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TODIM Method for Picture Fuzzy Multiple Attribute Decision Making

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Abstract. For this article, we shall expand the TODIM model to the MADM with the picture fuzzy numbers (PFNs). Firstly, the concept, comparative method and distance of PFNs are introduced and the traditional TODIM model is presented. Then, the expanded TODIM model is developed to solve MADM problems with PFNs. Finally, a numerical example is given to verify the proposed approach.

Key words: multiple attribute decision making (MADM), picture fuzzy sets (PFSs), picture fuzzy numbers (PFNs), TODIM, prospect theory.

1. Introduction

Atanassov (1986, 1989) proposed the concept of intuitionistic fuzzy sets (IFSs) based on fuzzy set by Zadeh (1965). Atanassov and Gargov (1989) and Atanassov (1994) defined interval-valued intuitionistic fuzzy sets (IVIFSs). The IFSs and IVIFSs have been investigated by many researchers (Bustince and Burillo, 1995; Atanassov *et al.*, 2005; Ronald, 2015; Xu, 2007; Xu and Yager, 2006; Konwar and Debnath, 2017; Wu and Chiclana, 2014; Wang, 2017; Chen, 2011; Chen and Li, 2011; Chen, 2014, 2016; Chen and Chiou, 2015; Garg, 2016; Li, 2011; Zhao and Wei, 2013; Liu, 2017b; Zhang, 2017; Song and Wang, 2017; Ye, 2009, 2010; Wei and Zhao, 2012; Liu, 2017a). Recently, Cuong (2013) developed picture fuzzy set (PFS) and studied the properties and basic operations laws of PFS. Singh (2014) studied the correlation coefficients for PFSs. Son (2015) and Thong (2015) proposed several clustering algorithms with PFSs. Thong (2015) proposed a hybrid method between PF clustering and IF recommender systems. Wei (2016) proposed the cross-entropy for MADM problems with PFNs. Wei (2017a) investigated the picture fuzzy aggregation operators for MADM problems. Wei *et al.* (2016b) gave the projection models for MADM with picture fuzzy information. Thong and Son (2016b) considered the improvement of FCM on the PFSs. Thong and Son (2016a) proposed the Automatic Picture Fuzzy Clustering (AFC-PFS). Son (2016) proposed a generalized picture distance measure. Son (2017) proposed the generalized picture distance measures and association measures. Son *et al.* (2017) proposed the picture inference system (PIS). Son and Thong (2017) developed two hybrid forecast models with picture fuzzy clustering.

Many previous studies have captured the DMs' attitudinal characters in the MADM problems (Gomes and Lima, 1992; Chen *et al.*, 2012; Liu *et al.*, 2014; Wu and Chiclana, 2014). In order to show the risk and uncertainty of the MADM problems simultaneously, more and more scholars have proposed some fuzzy TODIM approach (Konwar and Debnath, 2017; Fan *et al.*, 2013), the intuitionistic fuzzy TODIM approach (Lourenzutti and Krohling, 2013; Krohling *et al.*, 2013), Pythagorean fuzzy TODIM approach (Ren *et al.*, 2016)), multi-hesitant fuzzy linguistic TODIM approach (Wang *et al.*, 2016; Wei *et al.*, 2015), interval type-2 fuzzy sets-based TODIM method (Sang and Liu, 2016)), intuitionistic linguistic TODIM method (Wang and Liu, 2017) and 2-dimension uncertain linguistic TODIM method (Liu and Teng, 2016). But until now, no research extend TODIM model for PFNs. Therefore, it is necessary to investigate this issue. The purpose of this paper is to expand TODIM model to MADM with PFNs to overcome this limitation. The rest of this paper is organized as follows. In Section 2, we introduce the concepts of PFNs and classical TODIM model. In Section 3 we develop the TODIM model for MADM with PFNs. In Section 4, an illustrative example for potential evaluation of emerging technology commercialization is pointed out and some comparative analysis is conducted. In Section 5 we conclude this paper.

2. Preliminaries

Some definitions of PFSs are introduced. The operations of PFNs are also provided as they will be utilized in the rest of the paper. At the same time, the process of traditional TODIM approach in decision making is also presented.

2.1. *Picture Fuzzy Set Sets (PFSs)*

Definition 1. (See Atanassov, 1986, 1989.) An IFS A in X is given by

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\} \tag{1}$$

where $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$, where, $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$, $\forall x \in X$. The number $\mu_A(x)$ and $\nu_A(x)$ represents, respectively, the membership degree and non-membership degree of the element x to the set A.

Definition 2. (See Cuong, 2013.) A picture fuzzy set (PFS) A on the universe X is an object of the form

$$A = \{\langle x, \mu_A(x), \eta_A(x), \nu_A(x)\rangle \mid x \in X\} \tag{2}$$

where $\mu_A(x) \in [0, 1]$ is called the "degree of positive membership of A", $\eta_A(x) \in [0, 1]$ is called the "degree of neutral membership of A" and $\nu_A(x) \in [0, 1]$ is called the "degree of negative membership of A", and $\mu_A(x), \eta_A(x), \nu_A(x)$ satisfy the following condition: $0 \leqslant \mu_A(x) + \eta_A(x) + \nu_A(x) \leqslant 1$, $\forall x \in X$. Then for $x \in X$, $\pi_A(x) = 1 - (\mu_A(x) + \eta_A(x) + \nu_A(x))$ could be called the degree of refusal membership of x in X.

If $\pi_A(x) = 0$, then the picture fuzzy set reduces to the Atanassov's IFSs theory (Atanassov, 1986, 1989). Thus, the Atanassov's IFSs theory is a special form of the PFSs (Cuong, 2013).

DEFINITION 3. (See Abdellaoui *et al.*, 2017.) Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ be a PFN, the score value S of PFN is:

$$S(\alpha) = \frac{1 + \mu_\alpha - \nu_\alpha}{2}, \quad S(\alpha) \in [0, 1]. \tag{3}$$

DEFINITION 4. (See Wei, 2017a.) Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ be an accuracy function H of a PFN is:

$$H(\alpha) = \mu_\alpha + \eta_\alpha + \nu_\alpha, \quad H(\alpha) \in [0, 1]. \tag{4}$$

Wei (2018a) gave an order relation between two PFNs.

DEFINITION 5. (See Wei, 2017a.) Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \eta_\beta, \nu_\beta)$ be two PFNs, $S(\alpha) = \frac{1+\mu_\alpha-\nu_\alpha}{2}$ and $S(\beta) = \frac{1+\mu_\beta-\nu_\beta}{2}$ be the scores of α and β, respectively, and let $H(\alpha) = \mu_\alpha + \eta_\alpha + \nu_\alpha$ and $H(\beta) = \mu_\beta + \eta_\beta + \nu_\beta$ be the accuracy degrees of α and β, respectively, then if $S(\alpha) < S(\beta)$, then $\alpha < \beta$; if $S(\alpha) = S(\beta)$, then (1) if $H(\alpha) = H(\beta)$, then $\alpha = \beta$; (2) if $H(\alpha) < H(\beta)$, then $\alpha < \beta$.

DEFINITION 6. Let $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \eta_\beta, \nu_\beta)$ be two PFNs, then the normalized Hamming distance between $\alpha = (\mu_\alpha, \eta_\alpha, \nu_\alpha)$ and $\beta = (\mu_\beta, \eta_\beta, \nu_\beta)$ is:

$$d(\alpha, \beta) = \frac{1}{2}\big(|\mu_\alpha - \mu_\beta| + |\eta_\alpha - \eta_\beta| + |\nu_\alpha - \nu_\beta|\big). \tag{5}$$

2.2. *The TODIM Approach*

Let $G = \{G_1, G_2, \dots, G_n\}$ be the set of attributes, $w = (w_1, w_2, \dots, w_n)$ be the weight vector of attributes G_j, where $w_j \in [0, 1]$, $j = 1, 2, \dots, n,$, $\sum_{j=1}^{n} w_j = 1$. Let $A = \{A_1, A_2, \dots, A_m\}$ be a discrete set of alternatives. Suppose that $A = (a_{ij})_{m\times n}$ be a decision matrix, where a_{ij} is the attribute value, given by an expert, for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. We define $w_{jr} = w_j / w_r$ $(r, j = 1, 2, \dots, n)$ are the relative weight of the attribute G_j to G_r, and $w_r = \max\{w_j \mid j = 12, \dots, n\}$, and $0 \leqslant w_{jr} \leqslant 1$. Then the traditional TODIM model includes the following steps:

Step 1. Normalize the $A = (a_{ij})_{m\times n}$ into $B = (b_{ij})_{m\times n}$.

Step 2. Compute the dominance degree of A_i over each alternative A_t for G_j:

$$\delta(A_i, A_t) = \sum_{j=1}^{n} \phi_j(A_i, A_t) \quad (i, t = 1, 2, \dots, m) \tag{6}$$

where

$$\phi_j(A_j, A_t) = \begin{cases} \sqrt{\frac{w_{jr}(b_{ij}-b_{tj})}{\sum_{j=1}^{n} w_{jr}}}, & \text{if } b_{ij} > b_{tj}; \\ 0, & \text{if } b_{ij} = b_{tj}; \\ -\frac{1}{\theta}\sqrt{\frac{(b_{ij}-b_{tj})\sum_{j=1}^{n} w_{jr}}{w_{jr}}}, & \text{if } b_{ij} < b_{tj}, \end{cases} \tag{7}$$

and the parameter values θ depict the attenuation factor of the losses. If $b_{ij} - b_{tj} > 0$, then $\phi_j(A_i, A_t)$ represents a gain; if $b_{ij} - b_{tj} < 0$, then $\phi_j(A_i, A_t)$ signifies a loss.

Step 3. Compute the overall dominance of the alternative A_i with the following formula:

$$\phi(A_i) = \frac{\sum_{t=1}^{m} \delta(A_i, A_t) - \min_i \left\{ \sum_{t=1}^{m} \delta(A_i, A_t) \right\}}{\max_i \left\{ \sum_{t=1}^{m} \delta(A_i, A_t) \right\} - \min_i \left\{ \sum_{t=1}^{m} \delta(A_i, A_t) \right\}}, \quad i = 1, 2, \dots, m. \tag{8}$$

Step 4. Rank and select the best alternative by the overall values $\phi(A_i)$ $(i = 1, 2, \dots, m)$. The alternative with the minimum value is the worst. Inversely, the maximum value is the most desirable one.

3. TODIM Method for Picture Fuzzy MADM Problems

The following notations are utilized to show MADM problems with PFNs. Let $A = \{A_1, A_2, \dots, A_m\}$ be a set of alternatives, and $G = \{G_1, G_2, \dots, G_m\}$ be a set of attributes. Let $w = (w_1, w_1, \dots, w_1)$ be the weight vector of attributes, where $w_j \in [0, 1]$, $j = 1, 2, \dots, n$, $\sum_{j=1}^{n} w_j = 1$. Suppose that $R = (r_{ij})_{m\times n} = (\mu_{ij}, \eta_{ij}, \nu_{ij})_{m\times n}$ be a picture fuzzy decision matrix, where μ_{ij} indicates the degree of positive membership, η_{ij} indicates the degree of neutral membership, ν_{ij} indicates the degree of negative membership, $\mu_{ij} \in [0, 1]$, $\eta_{ij} \in [0, 1]$, $\nu_{ij} \in [0, 1]$, $\mu_{ij} + \eta_{ij} + \nu_{ij} \leqslant 1$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$.

Then, we extend the TODIM model to solve the MADM problem with PFNs.

Firstly, we can obtain the relative weight of G_j as:

$$w_{jr} = \frac{w_j}{w_r}, \quad j, r = 1, 2, \dots, n, \tag{9}$$

where $w_r = \max\{w_j \mid j = 1, 2, \dots, n\}$, and $0 \leqslant w_{jr} \leqslant 1$.

We calculate the dominance of A_i over alternative A_t under attribute G_j:

$$\phi_j(A_j, A_t) = \begin{cases} \sqrt{\frac{w_{jr} d(r_{ij}, r_{tj})}{\sum_{j=1}^{n} w_{jr}}}, & \text{if } r_{ij} > r_{tj}; \\ 0, & \text{if } r_{ij} = r_{tj}; \\ -\frac{1}{\theta}\sqrt{\frac{d(r_{ij}, d_{tj})\sum_{j=1}^{n} w_{jr}}{w_{jr}}}, & \text{if } r_{ij} < r_{tj}. \end{cases} \tag{10}$$

$$d(r_{ij}, r_{tj}) = \frac{1}{2}\left(|\mu_{ij} - \mu_{tj}| + |\eta_{ij} - \eta_{tj}| + |\nu_{ij} - \nu_{tj}|\right) \tag{11}$$

where the parameter θ is the attenuation factor of the losses.

In order to indicate the functions $\phi_j(A_j, A_t)$ clearly, we depict it in a matrix under attribute of G_j as:

$$\phi_j = \left(\phi_j(A_i, A_j)\right)_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{pmatrix} 0 & \phi_j(A_1, A_2) & \dots & \phi_j(A_1, A_m) \\ \phi_j(A_2, A_1) & 0 & \dots & \phi_j(A_2, A_m) \\ \vdots & \vdots & \cdots & \vdots \\ \phi_j(A_m, A_1) & \phi_j(A_m, A_2) & \cdots & 0 \end{pmatrix} \end{array} \tag{12}$$

where $j = 1, 2, \dots, n$, then we can derive the overall dominance degree of the alternative A_i over alternative A_j by

$$\delta(A_i, A_j) = \sum_{j=1}^{n} \phi_j(A_i, A_t), \quad i, t = 1, 2, \dots, m. \tag{13}$$

Thus, by Eq. (13), the overall dominance matrix is:

$$\delta_j = \left(\delta_j(A_i, A_j)\right)_{m\times m} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \cdots & A_m \end{array} \\ \begin{pmatrix} 0 & \delta_j(A_1, A_2) & \dots & \delta_j(A_1, A_m) \\ \delta_j(A_2, A_1) & 0 & \dots & \delta_j(A_2, A_m) \\ \vdots & \vdots & \cdots & \vdots \\ \delta_j(A_m, A_1) & \delta_j(A_m, A_2) & \cdots & 0 \end{pmatrix} \end{array}. \tag{14}$$

Finally, the overall value of A_i is:

$$\delta(A_i) = \frac{\sum_{t=1}^{m} \delta(A_i, A_t) - \min_i \left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\}}{\max_i \left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\} - \min_i \left\{\sum_{t=1}^{m} \delta(A_i, A_t)\right\}}, \quad i = 1, 2, \dots, m, \tag{15}$$

and rank all alternatives, the greater the overall value $\delta(A_i)$ $(i = 1, 2, \dots, m)$, the better the alternative A_i.

4. Numerical Example and Comparative Analysis

4.1. *Numerical Example*

With the rapid development of science and technology, the social life, national politics, the economy and the culture has also taken significant changes. Some theory in the traditional single field has been unable to guide the new practice. The new complex issues that appear in people's social practice can't be resolved by relying on the knowledge, theories and tools in a single field. Transdisciplinary research of emerging technologies appears on the scene. Evaluating transdisciplinary research of emerging technologies has important theoretical and practical significance. Thus, we shall give a numerical example for potential evaluation of emerging technology commercialization with PFNs. There are five possible emerging technology enterprises (ETES) A_i $(i = 1, 2, 3, 4, 5)$ to select. The expert selects four attributes to assess the five possible ETES: (1) G_1 is the human resources and financial conditions; (2) G_2 is the industrialization infrastructure; (3) G_3 is the technical advancement; (4) G_4 is the development of science and technology. The five possible ETES A_i $(i = 1, 2, 3, 4, 5)$ are to be assessed with PFNs according to four attributes (whose weighting vector $w = (0.2, 0.1, 0.3, 0.4)^T$), as listed as follows.

$$R = \begin{pmatrix} (0.89, 0.08, 0.03) & (0.42, 0.35, 0.18) & (0.08, 0.89, 0.02) & (0.80, 0.11, 0.05) \\ (0.23, 0.64, 0.11) & (0.03, 0.82, 0.13) & (0.73, 0.15, 0.08) & (0.73, 0.10, 0.14) \\ (0.52, 0.26, 0.05) & (0.04, 0.85, 0.10) & (0.68, 0.26, 0.06) & (0.43, 0.13, 0.25) \\ (0.74, 0.16, 0.10) & (0.02, 0.89, 0.05) & (0.08, 0.84, 0.06) & (0.85, 0.09, 0.05) \\ (0.68, 0.08, 0.21) & (0.05, 0.87, 0.06) & (0.13, 0.75, 0.09) & (0.65, 0.05, 0.02) \end{pmatrix}.$$

In the following, we utilize the approach developed for potential evaluation of emerging technology commercialization of five possible ETEs.

Firstly, since $w_4 = (w_1, w_2, w_3, w_4)$, then G_4 is the reference attribute and $w_r = 0.4$. Thus, $w_{1r} = 0.50$, $w_{2r} = 0.25$, $w_{3r} = 0.75$ and $w_{4r} = 1.00$. Then, we can calculate the dominance degree of the candidate A_i over each candidate A_t under G_j $(j = 1, 2, 3, 4)$. Let $\theta = 2.5$, we get:

$$\phi_1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{pmatrix} 0.0000 & 0.3606 & 0.2387 & 0.1732 & 0.1975 \\ -0.7211 & 0.0000 & -0.5404 & -0.6325 & -0.6663 \\ -0.4775 & 0.2702 & 0.0000 & -0.3847 & -0.4472 \\ -0.3464 & 0.3162 & 0.1924 & 0.0000 & 0.1581 \\ -0.3950 & 0.3332 & 0.2236 & -0.3162 & 0.0000 \end{pmatrix} \end{array},$$

$$\phi_2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{pmatrix} 0.0000 & 0.1724 & 0.1789 & 0.1889 & 0.1835 \\ -0.6967 & 0.0000 & -0.1932 & -0.2921 & -0.2733 \\ -0.7155 & 0.0483 & 0.0000 & -0.2422 & -0.1932 \\ -0.7544 & 0.0730 & 0.0606 & 0.0000 & -0.1789 \\ -0.7339 & 0.0683 & 0.0483 & -0.0447 & 0.0000 \end{pmatrix} \end{array},$$

$$\phi_3 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left(\begin{array}{ccccc} 0.0000 & -0.5077 & -0.4752 & 0.0949 & 0.1612 \\ 0.3808 & 0.0000 & 0.1342 & 0.3688 & 0.3479 \\ 0.3564 & -0.1789 & 0.0000 & 0.3435 & 0.3271 \\ -0.1265 & -0.4917 & -0.4580 & 0.0000 & -0.1738 \\ -0.2150 & -0.4638 & -0.4153 & 0.1304 & 0.0000 \end{array}\right) \end{array},$$

$$\phi_4 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left(\begin{array}{ccccc} 0.0000 & 0.1506 & 0.2805 & -0.0987 & 0.1789 \\ -0.1506 & 0.0000 & 0.2422 & -0.1247 & -0.1826 \\ -0.2805 & -0.2422 & 0.0000 & -0.2973 & -0.2658 \\ 0.0987 & 0.1724 & 0.2973 & 0.0000 & 0.1908 \\ -0.1789 & 0.1826 & 0.2658 & -0.1908 & 0.0000 \end{array}\right) \end{array}.$$

Secondly, by Eq. (13), and the overall dominance matrix is:

$$\delta = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \left(\begin{array}{ccccc} 0.0000 & 0.1776 & 0.2229 & 0.3583 & 0.7211 \\ -1.1875 & 0.0000 & -0.3572 & -0.7282 & -0.7743 \\ -1.1171 & -0.1206 & 0.0000 & -0.5807 & -0.5792 \\ -1.1297 & 0.0700 & 0.0922 & 0.0000 & -0.0038 \\ -1.5288 & 0.1203 & 0.1225 & -0.3319 & 0.0000 \end{array}\right) \end{array}.$$

Then, we can obtain $\delta(A_i)$ $(i = 1, 2, 3, 4, 5)$ by Eq. (14):

$$\delta(A_1) = 1.0000, \quad \delta(A_2) = 0.0000, \quad \delta(A_3) = 0.1475,$$
$$\delta(A_4) = 0.4586, \quad \delta(A_5) = 0.3170.$$

Finally, the order is: $A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$, and thus the best ETE is A_1.

4.2. *Comparative Analysis*

Then, we compare our method with picture fuzzy weighted averaging (PFWA) operator and picture fuzzy weighted geometric (PFWG) operator proposed by Wei (2017a) as follows:

Definition 7. (See Wei, 2017a.) Let $a_{ij} = (\mu_{ij}, \eta_{ij}, \nu_{ij})$ be a collection of PFNs, $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of a_{ij} $(j = 1, 2, \ldots, n)$, and $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$, then

$$\begin{aligned} r_i &= (\mu_i, \eta_i, \nu_i) \\ &= PFWA_w(r_{i1}, r_{i2}, \ldots, r_{im}) = \bigoplus_{j=1}^{n} (w_j r_{ij}) \end{aligned}$$

Table 1
The aggregating results of the ETEs by the PFWA (PFWG).

	PFWA	PFWG
A_1	(0.6880,0.2170,0.0390)	(0.3840,0.5363,0.0521)
A_2	(0.6216,0.2020,0.1120)	(0.4211,0.3729,0.1154)
A_3	(0.5121,0.2218,0.1077)	(0.4042,0.3270,0.1431)
A_4	(0.6547,0.2482,0.0607)	(0.2801,0.5696,0.0632)
A_5	(0.5008,0.1647,0.0561)	(0.3131,0.4816,0.0858)

Table 2
The score values of the ETEs.

	PFWA	PFWG
A_1	0.8245	0.6664)
A_2	0.7548	0.6529
A_3	0.7022	0.6305
A_4	0.7970	0.6084
A_5	0.7223	0.6137

Table 3
Ordering of the ETEs.

	Ordering
PFWA	$A_1 > A_4 > A_2 > A_5 > A_3$
PFWG	$A_1 > A_2 > A_3 > A_4 > A_1$

$$= \left(1 - \prod_{j=1}^{n}(1-\mu_{ij})^{w_i}, \prod_{j=1}^{n}(\eta_{ij})^{w_i}, \prod_{j=1}^{n}(\nu_{ij})^{w_i}\right), \tag{16}$$

$$\begin{aligned} r_i &= (\mu_i, \eta_i, \nu_i) \\ &= PFWG_w(r_{i1}, r_{i2}, \dots, r_{im}) = \bigotimes_{j=1}^{n}(r_{ij})^{w_j} \\ &= \left(\prod_{j=1}^{n}(\mu_{ij})^{w_i}, 1 - \prod_{j=1}^{n}(1-\eta_{ij})^{w_i}, 1 - \prod_{j=1}^{n}(1-\nu_{ij})^{w_i}\right). \end{aligned} \tag{17}$$

The calculating results are shown in Table 1.

According to Table 2, the score of the ETEs are listed in Table 3.

According to Table 3, the ordering is in Table 4, and the best ETE is A_1.

From Table 4, it can be seen that two methods have the same best ETE A_1 and two methods' ranking results are slightly different.

Essentially, these two approaches are discrepant for consideration of the DMs' psychological behaviours. The PFWA and PFWG operators based on the approaches can't

Table 4
Ordering of the ETEs by using different methods.

	Ordering
Picture fuzzy cross-entropy (Wei, 2016)	$A_1 > A_4 > A_2 > A_5 > A_3$
Picture fuzzy projection models (Wei *et al.*, 2016b)	$A_1 > A_2 > A_3 > A_4 > A_1$
Generalized picture fuzzy distance measure (Son, 2016)	$A_1 > A_4 > A_2 > A_3 > A_5$
Similarity measures for picture fuzzy sets (Wei, 2018b)	$A_1 > A_4 > A_2 > A_5 > A_3$
Cosine similarity measures for picture fuzzy sets (Wei, 2017c)	$A_1 > A_4 > A_2 > A_5 > A_3$

depict the DMs' psychological behaviours under risk. The picture fuzzy TODIM model can reasonably show the DMs' psychological behaviours under risk.

Furthermore, we compare our proposed method with picture fuzzy cross-entropy (Wei, 2016), picture fuzzy projection models (Wei *et al.*, 2016b), generalized picture fuzzy distance measure (Son, 2016), similarity measures for picture fuzzy sets (Wei, 2018b) and cosine similarity measures for picture fuzzy sets (Wei, 2017c) as shown in Table 4.

5. Conclusion

In this paper, we expand the TODIM model for MADM with the PFNs. Firstly, the definition, comparative method and distance of PFNs and the calculating steps of the traditional TODIM model are introduced. Then, the extended TODIM model is developed to solve MADM problems in which the attribute values are in the PFNs, and its important characteristic is that it can fully depict the decision makers' bounded rationality. Finally, an example for potential evaluation of emerging technology commercialization is considered to verify the developed model and a comparative analysis is also given. In subsequent works, more and more models with PFNs need to be investigated in uncertain decision making and risk analysis ((Zeng *et al.*, 2016; Wei *et al.*, 2018a; Wei, 2017b; Merigo and Casanovas, 2009; Wei and Wei, 2018; Wei and Lu, 2018b; Zeng, 2017; Wei and Lu, 2017; Wei *et al.*, 2018b); Gao *et al.*, 2018a, 2018b; Tang and Wei, 2018; Wei and Lu, 2018a; Wei *et al.*, 2016a; Wei and Zhang, 2018; Wei *et al.*, 2018c; Wang *et al.*, 2018).

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