment to obtain more accurate and effective aggregation results. Since the CODAS method was proposed, a large number of scholars have studied it. Pamucar et al. (2018) presented a linguistic neutrosophic CODAS model. Badi et al. (2018) studied the site selection of desalination plant in Libya by using CODAS method. Bolturk (2018) proposed an extended CODAS model to deal with Pythagorean fuzzy decision making problems and studied its application to supplier selection. Based on interval-valued intuitionistic fuzzy information, Bolturk and Kahraman (2018a) developed a novel CODAS model. To handle renewable energy selection, Bolturk and Karasan (2018b) proposed the interval-valued neutrosophic CODAS model. According to novel information measure, Peng and Garg (2018) studied the CODAS method and presented some novel algorithms under the interval-valued fuzzy soft set. Ren (2018) established the intuitionistic fuzzy CODAS model for MADM. Karasan et al. (2019) developed an integrated methodology based on the neutrosophic CODAS model.

As for the 2-tuple linguistic neutrosophic sets, based on the Hamy mean (HM) operator, Wu et al. (2018b) proposed some 2-tuple linguistic neutrosophic Hamy mean (2TLNNHM) operators and 2-tuple linguistic neutrosophic dual Hamy mean (2TLNNDHM) operators to fuse 2TLNNs. Wang et al. (2019b) developed some 2-tuple linguistic neutrosophic Muirhead mean (2TLNNMM) operators for MADM. Considering the Dombi operation laws and BM operators, Wei et al. (2019b) presented some novel aggregation operators. Wang et al. (2019h) combined the EDAS method with the 2-tuple linguistic neutrosophic set to build an extended EDAS model for MADM. Based on the single-valued neutrosophic 2-tuple linguistic set, Wang et al. (2019d) proposed some Muirhead mean (MM) aggregation operators, Wu et al. (2018a) defined some Hamcher aggregation operators, Ju et al. (2018) extended it to interval-valued environment and developed some MSM operators. Wang et al. (2018b) proposed the 2-tuple linguistic neutrosophic TODIM model. Thereafter, the 2TLNSs have been widely studied in MADM issues.
However, it is clear that there are no studies about the CODAS model with 2TLNNs information. Some scholars studied the CODAS model under neutrosophic and linguistic neutrosophic environment, but both of them cannot represent decision information in a convenient way. At the same time, in the area of 2TLNSs, the research mainly focuses on the aggregation operators, but there is a lack of research on 2TLN models. Hence, it is necessary to discuss the 2-tuple linguistic neutrosophic CODAS model. The goal of this paper is to develop a novel CODAS method based on the conventional CODAS model and 2-tuple linguistic neutrosophic information to study MADM problems more effectively.

3. Preliminaries

3.1. 2-Tuple Linguistic Neutrosophic Sets

Wang et al. (2018a) initially proposed the 2-tuple linguistic neutrosophic sets (2TLNSs), which consider the important characteristics of 2-tuple linguistic variables and single-valued neutrosophic sets (SVNSs), hence, can be more effective and accurate to evaluate the alternatives in multiple attribute decision making problems. To combine the 2TLSs and SVNSs, the definition of 2TLNSs can be expressed as follows.

**Definition 1.** Let $\delta_1, \delta_2, \ldots, \delta_k$ be a linguistic term set. Any label $\delta_i$ shows a possible linguistic scale, and $\delta = \{\delta_0 = \text{exceedingly terrible}, \delta_1 = \text{very terrible}, \delta_2 = \text{terrible}, \delta_3 = \text{medium}, \delta_4 = \text{well}, \delta_5 = \text{very well}, \delta_6 = \text{exceedingly well}\}$, then we can describe the 2TLNSs as:

$$\delta = \{(s_t, \alpha_1), (s_i, \beta), (s_f, \chi)\},$$

where $\Delta^{-1}(s_t, \alpha_1), \Delta^{-1}(s_i, \beta)$ and $\Delta^{-1}(s_f, \chi) \in [0, k]$ represent the truth membership function, the indeterminacy membership function and the falsity membership function, which are expressed by 2-tuple linguistic variables and satisfy the condition $0 \leq \Delta^{-1}(s_t, \phi) + \Delta^{-1}(s_f, \varphi) + \Delta^{-1}(s_f, \gamma) \leq 3k$.

**Definition 2 (See Wang et al., 2018a).** Let $\delta_1 = \langle(s_{t_1}, \alpha_{1_1}), (s_{i_1}, \beta_1), (s_{f_1}, \chi_1)\rangle$ and $\delta_2 = \langle(s_{t_2}, \alpha_{2_1}), (s_{i_2}, \beta_2), (s_{f_2}, \chi_2)\rangle$ be two 2-tuple linguistic neutrosophic numbers (2TLNNs), the operation formula of them can be defined:

$$\begin{align*}
\text{1) } \delta_1 \oplus \delta_2 &= \left\{ \begin{array}{l}
\Delta(k \left( \frac{\Delta^{-1}(s_{t_1}, \alpha_{1_1})}{k} + \frac{\Delta^{-1}(s_{t_2}, \alpha_{2_1})}{k} \right) + \frac{\Delta^{-1}(s_{t_2}, \alpha_{2_1})}{k} - \frac{\Delta^{-1}(s_{t_1}, \alpha_{1_1})}{k}, \frac{\Delta^{-1}(s_{t_2}, \alpha_{2_1})}{k} \right), \\
\Delta(k \left( \frac{\Delta^{-1}(s_{i_1}, \beta_1)}{k} \cdot \frac{\Delta^{-1}(s_{i_2}, \beta_2)}{k} \right)), \\
\Delta(k \left( \frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k} \cdot \frac{\Delta^{-1}(s_{f_2}, \chi_2)}{k} \right)) \right\};
\end{array} \right.
\end{align*}$$

$$\begin{align*}
\text{2) } \delta_1 \otimes \delta_2 &= \left\{ \begin{array}{l}
\Delta(k \left( \frac{\Delta^{-1}(s_{t_1}, \alpha_{1_1})}{k} \cdot \frac{\Delta^{-1}(s_{t_2}, \alpha_{2_1})}{k} \right)), \\
\Delta(k \left( \frac{\Delta^{-1}(s_{i_1}, \beta_1)}{k} \cdot \frac{\Delta^{-1}(s_{i_2}, \beta_2)}{k} \right) - \frac{\Delta^{-1}(s_{i_1}, \beta_1)}{k} - \frac{\Delta^{-1}(s_{i_2}, \beta_2)}{k}), \\
\Delta(k \left( \frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k} \cdot \frac{\Delta^{-1}(s_{f_2}, \chi_2)}{k} \right) - \frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k} - \frac{\Delta^{-1}(s_{f_2}, \chi_2)}{k}) \right\};
\end{array} \right.
\end{align*}$$
According to Definition 2, it is clear that the operation laws have the following properties.

\[
\begin{align*}
\delta_1 \oplus \delta_2 &= \delta_2 \oplus \delta_1, \quad \delta_1 \otimes \delta_2 = \delta_2 \otimes \delta_1, \quad ((\delta_1)^{\lambda_1})^{\lambda_2} = (\delta_1)^{\lambda_1 \lambda_2}, \\
\lambda(\delta_1 \oplus \delta_2) &= \lambda\delta_1 \oplus \lambda\delta_2, \quad (\delta_1 \otimes \delta_2)^{\lambda} = (\delta_1)^{\lambda} \otimes (\delta_2)^{\lambda}, \\
\lambda_1\delta_1 \oplus \lambda_2\delta_1 &= (\lambda_1 + \lambda_2)\delta_1, \quad (\delta_1)^{\lambda_1} \otimes (\delta_1)^{\lambda_2} = (\delta_1)^{(\lambda_1 + \lambda_2)}.
\end{align*}
\]

**Definition 3** (See Wang et al., 2018b). Let \(\delta = (s_t, \alpha), (s_i, \beta), (s_f, \chi)\) be a 2TLNN, the score and accuracy functions of \(\delta\) can be expressed:

\[
\begin{align*}
s(\delta) &= \frac{(2k + \Delta^{-1}(s_t, \alpha) - \Delta^{-1}(s_i, \beta) - \Delta^{-1}(s_f, \chi))}{3k}, \quad s(\delta) \in [0, 1], \\
 h(\delta) &= \frac{1}{k} (\Delta^{-1}(s_t, \alpha) - \Delta^{-1}(s_f, \chi)), \quad h(\delta) \in [-1, 1].
\end{align*}
\]

For two 2TLNNs \(\delta_1\) and \(\delta_2\), based on Definition 3, then

1. if \(s(\delta_1) < s(\delta_2)\), then \(\delta_1 < \delta_2\);
2. if \(s(\delta_1) > s(\delta_2)\), then \(\delta_1 > \delta_2\);
3. if \(s(\delta_1) = s(\delta_2), \quad h(\delta_1) < h(\delta_2), \quad \delta_1 < \delta_2\);
4. if \(s(\delta_1) = s(\delta_2), \quad h(\delta_1) > h(\delta_2), \quad \delta_1 > \delta_2\);
5. if \(s(\delta_1) = s(\delta_2), \quad h(\delta_1) = h(\delta_2), \quad \delta_1 = \delta_2\).

3.2. The Distance Measurement of 2TLNNs

**Definition 4.** Let \(\delta_1 = \{(s_{t1}, \alpha_1), (s_{i1}, \beta_1), (s_{f1}, \chi_1)\}\) and \(\delta_2 = \{(s_{t2}, \alpha_2), (s_{i2}, \beta_2), (s_{f2}, \chi_2)\}\) be two 2TLNNs, then we can get the normalized Hamming distance:

\[
d^H(\delta_1, \delta_2) = \frac{1}{3} \left( \frac{|\Delta^{-1}(s_{t1}, \alpha_1) - \Delta^{-1}(s_{t2}, \alpha_2)|}{k} + \frac{|\Delta^{-1}(s_{i1}, \beta_1) - \Delta^{-1}(s_{i2}, \beta_2)|}{k} + \frac{|\Delta^{-1}(s_{f1}, \chi_1) - \Delta^{-1}(s_{f2}, \chi_2)|}{k} \right). 
\]

\[(7)\]
Definition 5. Let \( \delta_1 = \{ (s_{t1}, \alpha_1), (s_{i1}, \beta_1), (s_{f1}, \chi_1) \} \) and \( \delta_2 = \{ (s_{t2}, \alpha_2), (s_{i2}, \beta_2), (s_{f2}, \chi_2) \} \) be two 2TLNNs, then we can get the normalized Euclidean distance:
\[
d_E(\delta_1, \delta_2) = \sqrt{\frac{1}{3} \left( \frac{\Delta^{-1}(s_{t1}, \alpha_1) - \Delta^{-1}(s_{t2}, \alpha_2)}{k} \right)^2 + \left( \frac{\Delta^{-1}(s_{f1}, \chi_1) - \Delta^{-1}(s_{f2}, \chi_2)}{k} \right)^2 }.
\]

3.3. The 2TLNNWA and 2TLNNWG Operators

Definition 6 (See Wang et al., 2018a). Let \( \delta_j = \{ (s_{tj}, \alpha_j), (s_{ij}, \beta_j), (s_{fj}, \chi_j) \} \), \( (j = 1, 2, \ldots, n) \) be a set of 2TLNNs, the 2TLNNWA and 2TLNNWG operators can be presented:
\[
2\text{TLNNWA}(\delta_1, \delta_2, \ldots, \delta_n) = w_1 \delta_1 \oplus w_2 \delta_2 \otimes \cdots \otimes w_n \delta_n = \bigoplus_{j=1}^{n} w_j \delta_j
\]
\[
= \left\{ \Delta\left(k \left( 1 - \prod_{j=1}^{n} \left( 1 - \frac{\Delta^{-1}(s_{tj}, \alpha_j)}{k} \right)^{w_j} \right) \right) \right\}, \Delta\left(k \prod_{j=1}^{n} \left( \frac{\Delta^{-1}(s_{fj}, \chi_j)}{k} \right)^{w_j} \right).
\]

and
\[
2\text{TLNNWG}(\delta_1, \delta_2, \ldots, \delta_n)
\]
\[
= (\delta_1)^{w_1} \otimes (\delta_2)^{w_2} \otimes \cdots \otimes (\delta_n)^{w_n} = \bigotimes_{j=1}^{n} (\delta_j)^{w_j}
\]
\[
= \left\{ \Delta\left(k \prod_{j=1}^{n} \left( \frac{\Delta^{-1}(s_{fj}, \chi_j)}{k} \right)^{w_j} \right) \right\}, \Delta\left(k \left( 1 - \prod_{j=1}^{n} \left( 1 - \frac{\Delta^{-1}(s_{fj}, \chi_j)}{k} \right)^{w_j} \right) \right).
\]

where \( w_j \) is weighting vector of \( \delta_j \), \( j = 1, 2, \ldots, n \), which satisfies \( 0 \leq w_j \leq 1 \), \( \sum_{j=1}^{n} w_j = 1 \).

4. The Traditional Fuzzy CODAS Model

The CODAS (combinative distance-based assessment) method, which was originally defined by Keshavarz Ghorabaee et al. (2016), uses the combinative form of two distance measurements, including Euclidean and Taxicab distances, which present accurate values to compute the assessment results of alternatives. However, this model cannot be applied in fuzzy environment. To overcome this disadvantage, Keshavarz Ghorabaee et
al. (2017) extended the CODAS method to fuzzy environment and proposed the fuzzy CODAS model which used fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED) rather than the crisp distances. Suppose there are alternatives \( \{\phi_1, \phi_2, \ldots, \phi_m\} \), \( n \) attributes \( \{O_1, O_2, \ldots, O_n\} \) and \( t \) experts \( \{d_1, d_2, \ldots, d_t\} \), then the decision making steps are expressed as follows.

**Step 1.** Construct the evaluation matrix \( R = [\phi_{ij}]_{m \times n}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) and calculate the average results matrix \( r = [\phi_{ij}]_{m \times n}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) which can be depicted as follows:

\[
R = \begin{bmatrix}
\phi^1_{11} & \phi^1_{12} & \cdots & \phi^1_{1n} \\
\phi^2_{11} & \phi^2_{12} & \cdots & \phi^2_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
\phi^m_{11} & \phi^m_{12} & \cdots & \phi^m_{1n}
\end{bmatrix} = 
\begin{bmatrix}
O_1 & O_2 & \cdots & O_n \\
\phi^1_{11} & \phi^1_{12} & \cdots & \phi^1_{1n} \\
\phi^2_{11} & \phi^2_{12} & \cdots & \phi^2_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
\phi^m_{11} & \phi^m_{12} & \cdots & \phi^m_{1n}
\end{bmatrix},
\]

\( \phi_{ij}^t = \phi_{ij}^1 \oplus \phi_{ij}^2 \oplus \cdots \oplus \phi_{ij}^t, \) \( i = 1, 2, \ldots, m, j = 1, 2, \ldots, n \) denotes the evaluation information of alternative \( \phi_i \) \( (i = 1, 2, \ldots, m) \) on attribute \( O_j \) \( (j = 1, 2, \ldots, n) \) by expert \( d^t \) and \( \phi_{ij} \) means the average values of alternative \( \phi_i \) with respect to attribute \( O_j \) \( (j = 1, 2, \ldots, n) \).

**Step 2.** Obtain the attribute’s fuzzy weighting vector \( W^t \) which is given by each expert with respect to all attributes and compute the average fuzzy weighting vector \( W \) as follows:

\[
W^t = [w^t_j]_{1 \times n},
\]

\[
W = [w_j]_{1 \times n},
\]

\[
w_j = w^1_j \oplus w^2_j \oplus \cdots \oplus w^t_j,
\]

where \( w^t_j \) denotes the fuzzy weight of attribute \( O_j \) \( (j = 1, 2, \ldots, n) \) by expert \( d^t \) and \( w_j \) \( (j = 1, 2, \ldots, n) \) means the average fuzzy weight values of a attribute \( O_j \) \( (j = 1, 2, \ldots, n) \).
Step 3. Normalize the average results matrix \( r = [\phi_{ij}]_{m \times n}, \ i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n \) based on the type of each attributes using the following formulae.

For benefit attributes:

\[
N_{ij} = \phi_{ij} / \max_i(\phi_{ij}), \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.
\]  

(17)

For cost attributes:

\[
N_{ij} = 1 - \phi_{ij} / \max_i(\phi_{ij}), \quad i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n.
\]  

(18)

Step 4. According to the normalized average matrix \( N_{ij} (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \) and average fuzzy weighting vector \( w_j (j = 1, 2, \ldots, n) \), the fuzzy weighted normalized average matrix \( WN_{ij} (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \) can be computed as:

\[
WN_{ij} = w_j \otimes N_{ij} \quad (i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n).
\]  

(19)

Step 5. Determine the fuzzy negative solution (NS) based on the equation (20):

\[
NS_j = \min_i(WN_{ij}) \quad (i = 1, 2, \ldots, m, \ j = 1, 2, \ldots, n).
\]  

(20)

Step 6. Calculate the fuzzy weighted Hamming distance \( HD_i \) and fuzzy weighted Euclidean distance \( ED_i \) between each alternatives and the negative solution (NS) according to Definition 4 and Definition 5:

\[
HD_i = \sum_{j=1}^{n} d^H(WN_{ij}, NS_j),
\]  

(21)

\[
ED_i = \sum_{j=1}^{n} d^E(WN_{ij}, NS_j).
\]  

(22)

Step 7. Determine the relative assessment (RA) matrix which is presented as follows:

\[
RA = [p_{il}]_{m \times m},
\]  

(23)

\[
p_{il} = (ED_i - ED_l) + (\lambda(ED_i - ED_l) \times (HD_i - HD_l)),
\]  

(24)

where \( i, l = 1, 2, \ldots, m \) and \( \lambda \) is a threshold function that can be defined:

\[
\lambda(x) = \begin{cases} 
1 & \text{if } |x| \geq \theta, \\
0 & \text{if } |x| < \theta.
\end{cases}
\]  

(25)

The threshold parameter \( \theta \) of this function can be set by the decision maker. In our paper, we let \( \theta = 0.02 \) for the calculations.
Step 8. Compute the values of assessment score (AS) based on each alternative’s using the following equation:

\[ AS_i = \sum_{l=1}^{m} p_{il}. \]  

(26)

Step 9. According to the calculation results of \( AS_i \), we can rank all the alternatives. The bigger the value of \( AS_i \) is, the better alternative will be selected.

5. The CODAS Model with 2-Tuple Linguistic Neutrosophic Information

By combining the CODAS method with 2-tuple linguistic neutrosophic information we can build the 2-tuple linguistic neutrosophic CODAS model where all the evaluation information and attribute’s weighting vector are presented with 2-tuple linguistic neutrosophic numbers (2TLNNs). Suppose there are \( m \) alternatives \( \{\phi_1, \phi_2, \ldots, \phi_m\} \), \( n \) attributes \( \{O_1, O_2, \ldots, O_n\} \) and \( t \) experts \( \{d_1, d_2, \ldots, d_t\} \), let expert’s weighting vector be \( \{a_1, a_2, \ldots, a_t\} \), then the decision making steps are expressed as follows.

Step 1. Construct the 2-tuple linguistic neutrosophic evaluation matrix \( R = [\phi_{ij}^t]_{m \times n}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \) and calculate the average results matrix

\[ r = [\phi_{ij}]_{m \times n}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n, \]

which can be depicted as follows:

\[ R = [\phi_{ij}^t]_{m \times n} = \begin{bmatrix}
O_1 & O_2 & \ldots & O_n \\
\phi_1 & \phi_{11}^t & \phi_{12}^t & \ldots & \phi_{1n}^t \\
\phi_2 & \phi_{21}^t & \phi_{22}^t & \ldots & \phi_{2n}^t \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\phi_m & \phi_{m1}^t & \phi_{m2}^t & \ldots & \phi_{mn}^t 
\end{bmatrix}, \]

(27)

\[ r = [\phi_{ij}]_{m \times n} = \begin{bmatrix}
O_1 & O_2 & \ldots & O_n \\
\phi_1 & \phi_{11} & \phi_{12} & \ldots & \phi_{1n} \\
\phi_2 & \phi_{21} & \phi_{22} & \ldots & \phi_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\phi_m & \phi_{m1} & \phi_{m2} & \ldots & \phi_{mn} 
\end{bmatrix}, \]

(28)

Based on the assessment information and expert’s weighting vector \( \{a_1, a_2, \ldots, a_t\} \), the

\[ r = [\phi_{ij}]_{m \times n} \]
can be calculated as in Wang et al. (2018c):

$$
\phi_{ij} = a_1\phi_{ij}^1 \oplus a_2\phi_{ij}^2 \oplus \cdots \oplus a_t\phi_{ij}^t
$$

$$
= \left\{ \Delta \left( k \left( 1 - \Pi_{d=1}^{t'} \left( 1 - \frac{\Delta^{-1}(s_{ij}, \alpha_{ij})^t}{k} \right)^{a_t} \right) \right), \right. \\
- \left. \left. \Delta \left( k \Pi_{d=1}^{t'} \left( \frac{\Delta^{-1}(s_{ij}, \beta_{ij})^t}{k} \right)^{a_t} \right) \right), \right. \\
- \left. \left. \Delta \left( k \Pi_{d=1}^{t'} \left( \frac{\Delta^{-1}(s_{ij}, \chi_{ij})^t}{k} \right)^{a_t} \right) \right) \right\},
$$

(29)

where $\phi_{ij}^t = \{(s_{ij}, \alpha_{ij})^t, (s_{ij}, \beta_{ij})^t, (s_{ij}, \chi_{ij})^t\}, (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ denotes the 2-tuple linguistic neutrosophic information of alternative $\phi_i (i = 1, 2, \ldots, m)$ on attribute $O_j (j = 1, 2, \ldots, n)$ by expert $d'$ and $\phi_{ij} = \{(s_{ij}, \alpha_{ij}), (s_{ij}, \beta_{ij}), (s_{ij}, \chi_{ij})\}, (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ means the average 2TLNNs values of alternative with respect to attribute $O_j (j = 1, 2, \ldots, n)$.

**Step 2.** Obtain the attribute’s fuzzy weighting vector $W^t$ which is given by each expert with respect to all attributes and compute the average fuzzy weighting vector $W$ as follows:

$$
W^t = [w^t_j]_{1 \times n} = \{(s_{ij}, \alpha_j)^t, (s_{ij}, \beta_j)^t, (s_{fj}, \chi_j)^t\}_{1 \times n},
$$

(30)

$$
W = [w_j]_{1 \times n} = \{(s_{ij}, \alpha_j), (s_{ij}, \beta_j), (s_{fj}, \chi_j)\}_{1 \times n}.
$$

(31)

Based on the operation rules of 2TLNNs, the $W = [w_j]_{1 \times n}$ can be calculated as:

$$
w_j = a_1w_j^1 \oplus a_2w_j^2 \oplus \cdots \oplus a_tw_j^t
$$

$$
= \left\{ \Delta \left( k \left( 1 - \Pi_{d=1}^{t'} \left( 1 - \frac{\Delta^{-1}(s_{ij}, \alpha_j)^t}{k} \right)^{a_t} \right) \right), \right. \\
- \left. \left. \Delta \left( k \Pi_{d=1}^{t'} \left( \frac{\Delta^{-1}(s_{ij}, \beta_j)^t}{k} \right)^{a_t} \right) \right), \right. \\
- \left. \left. \Delta \left( k \Pi_{d=1}^{t'} \left( \frac{\Delta^{-1}(s_{ij}, \chi_j)^t}{k} \right)^{a_t} \right) \right) \right\},
$$

(32)

where $w_j^t = w_j = \{(s_{ij}, \alpha_j)^t, (s_{ij}, \beta_j)^t, (s_{fj}, \chi_j)^t\} (j = 1, 2, \ldots, n)$ denotes the fuzzy weight of attribute $O_j (j = 1, 2, \ldots, n)$ by expert $d'$ and $w_j = \{(s_{ij}, \alpha_j), (s_{ij}, \beta_j), (s_{fj}, \chi_j)\}, (j = 1, 2, \ldots, n)$ means the average fuzzy weight values of attribute $O_j (j = 1, 2, \ldots, n)$.

**Step 3.** Normalize the average results matrix $r = [\phi_{ij}]_{m \times n}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n$ based on the type of each attributes using the following formulae.

For benefit attributes:

$$
N_{ij} = \phi_{ij} = \{(s_{ij}, \alpha_{ij})^t, (s_{ij}, \beta_{ij})^t, (s_{fj}, \chi_{ij})^t\}
$$

$$
= \{(s_{ij}, \alpha_{ij}), (s_{ij}, \beta_{ij}), (s_{fj}, \chi_{ij})\}, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n.
$$

(33)
For cost attributes:

\[
N_{ij} = k - \phi_{ij} = \begin{cases} 
(s_{ij}, \alpha_{ij})', \\
(s_{ij}, \beta_{ij})', \\
(s_{ij}, \chi_{ij})'
\end{cases} = \begin{cases} 
\Delta(k - \Delta^{-1}(s_{ij}, \alpha_{ij})), \\
\Delta(k - \Delta^{-1}(s_{ij}, \beta_{ij})), \\
\Delta(k - \Delta^{-1}(s_{ij}, \chi_{ij}))
\end{cases},
\]

\[i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.\]

**Step 4.** According to the normalized average matrix \(N_{ij} = \{(s_{ij}, \alpha_{ij})', (s_{ij}, \beta_{ij})', (s_{ij}, \chi_{ij})'\}\) and average fuzzy weighting vector \(w_j = \{(s_{ij}, \alpha_j), (s_{ij}, \beta_j), (s_{ij}, \chi_j)\}\) \((j = 1, 2, \ldots, n)\) the fuzzy weighted normalized average matrix \(WN_{ij} = \{(s_{ij}, \alpha_{ij})'', (s_{ij}, \beta_{ij})'', (s_{ij}, \chi_{ij})''\}\) \((i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)\) can be computed as:

\[
WN_{ij} = w_j \otimes N_{ij} = \left\{ \begin{array}{c}
\Delta\left(k \left( \frac{\Delta^{-1}(s_{ij}, \alpha_{ij})'}{k} \cdot \frac{\Delta^{-1}(s_{ij}, \alpha_j)}{k} \right) \right), \\
\Delta\left(1 - \left(1 - \frac{\Delta^{-1}(s_{ij}, \beta_{ij})'}{k} \cdot \frac{\Delta^{-1}(s_{ij}, \beta_j)}{k} \right) \right), \\
\Delta\left(1 - \left(1 - \frac{\Delta^{-1}(s_{ij}, \chi_{ij})'}{k} \cdot \frac{\Delta^{-1}(s_{ij}, \chi_j)}{k} \right) \right)
\end{array} \right\},
\]

\[i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.\]

**Step 5.** Determine the fuzzy negative solution (NS) based on the equation (20)

\[
NS_j = \min_i(WN_{ij}) = \left\{ \min_i(s_{ij}, \alpha_{ij})'', \max_i(s_{ij}, \beta_{ij})'', \max_i(s_{ij}, \chi_{ij})'' \right\},
\]

\[i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.\]

**Step 6.** Calculate the fuzzy weighted Hamming distance and fuzzy weighted Euclidean distance between each alternatives and the negative solution (NS) according to Definition 4 and Definition 5:

\[
HD_i = \sum_{j=1}^{n} d^H(WN_{ij}, NS_j)
\]

\[
= \sum_{j=1}^{n} \left( \frac{1}{3} \begin{pmatrix}
\left| \frac{\Delta^{-1}(s_{ij}, \alpha_{ij})'' - \min_i(s_{ij}, \alpha_{ij})''}{k} \right|
\\
\left| \frac{\Delta^{-1}(s_{ij}, \beta_{ij})'' - \max_i(s_{ij}, \beta_{ij})''}{k} \right|
\\
\left| \frac{\Delta^{-1}(s_{ij}, \chi_{ij})'' - \max_i(s_{ij}, \chi_{ij})''}{k} \right|
\end{pmatrix} \right),
\]

\[i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.\]

\[
ED_i = \sum_{j=1}^{n} d^E(WN_{ij}, NS_j)
\]
\[
\sum_{j=1}^{n} \left( \frac{1}{3} \left[ \frac{\Delta^{-1}(s_{ij}, \alpha_{ii})'' - \min_i \Delta^{-1}(s_{ij}, \alpha_{ij})''}{k} \right] + \frac{\Delta^{-1}(s_{ij}, \beta_{ij})'' - \max_i \Delta^{-1}(s_{ij}, \beta_{ij})''}{k} \right) \right) .
\]

(38)

Step 7. Determine the relative assessment (RA) matrix which is presented as follows.

\[
RA = [p_{il}]_{m \times m},
\]

(39)

\[
p_{il} = (ED_i - ED_l) + (\lambda(ED_i - ED_l) \times (HD_i - HD_l)),
\]

(40)

where \(i, l = 1, 2, \ldots, m\) and \(\lambda\) is a threshold function which can be defined:

\[
\lambda(x) = \begin{cases} 
1 & \text{if } |x| \geq \theta, \\
0 & \text{if } |x| < \theta.
\end{cases}
\]

(41)

The threshold parameter \(\theta\) of this function can be set by the decision maker. In our paper, we let \(\theta = 0.02\) for the calculations.

Step 8. Compute the values of assessment score (AS) based on each alternative’s \(p_{il}\) using the following equation:

\[
AS_i = \sum_{l=1}^{m} p_{il}.
\]

(42)

Step 9. According to the calculation results of \(AS_i\), we can rank all the alternatives. The bigger the value of \(AS_i\) is, the better alternative will be selected.

Thus, the decision making model can be described as:

Step 1. Construct the 2-tuple linguistic neutrosophic evaluation matrix and calculate the average results matrix by using the equation (29);

Step 2. Obtain the attribute’s fuzzy weighting vector \(W_t\) and compute the average fuzzy weighting vector \(W\) by using the equation (32);

Step 3. Normalize the average results matrix based on the type of each attributes by using the equations (33) and (34);

Step 4. According to the normalized average matrix and average fuzzy weighting vector, compute the fuzzy weighted normalized average matrix \(WN_{ij}\) by using the equation (35);

Step 5. Determine the fuzzy negative solution (NS) by using the equation (36);

Step 6. Calculate the fuzzy weighted Hamming distance and the fuzzy weighted Euclidean distance between each alternatives and the NS by using the equations (37) and (38);

Step 7. Determine the relative assessment (RA) matrix by using the equation (40);

Step 8. Compute the values of assessment score (AS) by using the equation (42);

Step 9. According to the calculation results of \(AS_i\), rank all the alternatives.
6. The Numerical Example

6.1. Numerical for 2TLNNs MAGDM Problems

The safety assessment of a construction project could be considered an MAGDM issue (Tang and Wei, 2019a; Tang et al., 2019; Wang et al., 2019f, 2019g). With the gradual progress of urbanization in China, the number of construction projects under development has increased. Thus, it is very important to evaluate the safety of construction projects. In this section, we provide a numerical example to select the best construction projects by using the CODAS model with 2-tuple linguistic neutrosophic information. In order to choose a suitable construction scheme for construction, assume there are five possible construction projects\( \phi_i \) (\( i = 1, 2, 3, 4, 5 \)), which are provided by five famous construction companies with different construction advantages. In order to select the best construction project, invite some experts with experience of construction engineering and fuzzy set theory to construct the evaluation system in order to assess these construction projects. The evaluation index includes: (1) \( O_1 \) is the human factor in construction projects; (2) \( O_2 \) is the energy cost factor; (3) \( O_3 \) is the building materials and equipment factor; (4) \( O_4 \) is the environmental factor. The five possible construction projects \( \phi_i \) (\( i = 1, 2, 3, 4, 5 \)) are to be evaluated with 2TLNNs according to the four criteria by three experts \( d^t \) (according to the professional years and the degree of authority of the expert, the weight of the expert is determined as \((0.3, 0.4, 0.3)\)).

**Step 1.** Construct the 2-tuple linguistic neutrosophic evaluation matrix \( R = [\phi_{ij}]_{m \times n} \), \( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \). Then according to equation (29) and expert’s weights, we can obtain the average results matrix \( r = [\phi_{ij}]_{m \times n} \), \( i = 1, 2, \ldots, m \), \( j = 1, 2, \ldots, n \) as follows. (Take \( \phi_{11} \) for example.)

According to Table 1, we can derive \( \phi_{11}^1 = \{(s_4, 0), (s_3, 0), (s_2, 0)\}, \phi_{11}^2 = \{(s_5, 0), (s_2, 0), (s_1, 0)\}, \phi_{11}^3 = \{(s_4, 0), (s_3, 0), (s_2, 0)\} \), then we can get (Tables 2, 3, 4)

\[
\phi_{11} = a_1\phi_{11}^1 \oplus a_2\phi_{11}^2 \oplus a_3\phi_{11}^3
\]

\[
= \left\{ \Delta(6(1 - (1 - \frac{4}{7})^{0.3} \times (1 - \frac{5}{8})^{0.4} \times (1 - \frac{4}{6})^{0.3})), \right. \\
\left. \Delta(6(\frac{3}{7})^{0.3} \times (\frac{2}{7})^{0.4} \times (\frac{5}{8})^{0.3}), \Delta(6(\frac{2}{7})^{0.3} \times (\frac{1}{7})^{0.4} \times (\frac{5}{8})^{0.3}) \right\}
\]

\[
= \{(s_4, 0.4843), (s_3, -0.4492), (s_2, -0.4843)\}.
\]

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>2-tuple linguistic neutrosophic numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceedingly Terrible – ET</td>
<td>{(s_4, 0), (s_5, 0), (s_6, 0)}</td>
</tr>
<tr>
<td>Very Terrible – VT</td>
<td>{(s_1, 0), (s_4, 0), (s_5, 0)}</td>
</tr>
<tr>
<td>Terrible – T</td>
<td>{(s_2, 0), (s_3, 0), (s_4, 0)}</td>
</tr>
<tr>
<td>Medium – M</td>
<td>{(s_3, 0), (s_3, 0), (s_3, 0)}</td>
</tr>
<tr>
<td>Well – W</td>
<td>{(s_4, 0), (s_3, 0), (s_2, 0)}</td>
</tr>
<tr>
<td>Very Well – VW</td>
<td>{(s_5, 0), (s_2, 0), (s_1, 0)}</td>
</tr>
<tr>
<td>Exceedingly Well – EW</td>
<td>{(s_6, 0), (s_1, 0), (s_0, 0)}</td>
</tr>
</tbody>
</table>
Step 2. Obtain the attribute’s fuzzy weighting vector $W^t$ which is given by each expert with respect to all attributes and compute the average fuzzy weighting vector $W$ as
Table 6
Attribute’s weighting vector given by decision-maker.

<table>
<thead>
<tr>
<th>$O_1$ (benefit)</th>
<th>$O_2$ (cost)</th>
<th>$O_3$ (benefit)</th>
<th>$O_4$ (benefit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d^1$ VT</td>
<td>W</td>
<td>VW</td>
<td>T</td>
</tr>
<tr>
<td>$d^2$ M</td>
<td>VT</td>
<td>W</td>
<td>M</td>
</tr>
<tr>
<td>$d^3$ VW</td>
<td>W</td>
<td>M</td>
<td>VT</td>
</tr>
</tbody>
</table>

Table 7
The normalized decision-making matrix $N_{ij}$.

<table>
<thead>
<tr>
<th>$O_1$</th>
<th>$O_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>${ (s_4, 0.4843), (s_3, -0.4492), (s_2, -0.4843) }$</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>${ (s_3, -0.4968), (s_3, 0.2704), (s_3, 0.4968) }$</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>${ (s_3, -0.2586), (s_3, 0.3659), (s_3, 0.2586) }$</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>${ (s_3, -0.4740), (s_3, 0.2704), (s_3, 0.4740) }$</td>
</tr>
<tr>
<td>$\phi_5$</td>
<td>${ (s_4, -0.1577), (s_3, -0.3436), (s_2, 0.1577) }$</td>
</tr>
</tbody>
</table>

follows (Table 6). According to expert’s weighting vector and the equation (32), the attribute’s weighting vector can be calculated as (take $W_1$ for example).

According to Table 1, we can derive $W^1 = \{ (s_1, 0), (s_4, 0), (s_5, 0) \}$, $W^2 = \{ (s_3, 0), (s_3, 0), (s_3, 0) \}$, $W^3 = \{ (s_5, 0), (s_2, 0), (s_1, 0) \}$, then we can get

$$W_1 = a_1 W^1 \oplus a_2 W^2 \oplus a_3 W^3$$

$$= \left\{ \Delta \left( 6 \left( 1 - \left( \frac{5}{6} \right)^{0.3} \times \left( 1 - \frac{3}{6} \right)^{0.4} \times \left( 1 - \frac{5}{6} \right)^{0.3} \right) \right), \Delta \left( 6 \left( \frac{4}{6} \right)^{0.3} \times \left( \frac{3}{6} \right)^{0.4} \times \left( \frac{2}{6} \right)^{0.3} \right), \Delta \left( 6 \left( \frac{5}{6} \right)^{0.3} \times \left( \frac{3}{6} \right)^{0.4} \times \left( \frac{1}{6} \right)^{0.3} \right) \right\}$$

$$= \{ (s_3, 0.4850), (s_3, -0.1042), (s_3, 0.4850) \}.$$ 

So the attribute weights are derived as:

$$W = \left\{ \{ (s_3, 0.4850), (s_3, -0.1042), (s_3, 0.4850) \}, \{ (s_3, 0.1146), (s_3, 0.3659), (s_3, -0.1146) \}, \{ (s_4, 0.1654), (s_3, -0.3436), (s_2, -0.1654) \}, \{ (s_2, 0.1880), (s_3, 0.2704), (s_4, -0.1880) \} \right\}.$$

Step 3. Normalize the average results matrix $r = [\phi_{ij}]_{m \times n}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ based on the type of each attribute by formulae (33) and (34).

Step 4. According to the normalized average matrix ($i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$) and average fuzzy weighting vector $w_j$ ($j = 1, 2, \ldots, n$), the fuzzy weighted nor-
Calculate the fuzzy weighted Hamming distance

Step 5.

WN_{ij} (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) can be computed as (take WN_{11} for example):

\[
WN_{11} = w_j \otimes N_{ij} = \left\{ \Delta \left( 6 \left( \frac{4.4843}{6} \cdot \frac{3.4850}{6} \right) \right), \Delta \left( 1 - \left( 1 - \frac{2.5508}{6} \right) \cdot \left( 1 - \frac{2.8958}{6} \right) \right) \right\} = \left\{ (s_3, -0.3954), (s_4, 0.2156), (s_3, 0.3954) \right\}.
\]

Thus, the weighted normalized average matrix WN_{ij} (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) is derived as follows (Table 8).

Step 5. Determine the fuzzy negative solution (NS):

\[
NS_j = \min_i (WN_{ij}) = \begin{cases} 
(s_1, 0.4539), (s_5, -0.3959), (s_5, -0.4539), \\
(s_1, -0.3150), (s_5, -0.0327), (s_5, 0.3150), \\
(s_1, -0.0810), (s_5, -0.2988), (s_5, 0.0810), \\
(s_1, 0.1011), (s_5, -0.3648), (s_5, -0.1011)
\end{cases}.
\]

Step 6. Calculate the fuzzy weighted Hamming distance (HD_i) and fuzzy weighted Euclidean distance (ED_i) between each alternatives and the negative solution (NS) according to Definition 4 and Definition 5. For example:

\[
HD_1 = \sum_{j=1}^{n} d^H(WN_{ij}, NS_j)
\]
Step 7. Determine the relative assessment (RA) matrix which is presented as follows (Table 9).

\[
\begin{array}{ccccc}
\phi_1 & \phi_2 & \phi_3 & \phi_4 & \phi_5 \\
0.0000 & 0.6437 & 0.2699 & -0.1937 & 0.1626 \\
-0.6437 & 0.0000 & -0.3738 & -0.8374 & -0.4811 \\
-0.2699 & 0.3738 & 0.0000 & -0.4636 & -0.1073 \\
0.1937 & 0.8374 & 0.4636 & 0.0000 & 0.3563 \\
-0.1626 & 0.4811 & 0.1073 & -0.3563 & 0.0000 \\
\end{array}
\]

\[
(\text{RA}) = \begin{pmatrix}
\frac{1}{3} \left[ \frac{2.6046 - 1.4539}{6} + \frac{4.2156 - 4.6372}{6} + \frac{3.3954 - 4.5461}{6} \right] \\
\frac{1}{3} \left[ \frac{0.9524 - 0.6850}{6} + \frac{4.8338 - 4.9673}{6} + \frac{5.0476 - 5.3150}{6} \right] \\
\frac{1}{3} \left[ \frac{0.9190 - 0.9190}{6} + \frac{4.7012 - 4.7012}{6} + \frac{5.0810 - 5.0810}{6} \right] \\
\frac{1}{3} \left[ \frac{1.4011 - 1.1011}{6} + \frac{4.4789 - 4.6352}{6} + \frac{4.5989 - 4.8989}{6} \right]
\end{pmatrix}
\]

\[= 0.2304,\]

\[
ED_1 = \sum_{j=1}^{n} d^E (WN_{ij}, NS_j)
\]

\[
= \begin{pmatrix}
\frac{1}{3} \left[ \frac{2.6046 - 1.4539}{6} + \frac{4.2156 - 4.6372}{6} + \frac{3.3954 - 4.5461}{6} \right]^2 \\
\frac{1}{3} \left[ \frac{0.9524 - 0.6850}{6} + \frac{4.8338 - 4.9673}{6} + \frac{5.0476 - 5.3150}{6} \right]^2 \\
\frac{1}{3} \left[ \frac{0.9190 - 0.9190}{6} + \frac{4.7012 - 4.7012}{6} + \frac{5.0810 - 5.0810}{6} \right]^2 \\
\frac{1}{3} \left[ \frac{1.4011 - 1.1011}{6} + \frac{4.4789 - 4.6352}{6} + \frac{4.5989 - 4.8989}{6} \right]^2
\end{pmatrix}
\]

\[= 0.2439.\]

Similarly, we can obtain

\[
HD_1 = 0.2304, \quad HD_2 = 0.5326, \quad HD_3 = 0.3554, \quad HD_4 = 0.1343, \\
HD_5 = 0.3123, \quad ED_1 = 0.2439, \quad ED_2 = 0.5854, \quad ED_3 = 0.3887, \\
ED_4 = 0.1463, \quad ED_5 = 0.3246.
\]

Step 8. Compute the values of assessment score (AS) based on each alternative’s \(p_{il}\)

\[
AS_1 = -0.8824, \quad AS_2 = 2.3360, \quad AS_3 = 0.4669, \\
AS_4 = -1.8510, \quad AS_5 = -0.0695.
\]
Table 10
Ordering by different parameter $\theta$.

<table>
<thead>
<tr>
<th>Parameter $\theta$</th>
<th>$AS_1$</th>
<th>$AS_1$</th>
<th>$AS_1$</th>
<th>$AS_1$</th>
<th>$AS_1$</th>
<th>Ordering</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0.05$</td>
<td>−0.8824</td>
<td>2.3360</td>
<td>0.4669</td>
<td>−1.8510</td>
<td>−0.0695</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
<tr>
<td>$\theta = 0.06$</td>
<td>−0.8824</td>
<td>2.3360</td>
<td>0.4669</td>
<td>−1.8510</td>
<td>−0.0695</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
<tr>
<td>$\theta = 0.07$</td>
<td>−0.8824</td>
<td>2.3360</td>
<td>0.4238</td>
<td>−1.8510</td>
<td>−0.0264</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
<tr>
<td>$\theta = 0.10$</td>
<td>−0.8966</td>
<td>2.3360</td>
<td>0.4238</td>
<td>−1.7549</td>
<td>−0.1083</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
<tr>
<td>$\theta = 0.20$</td>
<td>−0.7716</td>
<td>2.1588</td>
<td>0.4760</td>
<td>−1.5769</td>
<td>−0.2863</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
<tr>
<td>$\theta = 0.30$</td>
<td>−0.7716</td>
<td>1.9386</td>
<td>0.2549</td>
<td>−1.3558</td>
<td>−0.0660</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
<tr>
<td>$\theta = 0.40$</td>
<td>−0.4695</td>
<td>1.6364</td>
<td>0.2549</td>
<td>−1.3558</td>
<td>−0.0660</td>
<td>$\phi_2 &gt; \phi_3 &gt; \phi_5 &gt; \phi_1 &gt; \phi_4$</td>
</tr>
</tbody>
</table>

**Step 9.** According to the calculation results of $AS$, we can rank all the alternatives. The bigger the value of $AS$ is, the better alternative will be selected. Obviously, the rank of all alternatives is $\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$ and $\phi_2$ is the best alternative.

**6.2. Sensitivity Analysis**

To show the influence of the threshold parameter $\theta$ which is set by the decision maker, the ordering of the alternatives is shown as follows.

From Table 10, we can easily find that the ordering of alternatives is the same, which indicates that our proposed 2TLNN CODAS model is robust and effective. At the same time, when the threshold parameter $\theta \leq 0.06$, the assessment score ($AS$) remains the same, which indicates that the Hamming distance and the Euclidean distance are considered; when the threshold parameter $\theta \geq 0.50$, the assessment score ($AS$) also remains the same, which indicates only the Euclidean distance are considered. In other words, when the threshold parameter $0.06 \leq \theta \leq 0.50$, the Euclidean distance is considered absolutely and the Hamming distance is considered partly. In addition, the absolute values of assessment scores become smaller with the increase of the parameter. Thus, the decision maker can obtain different assessment scores by altering the threshold parameter.

**6.3. Compare 2TLNNs CODAS Method with Some 2TLNNs Aggregation Operators**

In this chapter, we compare our proposed 2-tuple linguistic neutrosophic CODAS method with the 2-tuple linguistic neutrosophic weighted average (2TLNNWA) operator and 2-tuple linguistic neutrosophic weighted geometric (2TLNNWG) operator. For the attribute’s weights that are presented by 2TLNNs we can use the score function to obtain the attribute’s weights with crisp number.

According to the value of the average attribute’s weighting vector $W$ which is listed as:

$$W = \begin{cases} 
(s_3, 0.4850), (s_3, -0.1042), (s_3, 0.4850), \\
(s_3, 0.1146), (s_3, 0.3659), (s_3, -0.1146), \\
(s_4, 0.1654), (s_3, -0.3436), (s_2, -0.1654), \\
(s_2, 0.1880), (s_3, 0.2704), (s_4, -0.1880) 
\end{cases}.$$
We can obtain the score results as:

\[ W_1 = 0.5597, \quad W_2 = 0.4924, \quad W_3 = 0.6486, \quad W_4 = 0.3948. \]

Then the normalized results \( w_j (j = 1, 2, \cdots, n) \) can be expressed as:

\[ w_j = W_i / \sum_{i=1}^{n} W_i, \quad (43) \]

\[ w_1 = 0.2671, \quad w_2 = 0.2350, \quad w_3 = 0.3095, \quad w_4 = 0.1884. \]

Based on the attribute's weight and the results in Table 7, the fused values by 2TLNNWA and 2TLNNWG operators are shown in Table 11.

According to the score function of 2TLNNs, we can obtain the alternative score results which are shown in Table 12.

The ranking of alternatives by some 2TLNNs aggregation operators are listed as follows (Table 13). Comparing the results of the 2-tuple linguistic neutrosophic CODAS model with 2TLNNWA and 2TLNNWG operators, it can be noted that the aggregation results are
slightly different in ranking of alternatives and the best alternatives are the same. However, 2-tuple linguistic neutrosophic CODAS model has important characteristics of using the combinative form of two distance measurements, including fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED) and can be more accurate and effective in the application of MADM problems.

7. Conclusion

In this paper, we present the 2-tuple linguistic neutrosophic CODAS model based on the traditional fuzzy CODAS (combinative distance-based assessment) model and some fundamental theories of 2-tuple linguistic neutrosophic information. Firstly, we briefly review the definition of 2-tuple linguistic neutrosophic sets (2TLNNSs) and introduce the score function, the accuracy function, operation laws and some aggregation operators of 2TLNNs. Then, the calculation steps of traditional fuzzy CODAS model are briefly presented. Furthermore, by combining the traditional fuzzy CODAS model with 2TLNNSs information, the 2-tuple linguistic neutrosophic CODAS model is established and the computing steps are simply depicted. Finally, a numerical example for safety assessment of a construction project is given to illustrate this new model and some comparisons between 2TLNNSs CODAS model and two 2TLNNSs aggregation operators are also made to further illustrate advantages of the new method. In actual decision making applications, our developed model has the advantage of considering the combinative form of two distance measurements, including fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED). However, it is difficult to obtain assessment information which is expressed by 2TLNNSs, so we need to continue to study this problem. In the future, the 2-tuple linguistic neutrosophic CODAS model can be applied to the risk analysis (Wei et al., 2019h, 2018), the MADM problems (Wei et al., 2019a; He et al., 2019a; Tang and Wei, 2019b; Wei, 2019a, 2019b; Wei et al., 2019c) and many other uncertain and fuzzy environments (Lu et al., 2019a; Wei et al., 2019c, 2019d, 2019f).

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References


**P. Wang** is a current master student at Institute of Technology at Sichuan Normal University, Chengdu, 610101, PR China.

**J. Wang** is a current master student at School of Business at Sichuan Normal University, Chengdu, 610101, PR China.

**G. Wei** has an MSc and a PhD degree in applied mathematics from SouthWest Petroleum University, business administration from school of Economics and Management at South-West Jiaotong University, China, respectively. From May 2010 to April 2012, he was a Postdoctoral Researcher with the School of Economics and Management, Tsinghua University, Beijing, China. He is a professor in the School of Business at Sichuan Normal University. He has published more than 100 papers in journals, books and conference proceedings including journals such as *Omega, Decision Support Systems, Expert Systems with Applications, Applied Soft Computing, Knowledge and Information Systems, Computers & Industrial Engineering, Knowledge-Based Systems, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, International Journal of Computational Intelligence Systems and Information: An International Interdisciplinary Journal*. He has published 1 book. He has participated in several scientific committees and serves as a reviewer in a wide range of journals including *Computers & Industrial Engineering, International Journal of Information Technology and Decision Making, Knowledge-Based Systems, Information Sciences, International Journal of Computational Intelligence Systems* and *European Journal of Operational Research*. He is currently interested in aggregation operators, decision making and computing with words.

**J. Wu** has a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is a professor in the School of Statistics at Southwestern University of Finance and Economics, Chengdu, 611130, PR China.

**C. Wei** has an MSc degree in applied mathematics from SouthWest Petroleum University. Now, he is a PhD student with School of Statistics, Southwestern University of Finance and Economics, Chengdu, 611130, PR China. He has published more than 10 papers in journals, such as *International Journal of Intelligent Systems, Journal of Intelligent and Fuzzy Systems, IEEE Access, Mathematics, Information*. He is currently interested in aggregation operators, decision making and computing with words.

**Y. Wei** has an MSc and a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is a professor in the School of Finance at Yunnan University of Finance and Economics. He has published more than 120 papers in journals, books and conference proceedings including journals such as *Journal of Forecasting, Journal of Banking & Finance, Empirical Economics, Energy Economics, Economic Modelling, Applied Economics Letters, International Review of Economics & Finance*. He has published two book (in Chinese). He has participated in several scientific committees and serves as a reviewer in a wide range of journals including *Energy Economics, Journal of Forecasting, Emerging Market Finance and Trade, Physica A*. He is currently interested in volatility modelling and forecasting in financial and energy markets, density forecasting and decision making in financial markets.