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A New Method of Multi-Criteria Analysis for Evaluation and Decision Making by Dominant Criterion

Miodrag M. ŽIŽOVIĆ[1], Miloljub ALBIJANIĆ[2]*, Verka JOVANOVIĆ[1], Mališa ŽIŽOVIĆ[1,3]
[1] *Singidunum University, Belgrade, Serbia*
[2] *FEFA, Metropolitan University, Belgrade, Serbia*
[3] *University of Kragujevac, Faculty of Technical Sciences in Čačak, Serbia*
e-mail: miodrgz@gmail.com, malbijanic@fefa.edu.rs, vjovanovic@singidunum.ac.rs, zizovic@gmail.com

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Abstract. This paper introduces a new method for multi-criteria analyses where the failure to meet the dominant criterion of an alternative causes low values for the entire alternative. In this method, the introduction of new alternatives into the multi-criteria model does not affect the existing alternatives in the model. The new method was applied for the rating of ten websites of dental clinics in Serbia, which provide prosthetic services to tourists. The dominant criterion was the amount of information provided by the site.

Key words: multi-criteria analysis, criteria, weight coefficients of the criteria, dominant criterion, website rating.

1. Introduction

Multi-criteria analysis or multi-criteria optimization, as a method and technique for evaluation and decision making, has been developed in different scientific disciplines, such as operational researches, statistics, management science, computer science (Ivanovic, 1973; Saaty, 1982; Mardani *et al.*, 2015a; Mardani *et al.*, 2015b; Brans and Vincke, 1985; Roberts and Goodwin, 2002). Multi-criteria decision analysis has been and continues to be applied in various fields of expertise, such as finance, logistics, transportation, marketing, public services, energy management, environmental planning (Dong and Xu, 2016; Huang *et al.*, 2011; Stanujkic *et al.*, 2017; Mardani *et al.*, 2015a, 2015b; Turskis and Zavadskas, 2011; Zavadskas *et al.*, 2017; Hashemkhani Zolfani *et al.*, 2013). In particular, in economics (Zavadskas *et al.*, 2016a, 2016b) and in engineering (Zavadskas *et al.*, 2016c; Šaparauskas *et al.*, 2011; Zavadskas *et al.*, 2014).

Today, many books are also available, in which an overview can be found of methods adjusted for various fields of application, since it is known that some of the methods

*Corresponding author.

Table 1
Initial decision matrix.

Alternatives	Criteria			
	C_1	C_2	$\cdots$	C_n
A_1	x_{11}	x_{12}	$\cdots$	x_{1n}
A_2	x_{21}	x_{22}	$\cdots$	x_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Am	x_{m1}	x_{m2}	$\cdots$	x_{mn}

can be more easily applied (or, they are better adjusted) to specific areas. Let us mention some of the published books (Zeleny, 1982; Figueira *et al.*, 2005; Hwang *et al.*, 1981; Triantaphyllou, 2000), among the recently published are (Huber *et al.*, 2019; Kaliszewski *et al.*, 2016; Köksalan *et al.*, 2011; Lee and Yang, 2017). See also conference proceedings (Sforza and Sterle, 2017).

Therefore, it should be noted that the most important factor in decision making, i.e. in choosing one of the alternatives or creating a preference order of alternatives, is, in fact, the decision maker. Namely, the decision maker has a decisive role in setting the weight coefficients for the criteria.

The criteria on the basis of which a decision is made are often conflicting; for some problems they are partially conflicting, and sometimes even completely conflicting. Also, they can be very diverse in their essence – sometimes those are cost values, sometimes various physical measurements, sometimes probabilities, sometimes subjective estimations (usually those of the decision maker) given in different possible scales, which are often created for a specific problem. Hence, we have measurement units, which are incomparable according to specific criteria.

We could say that the decision maker has the often-difficult task of comparing possible decisions.

The essential problem of a multi-criteria analysis can be shown in a simple manner through a decision-making matrix (Table 1): which contains n criteria $C_1, \ldots, C_n$, m alternatives $A_1, \ldots, A_m$ and values x_{ij}, $i = 1, \ldots, m$, $j = 1, \ldots, n$, where alternative A_i assumes according to criterion C_j. We also know, due to the nature of the problem, that criteria belong to the maximization type (bigger is better) or minimization type (smaller is better).

When it comes to solving real problems, it is obvious that almost never there is an alternative, which is optimal according to all of the criteria (an ideal alternative). Because of that, it is usually necessary to find a compromise, i.e. a "good" compromise or an alternative that will satisfy in the best manner possible, in a given situation, conditions set by the decision maker.

In order to achieve that, a valorization of criteria is performed, i.e. their importance is determined. Most commonly, the importance of criteria is established with weight coefficients (positive numbers whose sum equals one).

In almost all methods of multi-criteria analysis, the initial Table 1 is transformed into a numeric table, i.e. values according to criteria, which have a linguistic form, and which are translated, with the help of suitable transformation scales for linguistic expressions,

Table 2
Normalized decision matrix.

Alternatives	Criteria			
	C_1	C_2	$\cdots$	C_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
Am	a_{m1}	a_{m2}	$\cdots$	a_{mn}

into numbers. Thus, we can assume that Table 1 consists of numbers. The next step is to obtain, with Table 1 converted into numbers, a normalized table – Table 2, in which all values are unnamed numbers and all criteria are translated into a maximization type.

Hence, values a_{ij} are normalized, i.e. $0 \leqslant a_{ij} \leqslant 1$ and we also have, for example, that, $a_{ik} \leqslant a_{jk}$, $1 \leqslant i \leqslant m$, $1 \leqslant k \leqslant n$, $1 \leqslant j \leqslant m$.

This means that alternative A_i is weaker than alternative A_j according to criterion C_k.

Depending on how we perform the transformation of Table 1 into Table 2 and on how we now perform the evaluation of alternatives, from Table 2, by using weight coefficients of criteria whose sum equals one, we have a very large number of different models of multi-criteria analyses.

Of course, the above description does not include all available methods, an example is the method given in Žižović *et al.* (2016).

Descriptions of different types of normalizations can be found in the following papers: (Celen, 2014; Jahan and Edwards, 2015; Milani *et al.*, 2005).

Descriptions of different methods for finding weight coefficients can be found in the following papers: (Zavadskas and Podvezko, 2016; Krylovas *et al.*, 2017; Keshavarz Ghorabaee *et al.*, 2016).

2. A New Method

In the paper (Žižović *et al.*, 2017), the notion of a dominant criterion is defined: Criterion C_j is dominant for solving multi-criteria problems if the value of alternatives according to this criterion is extremely low; then there is no solution for that problem, or the solution obtained has an extremely small (weak) importance. ("Let C_j be the criterion which is the most dominant for the solution of multi-criteria problems, meaning that if its performance values of all alternatives are extremely low then problem should not be treated (it has no solution), or obtained solution of the problem has weak importance".)

In the paper (Žižović, 2018), a multi-criteria method is also given which disables the option for an alternative with a low value of the dominant criterion to obtain high value based on "good" values according to other criteria.

A new method of multi-criteria analysis is also given here, where an alternative with a low value of the dominant criterion has to have low general value as well, regardless of the values according to other criteria. That is to say that, the value of the dominant criterion, which is several times (more than two) lower than some other alternatives, cannot be compensated with higher values according to other criteria (those approximately similar can be compensated).

Let us assume that our starting point is the multi-criteria method given by the decision making matrix in Table 2, where the dominant criterion is C_1. Other criteria, $C_2, \dots, C_n$, are sorted according to their importance.

For the arbitrary alternative A_p, $1 \leqslant p \leqslant m$ we observe the comparison value in relation to other alternatives according to the formula:

$$V(A_p) = a_{p1} \cdot K_1 + \lambda \cdot a_{p1} \cdot (a_{p2} \cdot K_2 + a_{p3} \cdot K_3 + \cdots + a_{pn} \cdot K_n), \tag{1}$$

where a_{pi} are values which alternative A_p has according to criterion C_i $(i = 1, \dots, n)$; K_i is the weight coefficient of criterion C_i $(i = 1, \dots, n)$; λ – is a negative number, which determines the importance relation between the first dominant criterion and other criteria.

The comparison of alternatives A_p and A_q is performed according to the following rules:

Rule no. 1: Alternative A_p is better than alternative A_q if $V(A_p) > V(A_q)$;

Rule no. 2: If $V(A_p) = V(A_q)$ then A_p is better than A_q if $a_{p1} > a_{q1}$;

Rule no. 3: If $V(A_p) = V(A_q)$ and if $a_{p1} = a_{q1}$ then A_p is better than A_q if $a_{p2} > a_{q2}$;

Rule no. 4: If $V(A_p) = V(A_q)$ and $a_{p1} = a_{q1}$ and $a_{p2} = a_{q2}$ then A_p is better than A_q if $a_{p3} > a_{q3}$.

This proceeding continues up to the last comparison.

It is obvious that alternatives A_p and A_q would remain incomparable only if all the values are equal, i.e. if $a_{p1} = a_{q1}, a_{p2} = a_{q2}, \dots, a_{pn} = a_{qn}$.

3. Formula Analyses and Choice of Parametres

In formula (1), parameter λ has the important role of determining the influence of the first criterion in relation to other criteria in the model (or vice versa), or, in other words, determining the links and relations of the influence of the first criterion and other criteria on the defining of the ranking order of alternatives and, hence, decision making.

In order to have a better overview of this parameter and to make it easier for the decision maker to make his choice, let us notice, first, that for the arbitrary alternative

A_x with characteristics $a_{x1}, a_{x2}, \dots, a_{xn}$

according to formula (1) we get that

$$V(A_x) = a_{x1} \cdot k_1 + a_{x1} \cdot \lambda \cdot (a_{x2} \cdot k_2 + a_{x3} \cdot k_3 + \cdots + a_{xn} \cdot k_n) \geqslant a_{x1} \cdot k_1. \tag{2}$$

This is obvious from the condition which every alternative has to fulfil, and, of course, from the condition of parameter λ being set.

Let us just stress that this value is also realistically obtained for $a_2 = a_3 = \cdots = a_n = 0$ or for $\lambda = 0$.

The case of $\lambda = 0$ brings this method of multi-criteria analysis down to the familiar lexicographical order which is, basically, linear (the only case when two alternatives are not modified is when they have equal values according to all the criteria).

Value $V(A_x)$ can be limited on the upper side as well [alternative: can be bounded from above], that is to say, we get that

$$V(A_x) \leqslant a_{x1} \cdot (k_1 + \lambda \cdot (1 - k_1)). \tag{3}$$

This can be easily proven:

$$V(A_x) = a_{x1} \cdot k_1 + a_{x1} \cdot \lambda \cdot (a_{x2} \cdot k_2 + a_{x3} \cdot k_3 + \cdots + a_{xn} \cdot k_n).$$

If we know that $0 \leqslant a_{xi} \leqslant 1$, $i = 1, \ldots, n$ then

$$\begin{aligned} V(A_x) &\leqslant a_{x1} \cdot k_1 + a_{x1} \cdot \lambda \cdot (k_2 + k_3 + \cdots + k_n) \\ &= a_{x1} \cdot k_1 + a_{x1} \cdot \lambda \cdot (k_1 + k_2 + k_3 + \cdots + k_n - k_1) \\ &= a_{x1} \cdot k_1 + a_{x1} \cdot \lambda \cdot (1 - k_1) \\ &= a_{x1} \cdot (k_1 + \lambda \cdot (1 - k_1)). \end{aligned}$$

[Alternative: this upper bound is attained for:] The theoretical and the value on the right side can be reached for $a_{x2} = a_{x3} = \cdots = a_{xn} = 1$.

In the expression:

$$aA_{x1} \cdot k_1 \leqslant V(A_x) \leqslant a_{x1} \cdot \bigl(k_1 + \lambda \cdot (1 - k_1)\bigr) \tag{4}$$

with the choice of parameter λ we can pre-determine how large the right side would be in relation to the left side, if we observe the ratio of the right and the left side:

$$\bigl[a_{x1} \cdot \bigl(k_1 + \lambda \cdot (1 - k_1)\bigr)\bigr] \big/ (a_{x1} \cdot k_1) = \bigl[k_1 + \lambda \cdot (1 - k_1)\bigr] \big/ k_1 = t.$$

Here, it is obvious that $t > 0$ (it is assumed that $k_1 > 0$) and this number can be pre-determined. Hence, if we pre-determine, as a requirement, the relation between the maximum possible value and the measurement for an alternative according to the participation of the dominant criterion in that value, we can easily calculate the value for parameter λ:

$$\lambda = \bigl(k_1 \cdot (t - 1)\bigr) \big/ (1 - k_1) \tag{5}$$

for which we obtain the desired ratio.

Aside from this relation, it could be interesting for the decision maker to also have an estimate of the size of the difference of the maximal value for the alternative and the participation of the dominant criterion in that measurement compared to the contribution of the dominant criterion:

$$\theta = \bigl[a_{x1} \cdot \bigl(k_1 + \lambda \cdot (1 - k_1)\bigr) - a_{x1} \cdot k_1\bigr] \big/ (a_{x1} \cdot k_1) = \lambda \cdot (1 - k_1)/k_1. \tag{6}$$

Table 3
The choice of weight coefficient for the dominant criterion and quotient θ – an example.

k_1	θ			
	0.1	0.5	1	2
0.1	0.0111	0.0556	0.1111	0.2222
0.2	0.025	0.125	0.25	0.5
0.3	0.0429	0.2143	0.4286	0.8571
0.4	0.0667	0.3333	0.6667	1.3333
0.5	0.1	0.5	1	2
0.6	0.15	0.75	1.5	3
0.7	0.2333	1.1667	2.3333	4.6667
0.8	0.4	2	4	8
0.9	0.9	4.5	9	18

Here, we have the possibility of θ being greater than one – then the participation of other criteria in the maximum value is greater than the participation of the dominant criterion; that of θ being less than one – then the participation of other criterion in the maximum value is lower than the participation of the dominant criterion; and, finally, that $\theta = 1$ – here we have the equal participation of other criteria and the dominant criterion in the alternatives measuring. It is obvious that here, as well, we can easily get from (6) that

$$\lambda = \frac{\theta \cdot k_1}{1 - k_1} \tag{7}$$

and expressions (5) and (6) can be used in an equal manner for calculating the role of parameter λ and the influence it has on the connection in the participation of the first and other criteria in the measuring of the alternative preference order.

We find that expression (7) is more suitable for calculation, and the quotient θ is more suited for understanding; and as an illustration of values which were obtained, Table 3 provides examples of values calculated for parameter λ (rounded up to four decimals) depending on the choice of weight coefficient for the dominant criterion and quotient θ.

N.B.: When solving multi-criteria problems with this method, it is useful to observe the maximum value in the column corresponding to the dominant criterion C_1. Let us mark it with $a_{1\,\max}$ and compare values $a_{i1}\ (k_1 + (1 - k_1))$, $i = 1, \dots, m$.

With $a_{\max} \cdot k_1$.

In cases when

$$a_{i1}\big(k_1 + (1 - k_1)\big) < a_{\max} \cdot k_1.$$

Alternative A_i can be left out of the model, because it will be weaker than the alternative in which we have the highest value according to the dominant criterion.

4. Example

Medical tourism is very popular in the world of today. On one hand, we have people from countries with less developed medical systems going to those with more advanced health-care systems, looking for treatment for health problems which cannot be dealt with in their own countries.

On the other hand, we have people from highly developed countries, where medical care is expensive, going to countries where medical care is at the same quality level, but for a significantly lower price.

Today, the Republic of Serbia is a country whose medical staff performs high-quality medical services, but for a significantly lower price than those in developed countries. This is most prominent when it comes to dental care and cosmetic surgery, as well as some other medical specialities. Aside from the medical staff, hotel and restaurant owners, other people in the tourism economy are also interested in providing these services. The medical staff has been primarily interested in providing these services to our *Gastarbeiter* (people temporarily working in other countries). It was only recently that websites began to appear offering those services for foreign citizens as well. Because of the interest that tourism industry has here, we will perform the evaluation of the websites of several dental clinics targeting foreign citizens as well.

4.1. *Criteria for Rating Websites*

The following criteria were determined for rating websites:

C_1 – Information provided by the website;
C_2 – Search engine optimization;
C_3 – Design;
C_4 – Number of languages of the presentation;
C_5 – Activity on social networks;
C_6 – Visitors' ratings.

Within the first criterion, practical questions are evaluated, which could be asked by potential health tourists:

- What medical services can they expect?
- How much do those services cost?
- How long does it take to perform them?
- Is there a possibility to negotiate about dates that are convenient to customers?
- Can the beginning of the treatment be booked online?
- Can a tourist get recommendations on tourist attractions available if there is free time during the treatment?
- How much would that cost? etc.

In this evaluation, the first criterion is put in the dominant position, because, if it is not fulfilled, it is irrelevant or almost irrelevant whether other criteria were fulfilled.

The second criterion evaluates the position of the website after typing in either the subject or key words related to the subject of the search into a search engine. Marks for

Table 4
Initial decision matrix – an example.

Clinics	Criteria				
	C_1	C_2	C_3	C_4	C_5
O_1	8	10	9	8	2
O_2	9	9.5	10	7	8.5
O_3	9	8	8.5	7	8.5
O_4	7.5	8	9.5	7	6.5
O_5	9.5	7	10	7	10
O_6	7	10	6.5	8	8
O_7	7	9.5	6	8	7.5
O_8	7	8	8.5	7	8
O_9	5.5	7	6	8	2
O_{10}	9	6	9.5	7	10

this criterion are higher if the website in question is closer to the top of the page, i.e. if it is more visible in Google search (statistically speaking, sites which don't appear on the first or the second page in Google search are visited very rarely).

The third criterion evaluates the compatibility of the website with desktop computers or mobile devices. It also evaluates if the website is easy to use, as well as the information accessibility, modern design elements – if there are suitable maps, reservation databases, connections to social networks, etc.

The fourth criterion assumes that there are at least two languages in which the contents of a given site are presented. It is evaluated how equivalent those versions of the presentation are, but also the possibility of having equivalent opportunities for contact and communication regarding the services in those languages, and during the treatment as well.

The fifth criterion evaluates activities regarding the presentation of services and comments on those services on different social networks. Today, this criterion is considered as important, because it provides the potential user – tourist the possibility of gaining insight into the experiences of others.

The sixth criterion assumes that there are marks already given by previous visitors, and that those marks can be used as previously formed opinions.

In our case, it turned out that all the alternatives got maximum marks for the sixth criterion (with a small number of evaluators), hence, this criterion was left out in this particular case.

Marks according to all criteria were given in the range from 0 to 10, 0 being the absence of any value according to the given criterion, and 10, the highest possible value. The order of the criteria according to their importance is:

$$C_1 \rightarrow C_5 \rightarrow C_2 \rightarrow C_3 \rightarrow C_4.$$

This evaluation comprehended 10 dental clinics in Serbia, with designations $O_1, \dots, O_{10}$, and the marks are given in Table 4.

Data were normalized linearly (division by 10) and thus the following Table 5 is obtained.

Table 5
Normalized decision matrix – an example.

Clinics	Criteria				
	C_1	C_2	C_3	C_4	C_5
O_1	0.8	1	0.9	0.8	0.2
O_2	0.9	0.95	1	0.7	0.85
O_3	0.9	0.8	0.85	0.7	0.85
O_4	0.75	0.8	0.95	0.7	0.65
O_5	0.95	0.7	1	0.7	1
O_6	0.7	1	0.65	0.8	0.8
O_7	0.7	0.95	0.6	0.8	0.75
O_8	0.7	0.8	0.85	0.7	0.8
O_9	0.55	0.7	0.6	0.8	0.2
O_{10}	0.9	0.6	0.95	0.7	1

4.1.1.
If we add to those criteria the following weight coefficients

$$k_1 = 0.2; \quad k_2 = 0.2; \quad k_3 = 0.2; \quad k_4 = 0.1; \quad k_5 = 0.3$$

then, by calculations according to the formula

$$V(O_i) = a_{i1} \cdot 0.2 + a_{i1} \cdot \lambda \cdot (a_{i2} \cdot 0.2 + a_{i3} \cdot 0.2 + a_{i4} \cdot 0.1 + a_{i5} \cdot 0.3),$$

we get

$$V(O_1) = 0.16 + 0.64 \cdot \lambda \qquad V(O_2) = 0.18 + 0.685 \cdot \lambda$$
$$V(O_3) = 0.18 + 0.645 \cdot \lambda \qquad V(O_4) = 0.15 + 0.625 \cdot \lambda$$
$$V(O_5) = 0.19 + 0.65 \cdot \lambda \qquad V(O_6) = 0.14 + 0.65 \cdot \lambda$$
$$V(O_7) = 0.14 + 0.625 \cdot \lambda \qquad V(O_8) = 0.14 + 0.62 \cdot \lambda$$
$$V(O_9) = 0.11 + 0.52 \cdot \lambda \qquad V(O_{10}) = 0.18 + 0.62 \cdot \lambda$$

4.1.2.
For $\theta = 0.5$ we have $\lambda = 0.125$ and the order

$$O_5 \to O_2 \to O_3 \to O_{10} \to O_1 \to O_4 \to O_6 \to O_7 \to O_8 \to O_9.$$

4.1.3.
For $\theta = 1$ we have $\lambda = 0.25$ and the order

$$O_5 \to O_2 \to O_3 \to O_{10} \to O_1 \to O_4 \to O_6 \to O_7 \to O_8 \to O_9.$$

4.1.4.
For $\theta = 2$ we have $\lambda = 0.5$ and the order

$$O_2 \to O_5 \to O_3 \to O_{10} \to O_1 \to O_6 \to O_4 \to O_7 \to O_8 \to O_9.$$

4.1.5.
For $\theta = 4$ we have $\lambda = 1$ and the order

$$O_2 \to O_5 \to O_3 \to O_{10} \to O_1 \to O_6 \to O_4 \to O_7 \to O_8 \to O_9.$$

4.1.6.
For $\theta = 6$ we have $\lambda = 1.5$ and the order

$$O_2 \to O_5 \to O_3 \to O_1 \to O_6 \to O_{10} \to O_4 \to O_7 \to O_8 \to O_9.$$

4.2.

If we add to those criteria the following weight coefficients

$$k_1 = 0.5; \quad k_2 = 0.1; \quad k_3 = 0.15; \quad k_4 = 0.1; \quad k_5 = 0.15$$

(the importance of criteria)

$$C_1 \to C_5 \to C_3 \to C_2 \to C_4$$

and calculate values $V(O_i)$ according to the formula

$$V(O_i) = a_{i1} \cdot 0.5 + a_{i1} \cdot \lambda \cdot (a_{i2} \cdot 0.1 + a_{i3} \cdot 0.15 + a_{i4} \cdot 0.1 + a_{i5} \cdot 0.15)$$

we get

$$\begin{aligned} &V(O_1) = 0.4 + 0.375 \cdot \lambda && V(O_2) = 0.45 + 0.435 \cdot \lambda \\ &V(O_3) = 0.45 + 0.3975 \cdot \lambda && V(O_4) = 0.375 + 0.3925 \cdot \lambda, \\ &V(O_5) = 0.475 + 0.42 \cdot \lambda && V(O_6) = 0.35 + 0.3925 \cdot \lambda, \\ &V(O_7) = 0.35 + 0.38 \cdot \lambda && V(O_8) = 0.35 + 0.395 \cdot \lambda, \\ &V(O_9) = 0.275 + 0.3 \cdot \lambda && V(O_{10}) = 0.45 + 0.4075 \cdot \lambda. \end{aligned}$$

4.2.1.
For $\theta = 0.5$ we have $\lambda = 0.5$ and the order

$$O_5 \to O_2 \to O_{10} \to O_3 \to O_1 \to O_4 \to O_6 \to O_8 \to O_7 \to O_9.$$

4.2.2.
For $\theta = 1$ we have $\lambda = 1$ and the order

$$O_5 \to O_2 \to O_{10} \to O_3 \to O_1 \to O_4 \to O_8 \to O_6 \to O_7 \to O_9.$$

4.2.3.
For $\theta = 2$ we have $\lambda = 2$ and the order

$$O_5 \to O_2 \to O_{10} \to O_3 \to O_4 \to O_1 \to O_8 \to O_6 \to O_7 \to O_9.$$

4.2.4.
For $\theta = 4$ we have $\lambda = 4$ and the order

$$O_2 \to O_5 \to O_{10} \to O_3 \to O_4 \to O_8 \to O_6 \to O_1 \to O_7 \to O_9.$$

4.2.5.
For $\theta = 6$ we have $\lambda = 6$ and the order

$$O_2 \to O_5 \to O_{10} \to O_3 \to O_4 \to O_8 \to O_6 \to O_1 \to O_7 \to O_9.$$

N. B.: It is usually assumed that the dominant criterion is mostly fulfilled and it is not obligatory for it to have the largest weight coefficient (example 4.1)!

5. Discussion

We can say that the given multi-criteria problem (shown by the numerical decision making matrix (1)) is stable if one can define, for every criterion, domains $S_1, \ldots, S_n \subseteq R$ for the alternatives corresponding to criteria $C_1, \ldots, C_n$ and if for every $i = 1, \ldots, n$ one can define functions

$$f_i : S_i \to [0, 1],$$

while having $(s_{i1}, s_{i2}) \subset S_i$ such that $f_i : S_i \to [0, 1]$ is increasing (decreasing) mapping for the maximization (minimization) type of criterion C_i.

Values from S_i smaller than s_{i1} are mapped to 0 (1) and values S_i greater than s_{i2} are mapped to 1 (0) for the maximization (minimization) type of criterion C_i.

Theorem 1. *The rank of an alternative for the set of alternatives* $\{A_1, \ldots, A_m\}$ *in a stable multi-criteria model will not be changed according to the multi-criteria analysis method presented here if the given multi-criteria method is expanded with a new set of alternatives* $\{B_1, \ldots, B_r\}$.

Proof. We will observe three tables within this stabile multi-criteria model:
Table

$$\left[a_{ij}\right]_{i=1}^{m}{}_{j=1}^{n}$$

which corresponds to alternative $\{A_1, \ldots, A_m\}$ table

$$\left[b_{kj}\right]_{k=1}^{r}{}_{j=1}^{n}$$

which corresponds to alternative $\{B_1, \ldots, B_r\}$ i table

$$\left[c_{lj}\right]_{l=1}^{m+r}{}_{j=1}^{n}$$

which corresponds to alternative $\{A_1, \dots, A_m, B_1, \dots, B_r\}$ where

$$c_{lj} = \begin{cases} a_{lj}, & l = 1, \dots, m, \ j = 1, \dots, n, \\ b_{kj}, & l = m + k, \ k = 1, \dots r, \ j = 1, \dots, n. \end{cases}$$

It is obvious that the first two matrices are sub-matrices of the third matrix, hence, it follows that the order for the first matrix is a sub-order obtained from the third matrix. Therefore, introducing new alternatives did not change the established order obtained from the first matrix. □

Consequence 1. Adding new alternatives into a multi-criteria model does not imply favouring any of the existing alternatives in the model.

We should point out that many methods of multi-criteria analysis do not have this characteristic.

6. Conclusion

In this paper we presented a new method which has an important feature of preserving the order of initial alternatives when new alternatives are added, see Theorem 1. Most of the existing MCDM methods lack this property.

Another feature of our method is the following: alternatives in which dominant ciriteria are not satisfied are excluded. We should emphasize that selection of dominant criteria is crucial in any decision-making method.

Further research should incorporate the case of fuzzy entries in the decision matrix, which has obvious importance in applications.

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M.M. Žižović, MSc, PhD student at the Singidunum University, Belgrade, Serbia.

M. Albijanić obtained two PhD degrees. The first research paper, was successfully defended at the Singidunum University in 2011. He received his second PhD degree at the Faculty of Mathematics in Belgrade in 2016. Miloljub Albijanić is an assistant professor at the FEFA, Metropolitan University, Belgrade.

V. Jovanovic obtained MSc and PhD at the University of Belgrade, Faculty of Science (Department of Geography), 1994. Until 2000, she worked in the Geographic Institute of the Serbian Academy of Sciences and Arts. Now, she works at the University Singidunum as a full professor of geographical information systems in study programs: business systems in tourism and hospitality; advanced information technologies. Her current research interests include geographic information systems, decision support systems, spatial analysis, tourism and space.

M.R. Žižović is a full professor at Singidunum University Belgrade, Faculty of Health Legal and Business in Valjevo and also at the University of Kragujevac, Faculty of Technical Sciences in Čačak, Serbia. Editor in chief of the journal *Mathematica Moravica*.