INFORMATICA, 2020, Vol. 31, No. 1, 161–184

DOI: https://doi.org/10.15388/20-INFOR399

CODAS Method for Multiple Attribute Group Decision Making Under 2-Tuple Linguistic Neutrosophic Environment

Ping WANG[1], Jie WANG[2], Guiwu WEI[2], Jiang WU[3], Cun WEI[3], Yu WEI[4,*]

[1] *Institute of Technology, Sichuan Normal University, Chengdu 610101, PR China*
[2] *School of Business, Sichuan Normal University, Chengdu 610101, PR China*
[3] *School of Statistics, Southwestern University of Finance and Economics, Chengdu 611130, PR China*
[4] *School of Finance, Yunnan University of Finance and Economics, Kunming 650221, PR China*
e-mail: weiyusy@126.com

Received: March 2019; accepted: November 2019

Abstract. In this paper, we present the 2-tuple linguistic neutrosophic CODAS model based on the traditional fuzzy CODAS (combinative distance-based assessment) model and some fundamental theories of 2-tuple linguistic neutrosophic information. Firstly, we briefly review the definition of 2-tuple linguistic neutrosophic sets (2TLNSs) and introduce the score function, the accuracy function, operation laws and some aggregation operators of 2TLNNs. Then, the calculation steps of traditional fuzzy CODAS model are briefly presented. Furthermore, by combining the traditional fuzzy CODAS model with 2TLNNs information, the 2-tuple linguistic neutrosophic CODAS model is established and the computing steps for multiple attribute group decision making (MAGDM) are simply depicted. Our presented model is more accurate and effective for considering the combinative form of two distance measurements, including fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED). Finally, a numerical example for safety assessment of construction project has been given to illustrate this new model and some comparisons between 2TLNNs CODAS model and two 2TLNNs aggregation operators are also made to further illustrate the advantages of the new method.

Key words: multiple attribute group decision making (MAGDM) problems, 2-tuple linguistic neutrosophic sets (2TLNSs), CODAS model, 2-tuple linguistic neutrosophic number weighted average (2TLNNWA) operators, 2-tuple linguistic neutrosophic number weighted geometric (2TLNNWG) operators, 2TLNNs CODAS model, construction project.

1. Introduction

Due to the indeterminacy of DM's and the decision-making issues, we cannot always give accurate evaluation values of alternatives to select the best project in real MADM problems (Wang *et al.*, 2019a; Wang, 2019; Wu *et al.*, 2019a, 2019b). To conquer this disad-

*Corresponding author.

vantage, fuzzy set theory which was defined by Zadeh (1965) originally used the membership function to describe the estimation results rather than exact real-numbers. Atanassov (1986, 1989) presented another measurement index which named non-membership function as a complement. Smarandache (1999, 2003) introduced the neutrosophic set (NS). Then, Wang *et al.* (2010) introduced the definition and some operational rules of single-valued neutrosophic sets (SVNSs), where the evaluation information is depicted by truth membership degree, indeterminacy membership degree and falsity membership degree. Obviously, the SVNSs and their extensions can describe the fuzzy and uncertainty degree of a decision maker and can be more suitable for actual multiple attribute decision making problems (Wang *et al.*, 2019c, 2019e, 2019h, 2019j; Wei *et al.*, 2019e, 2019f).

However, the SVNSs can only represent quantitative decision making information and fail to depict the qualitative decision making information. As we all know, the 2-tuple linguistic set (2TLS) (Herrera and Martinez, 2001) can eliminate this limitation. In order to consider both qualitative and quantitative decision making information, Wang *et al.* (2018a) defined the 2-tuple linguistic neutrosophic sets (2TLNSs), where the truth membership function, indeterminacy membership function and falsity membership function are presented by 2TLNNs. Thus, the 2TLNNs are considered a useful tool to deal with practical MADM applications. In real decision making problems, finding a good way to denote evaluation information is only one aspect (Deng and Gao, 2019; Gao *et al.*, 2019; Li and Lu, 2019; Lu and Wei, 2019). It is also important to know how to deal with this information. To date, The CODAS (combinative distance-based assessment) method, which was originally defined by Keshavarz Ghorabaee *et al.* (2016), use the combinative form of two distance measurements, including Euclidean and Taxicab distances, which present accurate values to compute the assessment results of alternatives. However, this model cannot be applied in fuzzy environment. To overcome this disadvantage, Keshavarz Ghorabaee *et al.* (2017) extended the CODAS method to fuzzy environment and proposed a fuzzy CODAS model which used fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED) rather than the crisp distances. Thus, we can easily find that CODAS method can handle fuzzy decision making problems effectively.

Motivated by the 2-tuple linguistic neutrosophic sets (2TLNSs) and the traditional CODAS model, the research question and goal of this paper is to build an extended CODAS model to deal with the 2-tuple linguistic neutrosophic decision making problems. The main novelty and contribution of this paper is the proposition of the 2TLN CODAS model. On the one hand, the 2-tuple linguistic neutrosophic number can express the qualitative and quantitative fuzzy decision making information, on the other hand, the CODAS model has important merits mentioned above. Thus, we can derive accuracy assessment results for construction project by utilizing the 2TLN CODAS model. In order to elaborate the process of putting forward the 2TLN CODAS model, this article is structured in the following way: some related work about 2TLNSs and the CODAS method are given in Section 2. The definition, the score function, the accuracy function, operation rules and some aggregation operators of 2TLNNSs are briefly introduced in Section 3. The computing steps of traditional fuzzy CODAS model are briefly presented in Section 4. By combining the fuzzy traditional CODAS model with 2TLNNs information, the 2-tuple linguistic neutrosophic CODAS model is established and the computing steps for MAGDM problems are simply depicted in Section 5. A numerical example for safety assessment of construction project is given to illustrate this new model and some comparisons between 2-tuple linguistic neutrosophic CODAS model and two 2TLNNs aggregation operators are also made to further illustrate advantages of the new method in Section 6. Section 7 gives some conclusions of our work.

2. Related Work

Previously, a lot of decision-making models such as the VIKOR method (He *et al.*, 2019b; Opricovic and Tzeng, 2004; Wang *et al.*, 2018b), the ELECTRE method (Rashid *et al.*, 2018), the TOPSIS method (Chen, 2000; Lu *et al.*, 2019b), the PROMETHEE method (Balali *et al.*, 2014), the MABAC method (Pamucar and Cirovic, 2015), the EDAS method (Keshavarz Ghorabaee *et al.*, 2015; Wang *et al.*, 2019i; Zhang *et al.*, 2019) and the TODIM method (Gomes and Lima, 1979) have been studied extensively by numerous researchers. Compared with the existing literature, the CODAS model has the advantage of taking the combinative form of ED and HD into account with respect to the intangibility of decision maker (DM) and the uncertainty of decision-making environment to obtain more accurate and effective aggregation results. Since the CODAS method was proposed, a large number of scholars have studied it. Pamucar *et al.* (2018) presented a linguistic neutrosophic CODAS model. Badi *et al.* (2018) studied the site selection of desalination plant in Libya by using CODAS method. Bolturk (2018) proposed an extended CODAS model to deal with Pythagorean fuzzy decision making problems and studied its application to supplier selection. Based on interval-valued intuitionistic fuzzy information, Bolturk and Kahraman (2018a) developed a novel CODAS model. To handle renewable energy selection, Bolturk and Karasan (2018b) proposed the interval-valued neutrosophic CODAS model. According to novel information measure, Peng and Garg (2018) studied the CODAS method and presented some novel algorithms under the interval-valued fuzzy soft set. Ren (2018) established the intuitionistic fuzzy CODAS model for MADM. Karasan *et al.* (2019) developed an integrated methodology based on the neutrosophic CODAS model.

As for the 2-tuple linguistic neutrosophic sets, based on the Hamy mean (HM) operator, Wu *et al.* (2018b) proposed some 2-tuple linguistic neutrosophic Hamy mean (2TLNNHM) operators and 2-tuple linguistic neutrosophic dual Hamy mean (2TLNNDHM) operators to fuse 2TLNNs. Wang *et al.* (2019b) developed some 2-tuple linguistic neutrosophic Muirhead mean (2TLNNMM) operators for MADM. Considering the Dombi operation laws and BM operators, Wei *et al.* (2019b) presented some novel aggregation operators. Wang *et al.* (2019h) combined the EDAS method with the 2-tuple linguistic neutrosophic set to build an extended EDAS model for MADM. Based on the single-valued neutrosophic 2-tuple linguistic set, Wang *et al.* (2019d) proposed some Muirhead mean (MM) aggregation operators, Wu *et al.* (2018a) defined some Hamcher aggregation operators, Ju *et al.* (2018) extended it to interval-valued environment and developed some MSM operators. Wang *et al.* (2018b) proposed the 2-tuple linguistic neutrosophic TODIM model. Thereafter, the 2TLNSs have been widely studied in MADM issues.

However, it is clear that there are no studies about the CODAS model with 2TLNNs information. Some scholars studied the CODAS model under neutrosophic and linguistic neutrosophic environment, but both of them cannot represent decision information in a convenient way. At the same time, in the area of 2TLNSs, the research mainly focuses on the aggregation operators, but there is a lack of research on 2TLN models. Hence, it is necessary to discuss the 2-tuple linguistic neutrosophic CODAS model. The goal of this paper is to develop a novel CODAS method based on the conventional CODAS model and 2-tuple linguistic neutrosophic information to study MADM problems more effectively.

3. Preliminaries

3.1. *2-Tuple Linguistic Neutrosophic Sets*

Wang *et al.* (2018a) initially proposed the 2-tuple linguistic neutrosophic sets (2TLNSs), which consider the important characteristics of 2-tuple linguistic variables and single-valued neutrosophic sets (SVNSs), hence, can be more effective and accurate to evaluate the alternatives in multiple attribute decision making problems. To combine the 2TLSs and SVNSs, the definition of 2TLNSs can be expressed as follows.

Definition 1. Let $\delta_1, \delta_2, \dots, \delta_k$ be a linguistic term set. Any label δ_i shows a possible linguistic scale, and $\delta = \{\delta_0 = \textit{exceedingly terrible}, \delta_1 = \textit{very terrible}, \delta_2 = \textit{terrible}, \delta_3 = \textit{medium}, \delta_4 = \textit{well}, \delta_5 = \textit{very well}, \delta_6 = \textit{exceedingly well}\}$, then we can describe the 2TLNSs as:

$$\delta = \big\langle (s_t, \alpha,), (s_i, \beta), (s_f, \chi) \big\rangle, \tag{1}$$

where $\Delta^{-1}(s_t, \alpha,)$, $\Delta^{-1}(s_i, \beta)$ and $\Delta^{-1}(s_f, \chi) \in [0, k]$ represent the truth membership function, the indeterminacy membership function and the falsity membership function, which are expressed by 2-tuple linguistic variables and satisfy the condition $0 \leqslant \Delta^{-1}(s_t, \phi) + \Delta^{-1}(s_f, \varphi) + \Delta^{-1}(s_f, \gamma) \leqslant 3k$.

Definition 2 (*See* Wang *et al.*, 2018a). Let $\delta_1 = \langle (s_{t_1}, \alpha_1,), (s_{i_1}, \beta_1), (s_{f_1}, \chi_1) \rangle$ and $\delta_2 = \langle (s_{t_2}, \alpha_2,), (s_{i_2}, \beta_2), (s_{f_2}, \chi_2) \rangle$ be two 2-tuple linguistic neutrosophic numbers (2TLNNs), the operation formula of them can be defined:

$$(1)\ \delta_1 \oplus \delta_2 = \left\{ \begin{array}{l} \Delta\big(k\big(\frac{\Delta^{-1}(s_{t_1}, \alpha_1,)}{k} + \frac{\Delta^{-1}(s_{t_2}, \alpha_2,)}{k} - \frac{\Delta^{-1}(s_{t_1}, \alpha_1,)}{k} \cdot \frac{\Delta^{-1}(s_{t_2}, \alpha_2,)}{k}\big)\big), \\ \Delta\big(k\big(\frac{\Delta^{-1}(s_{i_1}, \beta_1)}{k} \cdot \frac{\Delta^{-1}(s_{i_2}, \beta_2)}{k}\big)\big), \\ \Delta\big(k\big(\frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k} \cdot \frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k}\big)\big) \end{array} \right\};$$

$$(2)\ \delta_1 \otimes \delta_2 = \left\{ \begin{array}{l} \Delta\big(k\big(\frac{\Delta^{-1}(s_{t_1}, \alpha_1,)}{k} \cdot \frac{\Delta^{-1}(s_{t_2}, \alpha_2,)}{k}\big)\big), \\ \Delta\big(k\big(\frac{\Delta^{-1}(s_{i_1}, \beta_1)}{k} + \frac{\Delta^{-1}(s_{i_2}, \beta_2)}{k} - \frac{\Delta^{-1}(s_{i_1}, \beta_1)}{k} \cdot \frac{\Delta^{-1}(s_{i_2}, \beta_2)}{k}\big)\big), \\ \Delta\big(k\big(\frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k} + \frac{\Delta^{-1}(s_{f_2}, \chi_2)}{k} - \frac{\Delta^{-1}(s_{f_1}, \chi_1)}{k} \cdot \frac{\Delta^{-1}(s_{f_2}, \chi_2)}{k}\big)\big) \end{array} \right\};$$

$$(3)\ \lambda\delta_1 = \left\{ \begin{array}{l} \Delta\big(k\big(1-\big(1-\frac{\Delta^{-1}(s_{t_1},\alpha_{1},)}{k}\big)^{\lambda}\big)\big), \\ \Delta\big(k\big(\frac{\Delta^{-1}(s_{i_1},\beta_1)}{k}\big)^{\lambda}\big), \\ \Delta\big(k\big(\frac{\Delta^{-1}(s_{f_1},\chi_1)}{k}\big)^{\lambda}\big) \end{array} \right\}, \quad \lambda > 0;$$

$$(4)\ \delta_1^{\lambda} = \left\{ \begin{array}{l} \Delta\big(k\big(\frac{\Delta^{-1}(s_{t_1},\alpha_{1},)}{k}\big)^{\lambda}\big), \\ \Delta\big(k\big(1-\big(1-\frac{\Delta^{-1}(s_{i_1},\beta_1)}{k}\big)^{\lambda}\big)\big), \\ \Delta\big(k\big(1-(1-\frac{\Delta^{-1}(s_{f_1},\chi_1)}{k})^{\lambda}\big)\big) \end{array} \right\}, \quad \lambda > 0.$$

According to Definition 2, it is clear that the operation laws have the following properties.

$$\delta_1 \oplus \delta_2 = \delta_2 \oplus \delta_1, \qquad \delta_1 \otimes \delta_2 = \delta_2 \otimes \delta_1, \qquad \big((\delta_1)^{\lambda_1}\big)^{\lambda_2} = (\delta_1)^{\lambda_1\lambda_2}, \tag{2}$$

$$\lambda(\delta_1 \oplus \delta_2) = \lambda\delta_1 \oplus \lambda\delta_2, \qquad (\delta_1 \otimes \delta_2)^{\lambda} = (\delta_1)^{\lambda} \otimes (\delta_2)^{\lambda}, \tag{3}$$

$$\lambda_1\delta_1 \oplus \lambda_2\delta_1 = (\lambda_1+\lambda_2)\delta_1, \qquad (\delta_1)^{\lambda_1} \otimes (\delta_1)^{\lambda_2} = (\delta_1)^{(\lambda_1+\lambda_2)}. \tag{4}$$

Definition 3 (*See* Wang *et al.*, 2018b). Let $\delta = \langle (s_t, \alpha,), (s_i, \beta), (s_f, \chi) \rangle$ be a 2TLNN, the score and accuracy functions of δ can be expressed:

$$s(\delta) = \frac{(2k + \Delta^{-1}(s_t, \alpha,) - \Delta^{-1}(s_i, \beta) - \Delta^{-1}(s_f, \chi))}{3k}, \quad s(\delta) \in [0, 1], \tag{5}$$

$$h(\delta) = \frac{1}{k}\big(\Delta^{-1}(s_t, \alpha,) - \Delta^{-1}(s_f, \chi)\big), \quad h(\delta) \in [-1, 1]. \tag{6}$$

For two 2TLNNs δ_1 and δ_2, based on Definition 3, then

(1) if $s(\delta_1) \prec s(\delta_2)$, then $\delta_1 \prec \delta_2$;
(2) if $s(\delta_1) \succ s(\delta_2)$, then $\delta_1 \succ \delta_2$;
(3) if $s(\delta_1) = s(\delta_2)$, $h(\delta_1) \prec h(\delta_2)$, then $\delta_1 \prec \delta_2$;
(4) if $s(\delta_1) = s(\delta_2)$, $h(\delta_1) \succ h(\delta_2)$, then $\delta_1 \succ \delta_2$;
(5) if $s(\delta_1) = s(\delta_2)$, $h(\delta_1) = h(\delta_2)$, then $\delta_1 = \delta_2$.

3.2. *The Distance Measurement of 2TLNNs*

Definition 4. Let $\delta_1 = \{(s_{t_1}, \alpha_1), (s_{i_1}, \beta_1), (s_{f_1}, \chi_1)\}$ and $\delta_2 = \{(s_{t_2}, \alpha_2), (s_{i_2}, \beta_2), (s_{f_2}\chi_2)\}$ be two 2TLNNs, then we can get the normalized Hamming distance:

$$d^H(\delta_1, \delta_2) = \frac{1}{3}\left(\begin{array}{l} \left|\frac{\Delta^{-1}(s_{t_1},\alpha_1)-\Delta^{-1}(s_{t_2},\alpha_2)}{k}\right| + \left|\frac{\Delta^{-1}(s_{i_1},\beta_1)-\Delta^{-1}(s_{i_2},\beta_2)}{k}\right| \\ + \left|\frac{\Delta^{-1}(s_{f_1},\chi_1)-\Delta^{-1}(s_{f_2},\chi_2)}{k}\right| \end{array} \right). \tag{7}$$

DEFINITION 5. Let $\delta_1 = \{(s_{t_1}, \alpha_1), (s_{i_1}, \beta_1), (s_{f_1}, \chi_1)\}$ and $\delta_2 = \{(s_{t_2}, \alpha_2), (s_{i_2}, \beta_2), (s_{f_2}\chi_2)\}$ be two 2TLNNs, then we can get the normalized Euclidean distance:

$$d^E(\delta_1, \delta_2) = \sqrt{\frac{1}{3}\left(\begin{array}{l}\left|\frac{\Delta^{-1}(s_{t_1},\alpha_1)-\Delta^{-1}(s_{t_2},\alpha_2)}{k}\right|^2 + \left|\frac{\Delta^{-1}(s_{i_1},\beta_1)-\Delta^{-1}(s_{i_2},\beta_2)}{k}\right|^2 \\ +\left|\frac{\Delta^{-1}(s_{f_1},\chi_1)-\Delta^{-1}(s_{f_2},\chi_2)}{k}\right|^2\end{array}\right)}. \tag{8}$$

3.3. *The 2TLNNWA and 2TLNNWG Operators*

DEFINITION 6 (*See* Wang *et al.*, 2018a). Let $\delta_j = \{(s_{t_j}, \alpha_j), (s_{i_j}, \beta_j), (s_{f_j}, \chi_j)\}$, $(j = 1, 2, \dots, n)$ be a set of 2TLNNs, the 2TLNNWA and 2TLNNWG operators can be presented:

$$\begin{aligned}&\mathrm{2TLNNWA}(\delta_1, \delta_2, \dots, \delta_n)\\ &\quad = w_1\delta_1 \oplus w_2\delta_2 \otimes \cdots \oplus w_n\delta_n = \bigoplus_{j=1}^{n} w_j\delta_j\\ &\quad = \left\langle \begin{array}{l}\Delta\left(k\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta^{-1}(s_{t_j},\alpha_j)}{k}\right)^{w_j}\right)\right), \Delta\left(k\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(s_{i_j},\beta_j)}{k}\right)^{w_j}\right),\\ \Delta\left(k\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(s_{f_j},\chi_j)}{k}\right)^{w_j}\right).\end{array}\right\rangle\end{aligned} \tag{9}$$

and

$$\begin{aligned}&\mathrm{2TLNNWG}(\delta_1, \delta_2, \dots, \delta_n)\\ &\quad = (\delta_1)^{w_1} \otimes (\delta_2)^{w_2} \otimes \cdots \otimes (\delta_n)^{w_n} = \bigotimes_{j=1}^{n} (\delta_j)^{w_j}\\ &\quad = \left\langle \begin{array}{l}\Delta\left(k\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(s_{t_j},\alpha_j)}{k}\right)^{w_j}\right), \Delta\left(k\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta^{-1}(s_{i_j},\beta_j)}{k}\right)^{w_j}\right)\right),\\ \Delta\left(k\left(1-\prod_{j=1}^{n}(1-\frac{\Delta^{-1}(s_{f_j},\chi_j)}{k})^{w_j}\right)\right).\end{array}\right\rangle,\end{aligned} \tag{10}$$

where w_j is weighting vector of δ_j, $j = 1, 2, \dots, n$, which satisfies $0 \leqslant w_j \leqslant 1$, $\sum_{j=1}^{n} w_j = 1$.

4. The Traditional Fuzzy CODAS Model

The CODAS (combinative distance-based assessment) method, which was originally defined by Keshavarz Ghorabaee *et al.* (2016), uses the combinative form of two distance measurements, including Euclidean and Taxicab distances, which present accurate values to compute the assessment results of alternatives. However, this model cannot be applied in fuzzy environment. To overcome this disadvantage, Keshavarz Ghorabaee *et* *al.* (2017) extended the CODAS method to fuzzy environment and proposed the fuzzy CODAS model which used fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED) rather than the crisp distances. Suppose there are alternatives $\{\phi_1, \phi_2, \dots, \phi_m\}$, n attributes $\{O_1, O_2, \dots, O_n\}$ and t experts $\{d_1, d_2, \dots, d_t\}$, then the decision making steps are expressed as follows.

Step 1. Construct the evaluation matrix $R = [\phi_{ij}^t]_{m\times n}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ and calculate the average results matrix

$$r = [\phi_{ij}]_{m\times n}, \quad i = 1, 2, \dots, m,\ j = 1, 2, \dots, n$$

which can be depicted as follows:

$$R = [\phi_{ij}^t]_{m\times n} = \begin{array}{c} \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{array} \begin{array}{c} \begin{array}{cccc} \mathrm{O}_1 & \mathrm{O}_2 & \dots & \mathrm{O}_n \end{array} \\ \begin{bmatrix} \phi_{11}^t & \phi_{12}^t & \cdots & \phi_{1n}^t \\ \phi_{21}^t & \phi_{22}^t & \cdots & \phi_{2n}^t \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{m1}^t & \phi_{m2}^t & \cdots & \phi_{mn}^t \end{bmatrix} \end{array}, \tag{11}$$

$$r = [\phi_{ij}]_{m\times n} = \begin{array}{c} \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{array} \begin{array}{c} \begin{array}{cccc} \mathrm{O}_1 & \mathrm{O}_2 & \dots & \mathrm{O}_n \end{array} \\ \begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{m1} & \phi_{m2} & \dots & \phi_{mn} \end{bmatrix} \end{array}, \tag{12}$$

$$\phi_{ij} = \phi_{ij}^1 \oplus \phi_{ij}^2 \oplus \cdots \oplus \phi_{ij}^t, \tag{13}$$

where ϕ_{ij}^t $(i = 1, 2, \dots, m,\ j = 1, 2, \dots, n)$ denotes the evaluation information of alternative ϕ_i $(i = 1, 2, \dots, m)$ on attribute O_j $(j = 1, 2, \dots, n)$ by expert d^t and ϕ_{ij} means the average values of alternative ϕ_i with respect to attribute O_j $(j = 1, 2, \dots, n)$.

Step 2. Obtain the attribute's fuzzy weighting vector W^t which is given by each expert with respect to all attributes and compute the average fuzzy weighting vector W as follows:

$$W^t = [w_j^t]_{1\times n}, \tag{14}$$

$$W = [w_j]_{1\times n}, \tag{15}$$

$$w_j = w_j^1 \oplus w_j^2 \oplus \cdots \oplus w_j^t, \tag{16}$$

where w_j^t denotes the fuzzy weight of attribute O_j $(j = 1, 2, \dots, n)$ by expert d^t and w_j $(j = 1, 2, \dots, n)$ means the average fuzzy weight values of a attribute O_j $(j = 1, 2, \dots, n)$.

Step 3. Normalize the average results matrix $r = [\phi_{ij}]_{m\times n}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ based on the type of each attributes using the following formulae.
For benefit attributes:

$$N_{ij} = \phi_{ij}/\max_i(\phi_{ij}), \quad i = 1, 2, \dots, m, \; j = 1, 2, \dots, n. \tag{17}$$

For cost attributes:

$$N_{ij} = 1 - \phi_{ij}/\max_i(\phi_{ij}), \quad i = 1, 2, \dots, m, \; j = 1, 2, \dots, n. \tag{18}$$

Step 4. According to the normalized average matrix N_{ij} $(i = 1, 2, \dots, m, j = 1, 2, \dots, n)$ and average fuzzy weighting vector w_j $(j = 1, 2, \dots, n)$, the fuzzy weighted normalized average matrix WN_{ij} $(i = 1, 2, \dots, m, \; j = 1, 2, \dots, n)$ can be computed as:

$$WN_{ij} = w_j \otimes N_{ij} \quad (i = 1, 2, \dots, m, \; j = 1, 2, \dots, n). \tag{19}$$

Step 5. Determine the fuzzy negative solution (NS) based on the equation (20):

$$NS_j = \min_i(WN_{ij}) \quad (i = 1, 2, \dots, m, \; j = 1, 2, \dots, n). \tag{20}$$

Step 6. Calculate the fuzzy weighted Hamming distance (HD_i) and fuzzy weighted Euclidean distance (ED_i) between each alternatives and the negative solution (NS) according to Definition 4 and Definition 5:

$$HD_i = \sum_{j=1}^{n} d^H(WN_{ij}, NS_j), \tag{21}$$

$$ED_i = \sum_{j=1}^{n} d^E(WN_{ij}, NS_j). \tag{22}$$

Step 7. Determine the relative assessment (RA) matrix which is presented as follows:

$$RA = [p_{il}]_{m\times m}, \tag{23}$$

$$p_{il} = (ED_i - ED_l) + \big(\lambda(ED_i - ED_l) \times (HD_i - HD_l)\big), \tag{24}$$

where $i, l = 1, 2, \dots, m$ and λ is a threshold function that can be defined:

$$\lambda(x) = \begin{cases} 1 & \text{if } |x| \geqslant \theta, \\ 0 & \text{if } |x| < \theta. \end{cases} \tag{25}$$

The threshold parameter θ of this function can be set by the decision maker. In our paper, we let $\theta = 0.02$ for the calculations.

Step 8. Compute the values of assessment score (AS) based on each alternative's using the following equation:

$$AS_i = \sum_{l=1}^{m} p_{il}. \tag{26}$$

Step 9. According to the calculation results of AS_i, we can rank all the alternatives. The bigger the value of AS_i is, the better alternative will be selected.

5. The CODAS Model with 2-Tuple Linguistic Neutrosophic Information

By combining the CODAS method with 2-tuple linguistic neutrosophic information we can build the 2-tuple linguistic neutrosophic CODAS model where all the evaluation information and attribute's weighting vector are presented with 2-tuple linguistic neutrosophic numbers (2TLNNs). Suppose there are m alternatives $\{\phi_1, \phi_2, \dots, \phi_m\}$, n attributes $\{O_1, O_2, \dots, O_n\}$ and t experts $\{d_1, d_2, \dots, d_t\}$, let expert's weighting vector be $\{a_1, a_2, \dots, a_t\}$, then the decision making steps are expressed as follows.

Step 1. Construct the 2-tuple linguistic neutrosophic evaluation matrix $R = [\phi_{ij}^t]_{m\times n}$, $i = 1, 2, \dots, m,\ j = 1, 2, \dots, n$ and calculate the average results matrix

$$r = [\phi_{ij}]_{m\times n}, \quad i = 1, 2, \dots, m,\ j = 1, 2, \dots, n,$$

which can be depicted as follows:

$$R = \left[\phi_{ij}^t\right]_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} \mathrm{O}_1 & \mathrm{O}_2 & \dots & \mathrm{O}_n \end{array} \\ \begin{array}{c} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{array} & \begin{bmatrix} \phi_{11}^t & \phi_{12}^t & \cdots & \phi_{1n}^t \\ \phi_{21}^t & \phi_{22}^t & \cdots & \phi_{2n}^t \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{m1}^t & \phi_{m2}^t & \cdots & \phi_{mn}^t \end{bmatrix} \end{array}, \tag{27}$$

$$r = [\phi_{ij}]_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} \mathrm{O}_1 & \mathrm{O}_2 & \dots & \mathrm{O}_n \end{array} \\ \begin{array}{c} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{array} & \begin{bmatrix} \phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\ \phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{m1} & \phi_{m2} & \cdots & \phi_{mn} \end{bmatrix} \end{array}. \tag{28}$$

Based on the assessment information and expert's weighting vector $\{a_1, a_2, \dots, a_t\}$, the

$$r = [\phi_{ij}]_{m\times n}$$

can be calculated as in Wang *et al.* (2018c):

$$\phi_{ij} = a_1\phi_{ij}^1 \oplus a_2\phi_{ij}^2 \oplus \cdots \oplus a_t\phi_{ij}^t$$
$$= \left\{ \begin{array}{l} \Delta\Big(k\Big(1 - \prod_{d=1}^{t} (1 - \frac{\Delta^{-1}(s_{t_{ij}},\alpha_{ij})^t}{k})^{a_t}\Big)\Big), \\ \Delta\Big(k\prod_{d=1}^{t} \Big(\frac{\Delta^{-1}(s_{i_{ij}},\beta_{ij})^t}{k}\Big)^{a_t}\Big), \\ \Delta\Big(k\prod_{d=1}^{t} \Big(\frac{\Delta^{-1}(s_{f_{ij}},\chi_{ij})^t}{k}\Big)^{a_t}\Big) \end{array} \right\}, \tag{29}$$

where $\phi_{ij}^t = \{(s_{t_{ij}}, \alpha_{ij})^t, (s_{i_{ij}}, \beta_{ij})^t, (s_{f_{ij}}, \chi_{ij})^t\}$, $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ denotes the 2-tuple linguistic neutrosophic information of alternative ϕ_i $(i = 1, 2, \ldots, m)$ on attribute O_j $(j = 1, 2, \ldots, n)$ by expert d^t and $\phi_{ij} = \{(s_{t_{ij}}, \alpha_{ij}), (s_{i_{ij}}, \beta_{ij}), (s_{f_{ij}}, \chi_{ij})\}$ $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ means the average 2TLNNs values of alternative with respect to attribute O_j $(j = 1, 2, \ldots, n)$.

Step 2. Obtain the attribute's fuzzy weighting vector W^t which is given by each expert with respect to all attributes and compute the average fuzzy weighting vector W as follows:

$$W^t = [w_j^t]_{1\times n} = \{(s_{t_j}, \alpha_j)^t, (s_{i_j}, \beta_j)^t, (s_{f_j}, \chi_j)^t\}_{1\times n}, \tag{30}$$

$$W = [w_j]_{1\times n} = \{(s_{t_j}, \alpha_j), (s_{i_j}, \beta_j), (s_{f_j}, \chi_j)\}_{1\times n}. \tag{31}$$

Based on the operation rules of 2TLNNs, the $W = [w_j]_{1\times n}$ can be calculated as:

$$w_j = a_1 w_j^1 \oplus a_2 w_j^2 \oplus \cdots \oplus a_t w_j^t$$
$$= \left\{ \begin{array}{l} \Delta\Big(k\Big(1 - \prod_{d=1}^{t} \Big(1 - \frac{\Delta^{-1}(s_{t_j},\alpha_j)^t}{k}\Big)^{a_t}\Big)\Big), \\ \Delta\Big(k\prod_{d=1}^{t} \Big(\frac{\Delta^{-1}(s_{i_j},\beta_j)^t}{k}\Big)^{a_t}\Big), \Delta\Big(k\prod_{d=1}^{t} \Big(\frac{\Delta^{-1}(s_{f_j},\chi_j)^t}{k}\Big)^{a_t}\Big) \end{array} \right\}, \tag{32}$$

where $w_j^t = w_j = \{(s_{t_j}, \alpha_j)^t, (s_{i_j}, \beta_j)^t, (s_{f_j}, \chi_j)^t\}$ $(j = 1, 2, \ldots, n)$ denotes the fuzzy weight of attribute O_j $(j = 1, 2, \ldots, n)$ by expert d^t and $w_j = \{(s_{t_j}, \alpha_j), (s_{i_j}, \beta_j), (s_{f_j}, \chi_j)\}$, $(j = 1, 2, \ldots, n)$ means the average fuzzy weight values of attribute O_j $(j = 1, 2, \ldots, n)$.

Step 3. Normalize the average results matrix $r = [\phi_{ij}]_{m\times n}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ based on the type of each attributes using the following formulae.
For benefit attributes:

$$N_{ij} = \phi_{ij} = \{(s_{t_{ij}}, \alpha_{ij})', (s_{i_{ij}}, \beta_{ij})', (s_{f_{ij}}, \chi_{ij})'\}$$
$$= \{(s_{t_{ij}}, \alpha_{ij}), (s_{i_{ij}}, \beta_{ij}), (s_{f_{ij}}, \chi_{ij})\}, \quad i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n. \tag{33}$$

For cost attributes:

$$N_{ij} = k - \phi_{ij} = \begin{Bmatrix} (s_{t_{ij}}, \alpha_{ij})', \\ (s_{i_{ij}}, \beta_{ij})', \\ (s_{f_{ij}}, \chi_{ij})' \end{Bmatrix} = \begin{Bmatrix} \Delta(k - \Delta^{-1}(s_{t_{ij}}, \alpha_{ij})), \\ \Delta(k - \Delta^{-1}(s_{i_{ij}}, \beta_{ij})), \\ \Delta(k - \Delta^{-1}(s_{f_{ij}}, \chi_{ij})), \end{Bmatrix}, \tag{34}$$

$i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.$

Step 4. According to the normalized average matrix $N_{ij} = \{(s_{t_{ij}}, \alpha_{ij})', (s_{i_{ij}}, \alpha_{ij})', (s_{f_{ij}}, \alpha_{ij})'\}$ and average fuzzy weighting vector $w_j = \{(s_{t_j}, \alpha_j), (s_{i_j}, \beta_j), (s_{f_j}, \chi_j)\}$ $(j = 1, 2, \ldots, n)$ the fuzzy weighted normalized average matrix $WN_{ij} = \{(s_{t_{ij}}, \alpha_{ij})'', (s_{i_{ij}}, \beta_{ij})'', (s_{f_{ij}}, \chi_{ij})''\}$ $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ can be computed as:

$$\begin{aligned} WN_{ij} &= w_j \otimes N_{ij} = \left\{(s_{t_{ij}}, \alpha_{ij})'', (s_{i_{ij}}, \beta_{ij})'', (s_{f_{ij}}, \chi_{ij})''\right\} \\ &= \left\{ \begin{array}{l} \Delta\left(k\left(\frac{\Delta^{-1}(s_{t_{ij}}, \alpha_{ij})'}{k} \cdot \frac{\Delta^{-1}(s_{t_j}, \alpha_j)}{k}\right)\right), \\ \Delta\left(1 - \left(1 - \frac{\Delta^{-1}(s_{i_{ij}}, \beta_{ij})'}{k}\right) \cdot \left(1 - \frac{\Delta^{-1}(s_{i_j}, \beta_j)}{k}\right)\right), \\ \Delta\left(1 - \left(1 - \frac{\Delta^{-1}(s_{f_{ij}}, \chi_{ij})'}{k}\right) \cdot \left(1 - \frac{\Delta^{-1}(s_{f_j}, \chi_j)}{k}\right)\right) \end{array} \right\}, \end{aligned} \tag{35}$$

where $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.$

Step 5. Determine the fuzzy negative solution (NS) based on the equation (20)

$$NS_j = \min_i(WN_{ij}) = \left\{\min_i (s_{t_{ij}}, \alpha_{ij})'', \max_i (s_{i_{ij}}, \beta_{ij})'', \max_i (s_{f_{ij}}, \chi_{ij})''\right\}, \tag{36}$$

where $i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n.$

Step 6. Calculate the fuzzy weighted Hamming distance and fuzzy weighted Euclidean distance between each alternatives and the negative solution (NS) according to Definition 4 and Definition 5:

$$\begin{aligned} HD_i &= \sum_{j=1}^{n} d^H(WN_{ij}, NS_j) \\ &= \sum_{j=1}^{n} \left(\frac{1}{3} \left\{ \begin{array}{l} \left|\frac{\Delta^{-1}(s_{t_{ij}}, \alpha_{ij})'' - \min_i \Delta^{-1}(s_{t_{ij}}, \alpha_{ij})''}{k}\right| \\ + \left|\frac{\Delta^{-1}(s_{i_{ij}}, \beta_{ij})'' - \max_i \Delta^{-1}(s_{i_{ij}}, \beta_{ij})''}{k}\right| \\ + \left|\frac{\Delta^{-1}(s_{f_{ij}}, \chi_{ij})'' - \max_i \Delta^{-1}(s_{f_{ij}}, \chi_{ij})''}{k}\right| \end{array} \right\} \right), \end{aligned} \tag{37}$$

$$ED_i = \sum_{j=1}^{n} d^E(WN_{ij}, NS_j)$$

$$= \sqrt{\sum_{j=1}^{n}\left(\frac{1}{3}\left\{\begin{array}{l}\left|\frac{\Delta^{-1}(s_{t_{ij}},\alpha_{ij})''-\min_i\Delta^{-1}(s_{t_{ij}},\alpha_{ij})''}{k}\right|^2\\ +\left|\frac{\Delta^{-1}(s_{i_{ij}},\beta_{ij})''-\max_i\Delta^{-1}(s_{i_{ij}},\beta_{ij})''}{k}\right|^2\\ +\left|\frac{\Delta^{-1}(s_{f_{ij}},\chi_{ij})''-\max_i\Delta^{-1}(s_{f_{ij}},\chi_{ij})''}{k}\right|^2\end{array}\right\}\right)}. \tag{38}$$

Step 7. Determine the relative assessment (RA) matrix which is presented as follows.

$$RA = [p_{il}]_{m\times m}, \tag{39}$$

$$p_{il} = (ED_i - ED_l) + \big(\lambda(ED_i - ED_l) \times (HD_i - HD_l)\big), \tag{40}$$

where $i, l = 1, 2, \dots, m$ and λ is a threshold function which can be defined:

$$\lambda(x) = \begin{cases} 1 & \text{if } |x| \geqslant \theta, \\ 0 & \text{if } |x| < \theta. \end{cases} \tag{41}$$

The threshold parameter θ of this function can be set by the decision maker. In our paper, we let $\theta = 0.02$ for the calculations.

Step 8. Compute the values of assessment score (AS) based on each alternative's p_{il} using the following equation:

$$AS_i = \sum\nolimits_{l=1}^{m} p_{il}. \tag{42}$$

Step 9. According to the calculation results of AS_i, we can rank all the alternatives. The bigger the value of AS_i is, the better alternative will be selected.

Thus, the decision making model can be described as:

Step 1. Construct the 2-tuple linguistic neutrosophic evaluation matrix and calculate the average results matrix by using the equation (29);
Step 2. Obtain the attribute's fuzzy weighting vector W^t and compute the average fuzzy weighting vector W by using the equation (32);
Step 3. Normalize the average results matrix based on the type of each attributes by using the equations (33) and (34);
Step 4. According to the normalized average matrix and average fuzzy weighting vector, compute the fuzzy weighted normalized average matrix WN_{ij} by using the equation (35);
Step 5. Determine the fuzzy negative solution (NS) by using the equation (36);
Step 6. Calculate the fuzzy weighted Hamming distance and the fuzzy weighted Euclidean distance between each alternatives and the NS by using the equations (37) and (38);
Step 7. Determine the relative assessment (RA) matrix by using the equation (40);
Step 8. Compute the values of assessment score (AS) by using the equation (42);
Step 9. According to the calculation results of AS_i, rank all the alternatives.

Table 1
Linguistic scale.

Linguistic terms	2-tuple linguistic neutrosophic numbers
Exceedingly Terrible – ET	$\{(s_0, 0), (s_5, 0), (s_6, 0)\}$
Very Terrible – VT	$\{(s_1, 0), (s_4, 0), (s_5, 0)\}$
Terrible – T	$\{(s_2, 0), (s_3, 0), (s_4, 0)\}$
Medium – M	$\{(s_3, 0), (s_3, 0), (s_3, 0)\}$
Well – W	$\{(s_4, 0), (s_3, 0), (s_2, 0)\}$
Very Well – VW	$\{(s_5, 0), (s_2, 0), (s_1, 0)\}$
Exceedingly Well – EW	$\{(s_6, 0), (s_1, 0), (s_0, 0)\}$

6. The Numerical Example

6.1. *Numerical for 2TLNNs MAGDM Problems*

The safety assessment of a construction project could be considered an MAGDM issue (Tang and Wei, 2019a; Tang *et al.*, 2019; Wang *et al.*, 2019f, 2019g). With the gradual progress of urbanization in China, the number of construction projects under development has increased. Thus, it is very important to evaluate the safety of construction projects. In this section, we provide a numerical example to select the best construction projects by using the CODAS model with 2-tuple linguistic neutrosophic information. In order to choose a suitable construction scheme for construction, assume there are five possible construction projects ϕ_i $(i = 1, 2, 3, 4, 5)$, which are provided by five famous construction companies with different construction advantages. In order to select the best construction project, invite some experts with experience of construction engineering and fuzzy set theory to construct the evaluation system in order to assess these construction projects. The evaluation index includes: (1) O_1 is the human factor in construction projects; (2) O_2 is the energy cost factor; (3) O_3 is the building materials and equipment factor; (4) O_4 is the environmental factor. The five possible construction projects ϕ_i $(i = 1, 2, 3, 4, 5)$ are to be evaluated with 2TLNNs according to the four criteria by three experts d^t (according to the professional years and the degree of authority of the expert, the weight of the expert is determined as $(0.3, 0.4, 0.3)$).

Step 1. Construct the 2-tuple linguistic neutrosophic evaluation matrix $R = [\phi_{ij}^t]_{m\times n}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. Then according to equation (29) and expert's weights, we can obtain the average results matrix $r = [\phi_{ij}]_{m\times n}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ as follows. (Take ϕ_{11} for example.)
According to Table 1, we can derive $\phi_{11}^1 = \{(s_4, 0), (s_3, 0), (s_2, 0)\}$, $\phi_{11}^2 = \{(s_5, 0), (s_2, 0), (s_1, 0)\}$, $\phi_{11}^3 = \{(s_4, 0), (s_3, 0), (s_2, 0)\}$, then we can get (Tables 2, 3, 4)

$$\begin{aligned}\phi_{11} &= a_1\phi_{11}^1 \oplus a_2\phi_{11}^2 \oplus a_3\phi_{11}^3\\ &= \left\{\begin{array}{l}\Delta\big(6\big(1-\big(1-\frac{4}{6}\big)^{0.3}\times\big(1-\frac{5}{6}\big)^{0.4}\times\big(1-\frac{4}{6}\big)^{0.3}\big)\big),\\ \Delta\big(6\big(\frac{3}{6}\big)^{0.3}\times\big(\frac{2}{6}\big)^{0.4}\times\big(\frac{3}{6}\big)^{0.3}\big), \Delta\big(6\big(\frac{2}{6}\big)^{0.3}\times\big(\frac{1}{6}\big)^{0.4}\times\big(\frac{2}{6}\big)^{0.3}\big)\end{array}\right\}\\ &= \big\{(s_4, 0.4843), (s_3, -0.4492), (s_2, -0.4843)\big\}.\end{aligned}$$

Table 2
Evaluation information by d^1.

	O_1 (benefit)	O_2 (cost)	O_3 (benefit)	O_4 (benefit)
ϕ_1	W	M	VT	VW
ϕ_2	M	T	VW	W
ϕ_3	W	VW	VT	M
ϕ_4	VT	VW	M	VT
ϕ_5	VW	VT	M	T

Table 3
Evaluation information by d^2.

	O_1 (benefit)	O_2 (cost)	O_3 (benefit)	O_4 (benefit)
ϕ_1	VW	W	VT	M
ϕ_2	M	T	VW	W
ϕ_3	VT	M	VW	T
ϕ_4	T	W	M	VW
ϕ_5	M	W	VT	VW

Table 4
Evaluation information by d^3.

	O_1 (benefit)	O_2 (cost)	O_3 (benefit)	O_4 (benefit)
ϕ_1	W	VW	T	M
ϕ_2	VT	VT	M	W
ϕ_3	M	T	VW	W
ϕ_4	W	VW	VT	T
ϕ_5	W	VW	VT	T

Table 5
The average results matrix.

	O_1	O_2
ϕ_1	$\{(s_4,\ 0.4843), (s_3, -0.4492), (s_2, -0.4843)\}$	$\{(s_4,\ 0.1654), (s_3, -0.3436), (s_2, -0.1654)\}$
ϕ_2	$\{(s_3,\ -0.4968), (s_3, 0.2704), (s_3, 0.4968)\}$	$\{(s_2,\ -0.2769), (s_3, 0.2704), (s_4, 0.2769)\}$
ϕ_3	$\{(s_3,\ -0.2586), (s_3, 0.3659), (s_3, 0.2586)\}$	$\{(s_4,\ -0.3522), (s_3, -0.3436), (s_2, 0.3522)\}$
ϕ_4	$\{(s_3,\ -0.4740), (s_3, 0.2704), (s_3, 0.4740)\}$	$\{(s_5,\ -0.3195), (s_2, 0.3522), (s_1, 0.3195)\}$
ϕ_5	$\{(s_4,\ -0.1577), (s_3, -0.3436), (s_2, 0.1577)\}$	$\{(s_3,\ 0.3672), (s_3, 0.2704), (s_3, -0.3672)\}$
	O_3	O_4
ϕ_1	$\{(s_1,\ 0.3238), (s_4, -0.3307), (s_5, -0.3238)\}$	$\{(s_4,\ -0.1577), (s_3, -0.3436), (s_2, 0.1577)\}$
ϕ_2	$\{(s_5,\ -0.3904), (s_2, 0.2587), (s_1, 0.3904)\}$	$\{(s_4,\ 0.0000), (s_3,\ 0.0000), (s_2,\ 0.0000)\}$
ϕ_3	$\{(s_4,\ 0.3793), (s_2, 0.4623), (s_2, -0.3793)\}$	$\{(s_3,\ 0.0199), (s_3, 0.0000), (s_3, -0.0199)\}$
ϕ_4	$\{(s_3,\ -0.4968), (s_3, 0.2704), (s_3, 0.4968)\}$	$\{(s_4,\ -0.4565), (s_3, -0.2192), (s_2, 0.4565)\}$
ϕ_5	$\{(s_2,\ -0.0118), (s_3, 0.3659), (s_4, 0.0118)\}$	$\{(s_4,\ -0.1074), (s_3, -0.4492), (s_2, 0.1074)\}$

Similarly, we can obtain other average results and all the average results are listed in Table 5.

Step 2. Obtain the attribute's fuzzy weighting vector W^t which is given by each expert with respect to all attributes and compute the average fuzzy weighting vector W as

Table 6
Attribute's weighting vector given by decision-maker.

	O_1 (benefit)	O_2 (cost)	O_3 (benefit)	O_4 (benefit)
d^1	VT	W	VW	T
d^2	M	VT	W	M
d^3	VW	W	M	VT

Table 7
The normalized decision-making matrix N_{ij}.

	O_1	O_2
ϕ_1	$\{(s_4, 0.4843), (s_3, -0.4492), (s_2, -0.4843)\}$	$\{(s_2, -0.1654), (s_3, 0.3436), (s_4, 0.1654)\}$
ϕ_2	$\{(s_3, -0.4968), (s_3, 0.2704), (s_3, 0.4968)\}$	$\{(s_4, 0.2769), (s_3, -0.2704), (s_2, -0.2769)\}$
ϕ_3	$\{(s_3, -0.2586), (s_3, 0.3659), (s_3, 0.2586)\}$	$\{(s_2, 0.3522), (s_3, 0.3436), (s_4, -0.3522)\}$
ϕ_4	$\{(s_3, -0.4740), (s_3, 0.2704), (s_3, 0.4740)\}$	$\{(s_1, 0.3195), (s_4, -0.3522), (s_5, -0.3195)\}$
ϕ_5	$\{(s_4, -0.1577), (s_3, -0.3436), (s_2, 0.1577)\}$	$\{(s_3, -0.3672), (s_3, -0.2704), (s_3, 0.3672)\}$
	O_3	O_4
ϕ_1	$\{(s_1, 0.3238), (s_4, -0.3307), (s_5, -0.3238)\}$	$\{(s_4, -0.1577), (s_3, -0.3436), (s_2, 0.1577)\}$
ϕ_2	$\{(s_5, -0.3904), (s_2, 0.2587), (s_1, 0.3904)\}$	$\{(s_4, 0.0000), (s_3, 0.0000), (s_2, 0.0000)\}$
ϕ_3	$\{(s_4, 0.3793), (s_2, 0.4623), (s_2, -0.3793)\}$	$\{(s_3, 0.0199), (s_3, 0.0000), (s_3, -0.0199)\}$
ϕ_4	$\{(s_3, -0.4968), (s_3, 0.2704), (s_3, 0.4968)\}$	$\{(s_4, -0.4565), (s_3, -0.2192), (s_2, 0.4565)\}$
ϕ_5	$\{(s_2, -0.0118), (s_3, 0.3659), (s_4, 0.0118)\}$	$\{(s_4, -0.1074), (s_3, -0.4492), (s_2, 0.1074)\}$

follows (Table 6). According to expert's weighting vector and the equation (32), the attribute's weighting vector can be calculated as (take W_1 for example).
According to Table 1, we can derive $W^1 = \{(s_1, 0), (s_4, 0), (s_5, 0)\}$, $W^2 = \{(s_3, 0), (s_3, 0), (s_3, 0)\}$, $W^3 = \{(s_5, 0), (s_2, 0), (s_1, 0)\}$, then we can get

$$\begin{aligned} W_1 &= a_1 W^1 \oplus a_2 W^2 \oplus a_3 W^3 \\ &= \left\{ \begin{array}{l} \Delta\left(6\left(1 - \left(1 - \frac{1}{6}\right)^{0.3} \times \left(1 - \frac{3}{6}\right)^{0.4} \times \left(1 - \frac{5}{6}\right)^{0.3}\right)\right), \\ \Delta\left(6\left(\frac{4}{6}\right)^{0.3} \times \left(\frac{3}{6}\right)^{0.4} \times \left(\frac{2}{6}\right)^{0.3}\right), \Delta\left(6\left(\frac{5}{6}\right)^{0.3} \times \left(\frac{3}{6}\right)^{0.4} \times \left(\frac{1}{6}\right)^{0.3}\right) \end{array} \right\} \\ &= \left\{(s_3,\ 0.4850), (s_3, -0.1042), (s_3, 0.4850)\right\}. \end{aligned}$$

So the attribute weights are derived as:

$$W = \left\{ \begin{array}{l} \{(s_3, 0.4850), (s_3, -0.1042), (s_3, 0.4850)\}, \\ \{(s_3, 0.1146), (s_3, 0.3659), (s_3, -0.1146)\}, \\ \{(s_4, 0.1654), (s_3, -0.3436), (s_2, -0.1654)\}, \\ \{(s_2, 0.1880), (s_3, 0.2704), (s_4, -0.1880)\}. \end{array} \right\}.$$

Step 3. Normalize the average results matrix $r = [\phi_{ij}]_{m \times n}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ based on the type of each attributes by formulae (33) and (34).

Step 4. According to the normalized average matrix ($i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$) and average fuzzy weighting vector w_j ($j = 1, 2, \ldots, n$), the fuzzy weighted nor-

Table 8
The fuzzy weighted normalized average matrix N_{ij}.

	O_1	O_2
ϕ_1	$\{(s_3,-0.3954),(s_4,0.2156),(s_3,0.3954)\}$	$\{(s_1,-0.0476),(s_5,-0.1662),(s_5,0.0476)\}$
ϕ_2	$\{(s_1,0.4539),(s_5,-0.4122),(s_4,-0.4539)\}$	$\{(s_2,0.2202),(s_5,-0.4358),(s_4,-0.2202)\}$
ϕ_3	$\{(s_2,-0.4077),(s_5,-0.3628),(s_4,0.4077)\}$	$\{(s_1,0.2210),(s_5,-0.1662),(s_5,-0.2210)\}$
ϕ_4	$\{(s_1,0.4672),(s_5,-0.4122),(s_5,-0.4672)\}$	$\{(s_1,-0.3150),(s_5,-0.0327),(s_5,0.3150)\}$
ϕ_5	$\{(s_2,0.2317),(s_4,0.2702),(s_4,-0.2317)\}$	$\{(s_1,0.3667),(s_5,-0.4358),(s_5,-0.3667)\}$
	O_3	O_4
ϕ_1	$\{(s_1,-0.0810),(s_5,-0.2988),(s_5,0.0810)\}$	$\{(s_1,0.4011),(s_4,0.4789),(s_5,-0.4011)\}$
ϕ_2	$\{(s_3,0.2001),(s_4,-0.0849),(s_3,-0.2001)\}$	$\{(s_1,0.4586),(s_5,-0.3648),(s_5,-0.4586)\}$
ϕ_3	$\{(s_3,0.0403),(s_4,0.0286),(s_3,-0.0403)\}$	$\{(s_1,0.1011),(s_5,-0.3648),(s_5,-0.1011)\}$
ϕ_4	$\{(s_2,-0.2622),(s_4,0.4789),(s_4,0.2622)\}$	$\{(s_1,0.2922),(s_5,-0.4645),(s_5,-0.2922)\}$
ϕ_5	$\{(s_1,0.3802),(s_5,-0.4679),(s_5,-0.3802)\}$	$\{(s_1,0.4195),(s_4,0.4309),(s_5,-0.4195)\}$

malized average matrix WN_{ij} $(i=1,2,\dots,m,\ j=1,2,\dots,n)$ can be computed as (take WN_{11} for example):

$$\begin{aligned}WN_{11} &= w_j \otimes N_{ij}\\ &= \left\{\begin{array}{l}\Delta\left(6\left(\frac{4.4843}{6}\cdot\frac{3.4850}{6}\right)\right), \Delta\left(1-\left(1-\frac{2.5508}{6}\right)\cdot\left(1-\frac{2.8958}{6}\right)\right),\\ \Delta\left(1-\left(1-\frac{1.5157}{6}\right)\cdot\left(1-\frac{3.4850}{6}\right)\right)\end{array}\right\}\\ &= \big\{(s_3,-0.3954),(s_4,0.2156),(s_3,0.3954)\big\}.\end{aligned}$$

Thus, the weighted normalized average matrix WN_{ij} $(i=1,2,\dots,m,\ j=1,2,\dots,n)$ is derived as follows (Table 8).

Step 5. Determine the fuzzy negative solution (NS):

$$NS_j = \min_i(WN_{ij}) = \left\{\begin{array}{l}\{(s_1,0.4539),(s_5,-0.3959),(s_5,-0.4539)\},\\ \{(s_1,-0.3150),(s_5,-0.0327),(s_5,0.3150)\},\\ \{(s_1,-0.0810),(s_5,-0.2988),(s_5,0.0810)\},\\ \{(s_1,0.1011),(s_5,-0.3648),(s_5,-0.1011)\}\end{array}\right\}.$$

Step 6. Calculate the fuzzy weighted Hamming distance (HD_i) and fuzzy weighted Euclidean distance (ED_i) between each alternatives and the negative solution (NS) according to Definition 4 and Definition 5.

For example:

$$HD_1 = \sum_{j=1}^{n} d^H(WN_{ij}, NS_j)$$

Table 9
The relative assessment matrix $(RA)_{m\times m}$.

	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
ϕ_1	0.0000	0.6437	0.2699	−0.1937	0.1626
ϕ_2	−0.6437	0.0000	−0.3738	−0.8374	−0.4811
ϕ_3	−0.2699	0.3738	0.0000	−0.4636	−0.1073
ϕ_4	0.1937	0.8374	0.4636	0.0000	0.3563
ϕ_5	−0.1626	0.4811	0.1073	−0.3563	0.0000

$$= \begin{pmatrix} \frac{1}{3}\left\{\left|\frac{2.6046-1.4539}{6}\right| + \left|\frac{4.2156-4.6372}{6}\right| + \left|\frac{3.3954-4.5461}{6}\right|\right\} \\ +\frac{1}{3}\left\{\left|\frac{0.9524-0.6850}{6}\right| + \left|\frac{4.8338-4.9673}{6}\right| + \left|\frac{5.0476-5.3150}{6}\right|\right\} \\ +\frac{1}{3}\left\{\left|\frac{0.9190-0.9190}{6}\right| + \left|\frac{4.7012-4.7012}{6}\right| + \left|\frac{5.0810-5.0810}{6}\right|\right\} \\ +\frac{1}{3}\left\{\left|\frac{1.4011-1.1011}{6}\right| + \left|\frac{4.4789-4.6352}{6}\right| + \left|\frac{4.5989-4.8989}{6}\right|\right\} \end{pmatrix}$$
$$= 0.2304,$$

$$ED_1 = \sum_{j=1}^{n} d^E(WN_{ij}, NS_j)$$
$$= \sqrt{\begin{pmatrix} \frac{1}{3}\left\{\left|\frac{2.6046-1.4539}{6}\right|^2 + \left|\frac{4.2156-4.6372}{6}\right|^2 + \left|\frac{3.3954-4.5461}{6}\right|^2\right\} \\ +\frac{1}{3}\left\{\left|\frac{0.9524-0.6850}{6}\right|^2 + \left|\frac{4.8338-4.9673}{6}\right|^2 + \left|\frac{5.0476-5.3150}{6}\right|^2\right\} \\ +\frac{1}{3}\left\{\left|\frac{0.9190-0.9190}{6}\right|^2 + \left|\frac{4.7012-4.7012}{6}\right|^2 + \left|\frac{5.0810-5.0810}{6}\right|^2\right\} \\ +\frac{1}{3}\left\{\left|\frac{1.4011-1.1011}{6}\right|^2 + \left|\frac{4.4789-4.6352}{6}\right|^2 + \left|\frac{4.5989-4.8989}{6}\right|^2\right\} \end{pmatrix}}$$
$$= 0.2439.$$

Similarly, we can obtain

$$HD_1 = 0.2304, \quad HD_2 = 0.5326, \quad HD_3 = 0.3554, \quad HD_4 = 0.1343,$$
$$HD_5 = 0.3123, \quad ED_1 = 0.2439, \quad ED_2 = 0.5854, \quad ED_3 = 0.3887,$$
$$ED_4 = 0.1463, \quad ED_5 = 0.3246.$$

Step 7. Determine the relative assessment (RA) matrix which is presented as follows (Table 9).

Step 8. Compute the values of assessment score (AS) based on each alternative's p_{il}

$$AS_1 = -0.8824, \quad AS_2 = 2.3360, \quad AS_3 = 0.4669,$$
$$AS_4 = -1.8510, \quad AS_5 = -0.0695.$$

Table 10
Ordering by different parameter θ.

Parameter	AS_1	AS_1	AS_1	AS_1	AS_1	Ordering
θ =0.05	−0.8824	2.3360	0.4669	−1.8510	−0.0695	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
θ =0.06	−0.8824	2.3360	0.4669	−1.8510	−0.0695	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
θ =0.07	−0.8824	2.3360	0.4238	−1.8510	−0.0264	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
θ =0.10	−0.8966	2.3360	0.4238	−1.7549	−0.1083	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
θ =0.20	−0.7716	2.1588	0.4760	−1.5769	−0.2863	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
θ =0.30	−0.7716	1.9386	0.2549	−1.3558	−0.0660	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
θ =0.40	−0.4695	1.6364	0.2549	−1.3558	−0.0660	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$

Step 9. According to the calculation results of AS, we can rank all the alternatives. The bigger the value of AS is, the better alternative will be selected. Obviously, the rank of all alternatives is $\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$ and ϕ_2 is the best alternative.

6.2. *Sensitivity Analysis*

To show the influence of the threshold parameter θ which is set by the decision maker, the ordering of the alternatives is shown as follows.

From Table 10, we can easily find that the ordering of alternatives is the same, which indicates that our proposed 2TLNN CODAS model is robust and effective. At the same time, when the threshold parameter $\theta \leqslant 0.06$, the assessment score (AS) remains the same, which indicates that the Hamming distance and the Euclidean distance are considered; when the threshold parameter $\theta \geqslant 0.50$, the assessment score (AS) also remains the same, which indicates only the Euclidean distance are considered. In other words, when the threshold parameter $0.06 \leqslant \theta \leqslant 0.50$, the Euclidean distance is considered absolutely and the Hamming distance is considered partly. In addition, the absolute values of assessment scores become smaller with the increase of the parameter. Thus, the decision maker can obtain different assessment scores by altering the threshold parameter.

6.3. *Compare 2TLNNs CODAS Method with Some 2TLNNs Aggregation Operators*

In this chapter, we compare our proposed 2-tuple linguistic neutrosophic CODAS method with the 2-tuple linguistic neutrosophic weighted average (2TLNNWA) operator and 2-tuple linguistic neutrosophic weighted geometric (2TLNNWG) operator. For the attribute's weights that are presented by 2TLNNs we can use the score function to obtain the attribute's weights with crisp number.

According to the value of the average attribute's weighting vector W which is listed as:

$$W = \left\{ \begin{array}{l} \{(s_3, 0.4850), (s_3, -0.1042), (s_3, 0.4850)\}, \\ \{(s_3, 0.1146), (s_3, 0.3659), (s_3, -0.1146)\}, \\ \{(s_4, 0.1654), (s_3, -0.3436), (s_2, -0.1654)\}, \\ \{(s_2, 0.1880), (s_3, 0.2704), (s_4, -0.1880)\} \end{array} \right\}.$$

Table 11
The fused values by using some 2TLNNs aggregation operator.

	2TLNNWA	2TLNNWG
ϕ_1	$\{(s_3, 0.0885), (s_3, 0.0654), (s_3, -0.0885)\}$	$\{(s_2, 0.4201), (s_3, 0.1435), (s_4, -0.4201)\}$
ϕ_2	$\{(s_4, -0.0034), (s_3, -0.2500), (s_2, 0.0034)\}$	$\{(s_4, -0.2538), (s_3, -0.1964), (s_2, 0.2538)\}$
ϕ_3	$\{(s_3, 0.3493), (s_3, -0.0147), (s_3, -0.3493)\}$	$\{(s_3, 0.1133), (s_3, 0.0367), (s_3, -0.1133)\}$
ϕ_4	$\{(s_3, -0.4976), (s_3, 0.2545), (s_3, 0.4976)\}$	$\{(s_2, 0.3048), (s_3, 0.2810), (s_4, -0.3048)\}$
ϕ_5	$\{(s_3, 0.1103), (s_3, -0.1452), (s_3, -0.1103)\}$	$\{(s_3, -0.1259), (s_3, -0.1077), (s_3, -0.1259)\}$

Table 12
Score results of alternatives ϕ_i.

	2TLNNWA	2TLNNWG
$s(\phi_1)$	0.5062	0.4276
$s(\phi_2)$	0.6246	0.5938
$s(\phi_3)$	0.5396	0.5106
$s(\phi_4)$	0.4306	0.4071
$s(\phi_1)$	0.5203	0.4920

Table 13
Rank of Alternatives by some 2TLNNs aggregation operators.

	order
2TLNNWA	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
2TLNNWG	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$
2TLNNs CODAS model	$\phi_2 > \phi_3 > \phi_5 > \phi_1 > \phi_4$

We can obtain the score results as:

$$W_1 = 0.5597, \qquad W_2 = 0.4924, \qquad W_3 = 0.6486, \qquad W_4 = 0.3948.$$

Then the normalized results w_j $(j = 1, 2, \cdots, n)$ can be expressed as:

$$w_j = W_i \Big/ \sum_{i=1}^{n} W_i, \tag{43}$$

$$w_1 = 0.2671, \qquad w_2 = 0.2350, \qquad w_3 = 0.3095, \qquad w_4 = 0.1884.$$

Based on the attribute's weight and the results in Table 7, the fused values by 2TLNNWA and 2TLNNWG operators are shown in Table 11.

According to the score function of 2TLNNs, we can obtain the alternative score results which are shown in Table 12.

The ranking of alternatives by some 2TLNNs aggregation operators are listed as follows (Table 13).

Comparing the results of the 2-tuple linguistic neutrosophic CODAS model with 2TLNNWA and 2TLNNWG operators, it can be noted that the aggregation results are slightly different in ranking of alternatives and the best alternatives are the same. However, 2-tuple linguistic neutrosophic CODAS model has important characteristics of using the combinative form of two distance measurements, including fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED) and can be more accurate and effective in the application of MADM problems.

7. Conclusion

In this paper, we present the 2-tuple linguistic neutrosophic CODAS model based on the traditional fuzzy CODAS (combinative distance-based assessment) model and some fundamental theories of 2-tuple linguistic neutrosophic information. Firstly, we briefly review the definition of 2-tuple linguistic neutrosophic sets (2TLNNSs) and introduce the score function, the accuracy function, operation laws and some aggregation operators of 2TLNNs. Then, the calculation steps of traditional fuzzy CODAS model are briefly presented. Furthermore, by combining the traditional fuzzy CODAS model with 2TLNNs information, the 2-tuple linguistic neutrosophic CODAS model is established and the computing steps are simply depicted. Finally, a numerical example for safety assessment of a construction project is given to illustrate this new model and some comparisons between 2TLNNs CODAS model and two 2TLNNs aggregation operators are also made to further illustrate advantages of the new method. In actual decision making applications, our developed model has the advantage of considering the combinative form of two distance measurements, including fuzzy weighted Hamming distance (HD) and fuzzy weighted Euclidean distance (ED). However, it is difficult to obtain assessment information which is expressed by 2TLNNs, so we need to continue to study this problem. In the future, the 2-tuple linguistic neutrosophic CODAS model can be applied to the risk analysis (Wei *et al.*, 2019h, 2018), the MADM problems (Wei *et al.*, 2019a; He *et al.*, 2019a; Tang and Wei, 2019b; Wei, 2019a, 2019b; Wei *et al.*, 2019c) and many other uncertain and fuzzy environments (Lu *et al.*, 2019a; Wei *et al.*, 2019c, 2019d, 2019f).

Funding

This paper is supported by the Natural Science Foundation of China (No. 71571128).

References

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87–96.
Atanassov, K. (1989). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 33, 37–46.
Badi, I., Ballem, M., Shetwan, A. (2018). Site selection of desalination plant in LIBYA by using combinative distance-based assessment (CODAS) method. *International Journal for Quality Research*, 12, 609–623.
Balali, V., Zahraie, B., Roozbahani, A. (2014). Integration of ELECTRE III and PROMETHEE II decision-making methods with an interval approach: application in selection of appropriate structural systems. *Journal of Computing in Civil Engineering*, 28, 297–314.
Bolturk, E. (2018). Pythagorean fuzzy CODAS and its application to supplier selection in a manufacturing firm. *Journal of Enterprise Information Management*, 31, 550–564.

Bolturk, E., Kahraman, C. (2018a). Interval-valued intuitionistic fuzzy CODAS method and its application to wave energy facility location selection problem. *Journal of Intelligent & Fuzzy Systems*, 35, 4865–4877.

Bolturk, E., Karasan, A. (2018b). *Interval Valued Neutrosophic CODAS Method for Renewable Energy Selection*, Vol. 11.

Chen, C.T. (2000). Extensions of the TOPSIS for group decision-making under fuzzy environment. *Fuzzy Sets and Systems*, 114, 1–9.

Deng, X.M., Gao, H. (2019). TODIM method for multiple attribute decision making with 2-tuple linguistic Pythagorean fuzzy information. *Journal of Intelligent & Fuzzy Systems*, 37, 1769–1780.

Gao, H., Lu, M., Wei, Y. (2019). Dual hesitant bipolar fuzzy hamacher aggregation operators and their applications to multiple attribute decision making. *Journal of Intelligent & Fuzzy Systems*, 37, 5755–5766.

Gomes, L., Lima, M. (1979). TODIM: basics and application to multicriteria ranking of projects with environmental impacts. *Foundations of Computing and Decision Sciences*, 16, 113–127.

He, T.T., Wei, G.W., Lu, J.P., Wei, C., Lin, R. (2019a). Pythagorean 2-tuple linguistic VIKOR method for evaluating human factors in construction project management. *Mathematics*, 7, 1149.

He, T.T., Wei, G.W., Lu, C.W., Wei, C., Lin, R. (2019b). Pythagorean 2-tuple linguistic taxonomy method for supplier selection in medical instrument industries. *International Journal of Environmental Research and Public Health*, 16, 4875.

Herrera, F., Martinez, L. (2001). The 2-tuple linguistic computational model. Advantages of its linguistic description, accuracy and consistency. *International Journal of Uncertainty Fuzziness and Knowledge-Based Systems*, 9, 33–48.

Ju, D.W., Ju, Y.B., Wang, A.H. (2018). Multiple attribute group decision making based on Maclaurin symmetric mean operator under single-valued neutrosophic interval 2-tuple linguistic environment. *Journal of Intelligent & Fuzzy Systems*, 34, 2579–2595.

Karasan, A., Bolturk, E., Kahraman, C. (2019). An integrated methodology using neutrosophic CODAS & fuzzy inference system: assessment of livability index of urban districts. *Journal of Intelligent & Fuzzy Systems*, 36, 5443–5455.

Keshavarz Ghorabaee, M., Zavadskas, E.K., Olfat, L., Turskis, Z. (2015). Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). *Informatica*, 26, 435–451.

Keshavarz Ghorabaee, M., Zavadskas, E.K., Turskis, Z., Antucheviciene, J. (2016). A new combinative distance distance-based assessment (CODAS) method for multi-criteria decision-making. *Economic Computation and Economic Cybernetics Studies and Research*, 50, 25–44.

Keshavarz Ghorabaee, M., Amiri, M., Zavadskas, E.K., Hooshmand, R., Antucheviciene, J. (2017). Fuzzy extension of the CODAS method for multi-criteria market segment evaluation. *Journal of Business Economics and Management*, 18, 1–19.

Li, Z.X., Lu, M. (2019). Some novel similarity and distance and measures of Pythagorean fuzzy sets and their applications. *Journal of Intelligent & Fuzzy Systems*, 37, 1781–1799.

Lu, J.P., Wei, C. (2019). TODIM method for performance appraisal on social-integration-based rural reconstruction with interval-valued intuitionistic fuzzy information. *Journal of Intelligent & Fuzzy Systems*, 37, 1731–1740.

Lu, J.P., Tang, X.Y., Wei, G.W., Wei, C., Wei, Y. (2019a). Bidirectional project method for dual hesitant Pythagorean fuzzy multiple attribute decision-making and their application to performance assessment of new rural construction. *International Journal of Intelligent Systems*, 34, 1920–1934.

Lu, J.P., Wei, C., Wu, J., Wei, G.W. (2019b). TOPSIS method for probabilistic linguistic MAGDM with entropy weight and its application to supplier selection of new agricultural machinery products. *Entropy*, 21, 953.

Opricovic, S., Tzeng, G.H. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156, 445–455.

Pamucar, D., Cirovic, G. (2015). The selection of transport and handling resources in logistics centers using multi-attributive border approximation area comparison (MABAC). *Expert Systems with Applications*, 42, 3016–3028.

Pamucar, D., Badi, I., Sanja, K., Obradovic, R. (2018). A novel approach for the selection of power-generation technology using a linguistic neutrosophic CODAS method: a case study in Libya. *Energies*, 11.

Peng, X.D., Garg, H. (2018). Algorithms for interval-valued fuzzy soft sets in emergency decision making based on WDBA and CODAS with new information measure. *Computers & Industrial Engineering*, 119, 439–452.

Rashid, T., Faizi, S., Xu, Z.S., Zafar, S. (2018). ELECTRE-based outranking method for multi-criteria decision making using hesitant intuitionistic fuzzy linguistic term sets. *International Journal of Fuzzy Systems*, 20, 78–92.

Ren, J.Z. (2018). Sustainability prioritization of energy storage technologies for promoting the development of renewable energy: a novel intuitionistic fuzzy combinative distance-based assessment approach. *Renewable Energy*, 121, 666–676.

Smarandache, F. (1999). *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*. American Research Press, Rehoboth, NM, USA.

Smarandache, F. (2003). *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*. American Research Press, Phoenix, AZ, USA.

Tang, X.Y., Wei, G.W. (2019a). Dual hesitant Pythagorean fuzzy Bonferroni mean operators in multi-attribute decision making. *Archives of Control Sciences*, 29, 339–386.

Tang, X.Y., Wei, G.W. (2019b). Multiple attribute decision making with dual hesitant Pythagorean fuzzy information. *Cognitive Computation*, 11, 193–211.

Tang, X.Y., Wei, G.W., Gao, H. (2019). Models for multiple attribute decision making with interval-valued Pythagorean fuzzy muirhead mean operators and their application to green suppliers selection. *Informatica*, 30, 153–186.

Wang, R. (2019). Research on the application of the financial investment risk appraisal models with some interval number muirhead mean operators. *Journal of Intelligent & Fuzzy Systems*, 37, 1741–1752.

Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace Multistruct*, 4, 410–413.

Wang, J., Wei, G., Wei, Y. (2018a). Models for Green supplier selection with some 2-tuple linguistic neutrosophic number Bonferroni mean operators. *Symmetry-Basel*, 10.

Wang, J., Wei, G.W., Lu, M. (2018b). An extended VIKOR method for multiple criteria group decision making with triangular fuzzy neutrosophic numbers. *Symmetry-Basel*, 10, 497.

Wang, J., Wei, G.W., Lu, M. (2018c). TODIM method for multiple attribute group decision making under 2-tuple linguistic neutrosophic environment. *Symmetry-Basel*, 10.

Wang, J., Gao, H., Lu, M. (2019a). Approaches to strategic supplier selection under interval neutrosophic environment. *Journal of Intelligent & Fuzzy Systems*, 37, 1707–1730.

Wang, J., Gao, H., Wei, G. (2019b). Some 2-tuple linguistic neutrosophic number Muirhead mean operators and their applications to multiple attribute decision making. *Journal of Experimental & Theoretical Artificial Intelligence*, 31, 409–439.

Wang, J., Gao, H., Wei, G.W. (2019c). Some 2-tuple linguistic neutrosophic number Muirhead mean operators and their applications to multiple attribute decision making. *Journal of Experimental & Theoretical Artificial Intelligence*, 31, 409–439.

Wang, J., Lu, J., Wei, G., Lin, R., Wei, C. (2019d). Models for MADM with single-valued neutrosophic 2-tuple linguistic muirhead mean operators. *Mathematics*, 7.

Wang, J., Lu, J.P., Wei, G.W., Lin, R., Wei, C. (2019e). Models for MADM with single-valued neutrosophic 2-tuple linguistic muirhead mean operators. *Mathematics*, 7. https://doi.org/10.3390/math7050442.

Wang, J., Wei, G.W., Lu, J.P., Alsaadi, F.E., Hayat, T., Wei, C., Zhang, Y. (2019f). Some q-rung orthopair fuzzy Hamy mean operators in multiple attribute decision-making and their application to enterprise resource planning systems selection. *International Journal of Intelligent Systems*, 34, 2429–2458.

Wang, J., Wei, G.W., Wei, C., Wei, Y. (2019g). Dual hesitant q-rung orthopair fuzzy muirhead mean operators in multiple attribute decision making. *IEEE Access*, 7, 67139–67166.

Wang, P., Wang, J., Wei, G. (2019h). EDAS method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment. *Journal of Intelligent & Fuzzy Systems*, 37, 1597–1608.

Wang, P., Wang, J., Wei, G.W. (2019i). EDAS method for multiple criteria group decision making under 2-tuple linguistic neutrosophic environment. *Journal of Intelligent & Fuzzy Systems*, 37, 1597–1608.

Wang, P., Wang, J., Wei, G.W., Wei, C., Wei, Y. (2019j). The multi-attributive border approximation area comparison (MABAC) for multiple attribute group decision making under 2-tuple linguistic neutrosophic environment. *Informatica*, 30, 799–818.

Wei, Y., Yu, Q., Liu, J., Cao, Y. (2018). Hot money and China's stock market volatility: Further evidence using the GARCH-MIDAS model. *Physica A: Statistical Mechanics and its Applications*, 492, 923–930.

Wei, G.W. (2019a). 2-tuple intuitionistic fuzzy linguistic aggregation operators in multiple attribute decision making. *Iranian Journal of Fuzzy Systems*, 16, 159–174.

Wei, G.W. (2019b). The generalized dice similarity measures for multiple attribute decision making with hesitant fuzzy linguistic information. *Economic Research-Ekonomska Istrazivanja*, 32, 1498–1520.

Wei, G.W., Wei, C., Wu, J., Wang, H.J. (2019a). Supplier selection of medical consumption products with a probabilistic linguistic MABAC method. *International Journal of Environmental Research and Public Health*, 16, 5082.

Wei, G., Wu, J., Wei, C., Wang, J., Lu, J. (2019b). Models for MADM with 2-tuple linguistic neutrosophic Dombi Bonferroni mean operators. *IEEE Access*, 7, 108878–108905.

Wei, G.W., Wang, J., Lu, M., Wu, J., Wei, C. (2019c). Similarity measures of spherical fuzzy sets based on cosine function and their applications. *IEEE Access*, 7, 159069–159080.

Wei, G.W., Wang, J., Wei, C., Wei, Y., Zhang, Y. (2019d). Dual hesitant Pythagorean fuzzy hamy mean operators in multiple attribute decision making. *IEEE Access*, 7, 86697–86716.

Wei, G.W., Wang, R., Wang, J., Wei, C., Zhang, Y. (2019e). Methods for evaluating the technological innovation capability for the high-tech enterprises with generalized interval neutrosophic number Bonferroni mean operators. *IEEE Access*, 7, 86473–86492.

Wei, G.W., Wu, J., Wei, C., Wang, J., Lu, J.P. (2019f). Models for MADM with 2-tuple linguistic neutrosophic Dombi Bonferroni mean operators. *IEEE Access*, 7, 108878–108905.

Wei, G.W., Zhang, S.Q., Lu, J.P., Wu, J., Wei, C. (2019g). An extended bidirectional projection method for picture fuzzy MAGDM and its application to safety assessment of construction project. *IEEE Access*, 7, 166138–166147.

Wei, Y., Qin, S., Li, X., Zhu, S.R., Wei, G. (2019h). Oil price fluctuation, stock market and macroeconomic fundamentals: Evidence from China before and after the financial crisis. *Finance Research Letters*, 30, 23–29.

Wu, Q., Wu, P., Zhou, L.G., Chen, H.Y., Guan, X.J. (2018a). Some new Hamacher aggregation operators under single-valued neutrosophic 2-tuple linguistic environment and their applications to multi-attribute group decision making. *Computers & Industrial Engineering*, 116, 144–162.

Wu, S., Wang, J., Wei, G., Wei, Y. (2018b). Research on construction engineering project risk assessment with some 2-tuple linguistic neutrosophic Hamy mean operators. *Sustainability*, 10.

Wu, L.P., Gao, H., Wei, C. (2019a). VIKOR method for financing risk assessment of rural tourism projects under interval-valued intuitionistic fuzzy environment. *Journal of Intelligent & Fuzzy Systems*, 37, 2001–2008.

Wu, L.P., Wang, J., Gao, H. (2019b). Models for competiveness evaluation of tourist destination with some interval-valued intuitionistic fuzzy Hamy mean operators. *Journal of Intelligent & Fuzzy Systems*, 36, 5693–5709.

Zadeh, L.A. (1965). Fuzzy Sets. *Information and Control*, 8, 338–356.

Zhang, S.Q., Wei, G.W., Gao, H., Wei, C., Wei, Y. (2019). EDAS method for multiple criteria group decision making with picture fuzzy information and its application to green suppliers selections. *Technological and Economic Development of Economy*, 26, 1123–1138.

P. Wang is a current master student at Institute of Technology at Sichuan Normal University, Chengdu, 610101, PR China.

J. Wang is a current master student at School of Business at Sichuan Normal University, Chengdu, 610101, PR China.

G. Wei has an MSc and a PhD degree in applied mathematics from SouthWest Petroleum University, business administration from school of Economics and Management at SouthWest Jiaotong University, China, respectively. From May 2010 to April 2012, he was a Postdoctoral Researcher with the School of Economics and Management, Tsinghua University, Beijing, China. He is a professor in the School of Business at Sichuan Normal University. He has published more than 100 papers in journals, books and conference proceedings including journals such as *Omega*, *Decision Support Systems*, *Expert Systems with Applications*, *Applied Soft Computing*, *Knowledge and Information Systems*, *Computers & Industrial Engineering*, *Knowledge-Based Systems*, *International Journal of Uncertainty*, *Fuzziness and Knowledge-Based Systems*, *International Journal of Computational Intelligence Systems and Information*: *An International Interdisciplinary Journal*. He has published 1 book. He has participated in several scientific committees and serves as a reviewer in a wide range of journals including *Computers & Industrial Engineering*, *International Journal of Information Technology and Decision Making*, *Knowledge-Based Systems*, *Information Sciences*, *International Journal of Computational Intelligence Systems* and *European Journal of Operational Research*. He is currently interested in aggregation operators, decision making and computing with words.

J. Wu has a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is a professor in the School of Statistics at Southwestern University of Finance and Economics, Chengdu, 611130, PR China.

C. Wei has an MSc degree in applied mathematics from SouthWest Petroleum University. Now, he is a PhD student with School of Statistics, Southwestern University of Finance and Economics, Chengdu, 611130, PR China. He has published more than 10 papers in journals, such as *International Journal of Intelligent Systems*, *Journal of Intelligent and Fuzzy Systems*, *IEEE Access*, *Mathematics*, *Information*. He is currently interested in aggregation operators, decision making and computing with words.

Y. Wei has an MSc and a PhD degree in management science and engineering from Southwest Jiaotong University, China. He is a professor in the School of Finance at Yunnan University of Finance and Economics. He has published more than 120 papers in journals, books and conference proceedings including journals such as *Journal of Forecasting*, *Journal of Banking & Finance*, *Empirical Economics*, *Energy Economics*, *Economic Modelling*, *Applied Economics Letters*, *International Review of Economics & Finance*. He has published two book (in Chinese). He has participated in several scientific committees and serves as a reviewer in a wide range of journals including *Energy Economics*, *Journal of Forecasting*, *Emerging Market Finance and Trade*, *Physica A*. He is currently interested in volatility modelling and forecasting in financial and energy markets, density forecasting and decision making in financial markets.