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Pythagorean Fuzzy Information Aggregation Based on Weighted Induced Operator and Its Application to R&D Projections Selection

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Abstract. In this paper, by unifying the dual roles of order-inducing variables, a PF weighted induced generalized weighted averaging (PFWIGOWA) operator is presented to facilitate the PF information. The key feature of the proposed operator is that it can improve the existing aggregation operators by the dual roles of its order-inducing variables. In addition, the PFWIGOWA's desirable properties and different families are also discussed. Furthermore, an approach based on the developed operator is presented for solving multi-attribute group decision making (MAGDM) problems with PF information. Finally, the usefulness of the proposed method is illustrated in a research and development (R&D) projects selection problem.

Key words: Pythagorean fuzzy set, induced aggregation operator, weighted, MAGDM, R&D projects selection.

1. Introduction

As an extension of a fuzzy set (Zadeh, 1965), the intuitionistic fuzzy set (IFS) proposed by Atanassov (1986) has been developed to handle imprecise and ambiguous information in various practical problems and applications. The IFS is characterized by a membership degree (μ), a non-membership degree (v), satisfying $\mu, v \in [0, 1]$ and $0 \leqslant \mu + v \leqslant 1$. Over the past thirty years, IFS has been broadly applied in kinds of multi-attribute group decision making (MAGDM) problems. Recently, Yu and Liao (2016) and Liu and Liao (2017) carried out attractive scientometric reviews on the development and application of IFS from various perspectives.

In 2014, Yager (2014) proposed a new extension of a fuzzy set, namely Pythagorean fuzzy set (PFS), which is also represented by the values of membership degree (μ) and non-membership degree (v). However, the restriction of these values is extended from $0 \leqslant \mu + v \leqslant 1$ to $\mu^2 + v^2 \leqslant 1$. Obviously, PFS is a more powerful tool than IFS because it can depict imprecise and ambiguous information that the latter one cannot. Consequently,

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the PFS theory has been viewed as an appropriate approach to handle and model uncertainty in decision making problems. Yager (2014) defined some PFS aggregation operations and further introduced a method to solve MAGDM with PFS information. Peng and Yang (2015) proposed a superiority and inferiority ranking method for PFS and studied its application in MAGDM problems. Meanwhile, Peng and Yang (2016) developed a Choquet integral method to aggregate PFSs. Zeng *et al.* (2016a) developed a hybrid based on distance measures for PF MAGDM problems. Chen (2018) developed a novel PF VIKOR-based approach for multiple criteria decision problems. Considering the four parameters of PFSs, Li and Zeng (2018) developed several kinds of distance measures between PFSs. Wei and Lu (2018) represented a variety of PF power aggregation operators and applied them to decision making problems. Garg (2017) developed a method based on confidence levels for PF decision making and Zeng (2017) developed a probabilistic method to handle PF information.

Information aggregation plays a vital role in information fusion of decision making problems. To date, numerous aggregation operators have been represented to aggregate information. Quite well-known one is the induced ordered weighted averaging (IOWA) operator (Yager and Filev, 1999) in which the ordering of arguments depend upon the order-inducing variables. Thus, the IOWA operator differs from the ordered weighted averaging (OWA) operator (Yager, 1988) in that the reordering relies on the values of the associated order-inducing variables, but not the values of the arguments. So far, the IOWA operator has been extensively studied in the context of decision making (Beliakov *et al.*, 2007; Chen and Zhou, 2011; Merigó, 2011; Merigó and Gil-Lafuente, 2009; Zeng and Chen, 2015). As for PFSs, combining the advantages of the IOWA and generalized means (Merigó, 2011), Xu *et al.* (2017) developed the PF induced generalized OWA (PFIGOWA) operator. Zeng *et al.* (2018) presented a PF IOWA weighted average (PFIOWAWA) operator by unifying the IOWA and weighted average method.

The aforementioned studies show that the PFIGOWA operator has been proved to be a powerful tool to deal with PF information. However, the different magnitudes of order-inducing variables in the PFIGOWA operator cannot be reflected in the final results, which often causes information loss in the aggregation process. In order to ameliorate the defect, in this paper, we will develop a new PF induced aggregation operator, namely PF weighted induced generalized OWA (PFWIGOWA) operator. In addition, the key features and particular cases of the PFWIGOWA operator are also investigated. Finally, the application of the developed operator in MAGDM comprising R&D projects selection is presented.

The remainder of this paper is carried out as follows. In next section, we briefly review the PFS theory, the IOWA and the PFIGOWA operator. We introduce the PFWIGOWA operator in Section 3. A MAGDM model based on the proposed operator and its application in R&D selection problem are presented in Section 4 and Section 5, respectively. In last section, we reach the main conclusions of the paper.

2. Preliminaries

2.1. *Pythagorean Fuzzy Set*

Yager (2014) developed the Pythagorean Fuzzy Set (PFS) and gave its definition as follows.

Definition 1. Let a set $Z = \{z_1, z_2, \dots, z_n\}$ be a fixed set, a PFS P in Z has the following form:

$$P = \{\langle z, P(\mu_P(z), v_P(z))\rangle | z \in Z\}, \tag{1}$$

where the numbers $\mu_P(z)$ and $v_P(z)$ indicate the membership degree and non-membership degree of the element z to the set P, respectively. For any PFS P and $z \in Z$, $(\mu_P(z))^2 + (v_P(z))^2 \leqslant 1$, and the pair $(\mu_P(z), v_P(z))$ is called a PF number (PFN), denoted as $\alpha = (\mu_\alpha, v_\alpha)$ (Zhang and Xu, 2014), where $0 \leqslant \mu_\alpha \leqslant 1$, $0 \leqslant v_\alpha \leqslant 1$ and $(\mu_\alpha)^2 + (v_\alpha)^2 \leqslant 1$.

For any three PFNs $\alpha = (\mu_\alpha, v_\alpha)$, $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$, their operational rules were given as follows (Yager, 2014; Zhang and Xu, 2014):

(1) $\alpha_1 \oplus \alpha_2 = (\sqrt{\mu_{\alpha_1}^2 + \mu_{\alpha_2}^2 - \mu_{\alpha_1}^2 \cdot \mu_{\alpha_2}^2}, v_{\alpha_1} \cdot v_{\alpha_2})$;
(2) $\alpha_1 \otimes \alpha_2 = (\mu_{\alpha_1} \cdot \mu_{\alpha_2}, \sqrt{v_{\alpha_1}^2 + v_{\alpha_2}^2 - v_{\alpha_1}^2 \cdot v_{\alpha_2}^2})$;
(3) $\lambda\alpha = (1 - (1 - \mu_\alpha^2)^\lambda, (v_\alpha)^\lambda)$, $\lambda > 0$;
(4) $\alpha^\lambda = ((\mu_\alpha)^\lambda, 1 - (1 - v_\alpha^2)^\lambda)$, $\lambda > 0$.

The comparative rules of PFNs are defined as follows:

Definition 2. For a PFN $\alpha = (\mu_\alpha, v_\alpha)$, $s(\alpha) = (\mu_\alpha)^2 - (v_\alpha)^2$ and $h(\alpha) = (\mu_\alpha)^2 + (v_\alpha)^2$ are called the score function and the accuracy function of α. For $\alpha_1 = (\mu_{\alpha_1}, v_{\alpha_1})$ and $\alpha_2 = (\mu_{\alpha_2}, v_{\alpha_2})$, if $s(\alpha_1) < s(\alpha_2)$, then $\alpha_1 < \alpha_2$; if $s(\alpha_1) = s(\alpha_2)$, then $\begin{cases} h(\alpha_1) < h(\alpha_2) \Rightarrow \alpha_1 < \alpha_2, \\ h(\alpha_1) = h(\alpha_2) \Rightarrow \alpha_1 = \alpha_2. \end{cases}$

Based on these operational laws, Yager (2014) defined the PF weighted average (PFWA) operator:

Definition 3. Let $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j})$ $(j = 1, 2, \dots, n)$ be a collection of PFNs with a weight vector $W = (w_1, w_2, \dots, w_n)$ meeting $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$. A PFWA is defined as:

$$PFWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \bigoplus_{j=1}^n w_j\alpha_j = \left(\sqrt{1 - \prod_{j=1}^n \left(1 - \mu_{\alpha_j}^2\right)^{w_j}}, \prod_{j=1}^n v_{\alpha_j}^{w_j}\right). \tag{2}$$

2.2. *The IOWA Operator*

The IOWA operator was defined by Yager and Filev (1999):

Definition 4. An IOWA operator is defined by a weight vector $W = (w_1, w_2, \dots, w_n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$; and an order-inducing vector $U = (u_1, \dots, u_n)$, such that:

$$IOWA\big(\langle u_1, x_1\rangle, \langle u_2, x_2\rangle, \dots, \langle u_n, x_n\rangle\big) = \sum_{j=1}^{n} w_j y_j. \tag{3}$$

where $(y_1, \dots, y_n)$ is reordered $(x_1, \dots, x_n)$ as per decreasing order of the $(u_1, \dots, u_n)$.

2.3. *The PFIGOWA Operator*

By combining the IOWA operator and generalized means, Xu *et al.* (2017) developed the PFIGOWA operator to handle PFNs.

Definition 5. Let $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j})$ $(j = 1, 2, \dots, n)$ be a collection of PFNs, the PFIGOWA operator is defined by a weight vector $W = (w_1, w_2, \dots, w_n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$; and an order-inducing vector $U = (u_1, \dots, u_n)$, such that:

$$PFIGOWA\big(\langle u_1, \alpha_1\rangle, \langle u_2, \alpha_2\rangle, \dots, \langle u_n, \alpha_n\rangle\big) = \left(\sum_{j=1}^{n} w_j \beta_j^{\lambda}\right)^{1/\lambda}, \tag{4}$$

where $(\beta_1, \dots, \beta_n)$ is reordered $(\alpha_1, \dots, \alpha_n)$ as per decreasing order of $(u_1, \dots, u_n)$. λ is a parameter satisfying $\lambda \in (-\infty, \infty)-\{0\}$.

As shown in the above analysis, the order-inducing variables in the PFIGOWA operator as well as the IOWA operator aren't involved in actual aggregation results, thus information loss inevitably occurs in the aggregation results. So, in the following section, we will develop a new PF induced aggregation approach to overcome this drawback.

3. PFWIGOWA Operator

3.1. *The Definition of the PFWIGOWA Operator*

The significant advantage of the PFWIGOWA operator is that it provides an associated weight related to the order-inducing variables, enabling us to capture the variations in the final aggregation results caused by the order-inducing variables. On the other hand, it can be viewed as an extension of the weighted IOWA (Aggarwal, 2015) operator by using the generalized means and uncertain information that can be represented with PFNs.

DEFINITION 6. Let $\alpha_j = (\mu_{\alpha_j}, v_{\alpha_j})$ $(j = 1, 2, \ldots, n)$ be a collection of PFNs, a PFWIGOWA operator is defined by $W = (w_1, w_2, \ldots, w_n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$; and an order-inducing vector $U = (u_1, \ldots, u_n)$, such that:

$$PFWIGOWA\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \left(\sum_{j=1}^{n} \varpi_j \beta_j^{\lambda}\right)^{1/\lambda}, \tag{5}$$

where β_j is α_i value of the PFWIGOWA pair $\langle u_i, \alpha_i\rangle$ having the j-th largest u_i, λ is a parameter satisfying $\lambda \in (-\infty, \infty) - \{0\}$. ϖ_j is a moderated weight that is closely related to the order-inducing variable $u_j \in U$ and with $w_j \in W$ $(j = 1, 2, \ldots, n)$, defined as:

$$\varpi_j = \frac{w_j u_{\sigma(j)}}{\sum_{j=1}^{n} w_j u_{\sigma(j)}}, \tag{6}$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is any possible permutation of $(1, 2, \ldots, n)$ satisfying $u_{\sigma(j-1)} \geqslant u_{\sigma(j)}$ for all $j > 1$.

In what follows, a numerical example is provided to demonstrate the PFWIGOWA's application in aggregation process.

EXAMPLE 1. Let the aggregated argument by means of $\langle u_i, \alpha_i\rangle$ be

$$\langle u_1, \alpha_1\rangle = \big\langle 0.8, (0.6, 0.7)\big\rangle, \qquad \langle u_2, \alpha_2\rangle = \big\langle 0.5, (0.9, 0.2)\big\rangle,$$
$$\langle u_3, \alpha_3\rangle = \big\langle 0.6, (0.8, 0.1)\big\rangle, \qquad \langle u_4, \alpha_4\rangle = \big\langle 0.9, (0.7, 0.5)\big\rangle.$$

Let $W = (0.3, 0.2, 0.1, 0.4)$ be the weight vector of the PFWIGOWA operator, then we can calculate the weight ϖ_j using Eq. (6):

$$\varpi_1 = \frac{w_1 u_{\sigma(1)}}{\sum_{j=1}^{4} w_j u_{\sigma(j)}} = \frac{0.3 \times 0.9}{0.3 \times 0.9 + 0.2 \times 0.8 + 0.1 \times 0.6 + 0.4 \times 0.5} = 0.391.$$

Similarly,

$$\varpi_2 = 0.232, \quad \varpi_3 = 0.087, \quad \varpi_4 = 0.290.$$

Without loss of generality, let $\lambda = 2$, then we can obtain the aggregation value by the PFWIGOWA operator:

$$\begin{aligned} &PFWIGOWA\big(\langle 0.8, (0.6, 0.7)\rangle, \langle 0.5, (0.9, 0.2)\rangle, \langle 0.6, (0.8, 0.1)\rangle, \langle 0.9, (0.7, 0.5)\rangle\big) \\ &\quad = \big(0.39 \times (0.7, 0.5)^2 \oplus 0.232 \times (0.6, 0.7)^2 \oplus 0.087 \\ &\qquad \times (0.8, 0.1)^2 \oplus 0.290 \times (0.9, 0.2)^2\big)^{1/2} \\ &\quad = (0.7904, 0.3497). \end{aligned}$$

To perform a comparative analysis with the (previous) PFIGOWA operator, the corresponding aggregation result of the PFIGOWA ($\lambda = 2$) is listed as follows:

$$\begin{aligned}&\mathit{PFIGOWA}\big(\langle 0.8, (0.6, 0.7)\rangle, \langle 0.5, (0.9, 0.2)\rangle, \langle 0.6, (0.8, 0.1)\rangle, \langle 0.9, (0.7, 0.5)\rangle\big)\\&\quad = \big(0.3 \times (0.7, 0.5)^2 \oplus 0.2 \times (0.6, 0.7)^2 \oplus 0.1 \times (0.8, 0.1)^2 \oplus 0.4 \times (0.9, 0.2)^2\big)^{1/2}\\&\quad = (0.8170, 0.3068).\end{aligned}$$

As can be observed, in contrast to the PFWIGOWA operator, we get a different aggregation result for the PFIGOWA operator, the reason is that the order-inducing variable values in the PFWIGOWA operator not only induce the ordering of the arguments but also act the moderated weights in aggregate process.

3.2. *The Properties of the PFWIGOWA Operator*

Next, we investigate some main features of the PFWIGOWA operator, including the idempotency, boundedness, monotonicity and commutativity.

Theorem 1 (Idempotency). *Let F be the PFWIGOWA operator, if all $\alpha_i = \alpha = (\mu_{\alpha_i}, v_{\alpha_i})$ for all i, then*

$$F\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \alpha. \tag{7}$$

Proof. As $\alpha_i = \alpha = (\mu_{\alpha_i}, v_{\alpha_i})$, then we have

$$F\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \left(\sum_{j=1}^{n} \varpi_j \alpha^{\lambda}\right)^{1/\lambda} = \left(\alpha^{\lambda} \sum_{j=1}^{n} \varpi_j\right)^{1/\lambda}.$$

As $\sum_{j=1}^{n} \varpi_j = 1$, then we get

$$F\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \left(\alpha^{\lambda} \sum_{j=1}^{n} \varpi_j\right)^{1/\lambda} = \big(\alpha^{\lambda}\big)^{1/\lambda} = \alpha. \quad \square$$

Theorem 2 (Boundedness). *Let $\min_i(\alpha_i) = m$ and $\max_i(\alpha_i) = M$, then*

$$m \leqslant F\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) \leqslant M. \tag{8}$$

Proof. As $\sum_{j=1}^{n} \varpi_j = 1$ and $\varpi_j \in [0, 1]$, then

$$F\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \left(\sum_{j=1}^{n} \varpi_j {\beta_j}^{\lambda}\right)^{1/\lambda} \leqslant \left(\sum_{j=1}^{n} \varpi_j M^{\lambda}\right)^{1/\lambda} = \big(M^{\lambda}\big)^{1/\lambda} = M.$$

Similarly,

$$F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big)=\left(\sum_{j=1}^{n}\varpi_j{\beta_j}^{\lambda}\right)^{1/\lambda}\geqslant\left(\sum_{j=1}^{n}\varpi_j m^{\lambda}\right)^{1/\lambda}=\big(m^{\lambda}\big)^{1/\lambda}=m.$$

Thus,

$$m\leqslant F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big)\leqslant M. \quad \square$$

Theorem 3 (Monotonicity). *If* $\alpha_i\geqslant\varphi_i$ *for all* i*, then*

$$F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big)\geqslant F\big(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle\big). \tag{9}$$

Proof. Let

$$F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big)=\left(\sum_{j=1}^{n}\varpi_j{\beta_j}^{\lambda}\right)^{1/\lambda},$$

$$F\big(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle\big)=\left(\sum_{j=1}^{n}\varpi_j{\xi_j}^{\lambda}\right)^{1/\lambda},$$

where $(\xi_1,\ldots,\xi_n)$ is a reordered $(\varphi_1,\ldots,\varphi_n)$ as per decreasing order of $(u_1,\ldots,u_n)$. As $\alpha_i\geqslant\varphi_i$ for all i, it follows $\beta_j\geqslant\xi_j$ for all j, therefore

$$\begin{aligned}F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big) &= \left(\sum_{j=1}^{n}\varpi_j{\beta_j}^{\lambda}\right)^{1/\lambda}\\ &\geqslant \left(\sum_{j=1}^{n}\varpi_j{\xi_j}^{\lambda}\right)^{1/\lambda}=F\big(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle\big). \quad \square\end{aligned}$$

Theorem 4 (Commutativity). *Let* $(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle)$ $(i=1,2,\ldots,n)$ *is any permutation of* $(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle)$*, then*

$$F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big)=F\big(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle\big). \tag{10}$$

Proof. As $(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle)$ $(i=1,2,\ldots,n)$ is any permutation of $(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle)$, it follows that the corresponding reordered arguments $\beta_j=\xi_j$ for all j, therefore

$$\begin{aligned}F\big(\langle u_1,\alpha_1\rangle,\ldots,\langle u_n,\alpha_n\rangle\big) &= \left(\sum_{j=1}^{n}\varpi_j{\beta_j}^{\lambda}\right)^{1/\lambda}\\ &= \left(\sum_{j=1}^{n}\varpi_j{\xi_j}^{\lambda}\right)^{1/\lambda}=F\big(\langle u_1,\varphi_1\rangle,\ldots,\langle u_n,\varphi_n\rangle\big). \quad \square\end{aligned}$$

3.3. *The Particular Cases of the PFWIGOWA Operator*

The PFWIGOWA operator is a general model that generalizes a wide range of PF aggregation operators. Generally speaking, we can distinguish different types of PFWIGOWA operators from the order-inducing vector U, the parameter λ and the weight vector W.

(1) If $u_1 = u_2 = \cdots = u_n = u$, we obtain the PF generalized weighted average (PFGWA) operator:

$$PFGWA(\alpha_1, \ldots, \alpha_n) = \left(\sum_{j=1}^{n} w_j \alpha_j^{\lambda}\right)^{1/\lambda}. \tag{11}$$

Note that in this case, we have the PFWA operator if $\lambda = 1$.

(2) If $\lambda = 1$, the PFWGIOWA operator reduces to the PF weighted induced weighted averaging (PFWIOWA) operator:

$$PFWIOWA\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \sum_{j=1}^{n} \varpi_j \beta_j. \tag{12}$$

(3) If $\lambda = 2$, the PFWGIOWA operator reduces to the PF weighted induced Euclidean weighted averaging (PFWIEOWA) operator:

$$PFWIEOWA\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \left(\sum_{j=1}^{n} \varpi_j {\beta_j}^2\right)^{1/2}. \tag{13}$$

(4) If $\lambda \to 0$, we form the PF weighted induced weighted geometric (PFWIOWG) operator:

$$PFWIOWG\big(\langle u_1, \alpha_1\rangle, \ldots, \langle u_n, \alpha_n\rangle\big) = \prod_{j=1}^{n} {\beta_j}^{\varpi_j}. \tag{14}$$

(5) The PF weighted max (PFWMax) operator is obtained if $w_p = 1$, $w_j = 0$ for all $j \neq p$, and $u_p = \max(\alpha_i)$.

(6) The PF weighted min (PFWMin) operator is obtained if $w_p = 1$, $w_j = 0$ for all $j \neq p$, and $u_p = \min(\alpha_i)$.

Moreover, by manipulating the similar method commonly used in the references (Merigó *et al.*, 2015; Zeng *et al.*, 2016b; Yu, 2015; Li *et al.*, 2015), we could develop different kinds of the PFWIGOWA operators.

4. MAGDM Method with the PFWIGOWA Operator

In this section, we explore the application of the PFWIGOWA operator in the context of the PF MAGDM problem. In real-life situations, multiple experts are invited to evaluate a certain problem against multiple attribute. Thus, by conducting this type of group decision analysis, one can effectively aggregate and deal with the evaluation values. Let us assume that $A = \{A_1, A_2, \dots, A_m\}$ be the m alternatives, $C = \{C_1, C_2, \dots, C_n\}$ be the n attributes. Assume $E = \{e_1, e_2, \dots, e_t\}$ be the t decision makers (whose weight vector is $\omega = (\omega_1, \omega_2, \dots, \omega_t)$, $\omega_k \geqslant 0$ and $\sum_{k=1}^{t} \omega_k = 1$). The MAGDM process based on the proposed operator can be described as follows:

Step 1. Each decision maker provides his/her own PF decision information related to the alternative A_i under the attribute C_j, thus forming the individual PF decision matrix $D^{(k)} = (\alpha_{ij}^{(k)})_{m\times n}$ $(k = 1, 2, \dots, t)$, where $\alpha_{ij}^{(k)}$ is PFN denoted by: $\alpha_{ij}^{(k)} = (\mu_{ij}^{(k)}, v_{ij}^{(k)})$.

Step 2. The PFWA operator is used to convert all the individual information into a collective one, obtaining the collective PF decision matrix $D = (\alpha_{ij})_{m\times n}$, where

$$\alpha_{ij} = \omega_1\alpha_{ij}^{(1)} \oplus \omega_2\omega_{ij}^{(2)} \oplus \cdots \oplus \omega_k\alpha_{ij}^{(k)}, \quad i = 1, 2, \dots, m; \ j = 1, 2, \dots, n. \tag{15}$$

Step 3. The weight vector is calculated by using Eq. (6) for the PFWIGOWA operator.

Step 4. Calculate the comprehensive evaluation value α_i for each alternative A_i $(i = 1, \dots, m)$ by means of the PFIGOWA operator as defined by Eq. (5). It should be noted that it is possible to consider several PFWIGOWA operators described in Section 3 for specific applications.

Step 5. The alternatives are ranked and selected based on the results obtained in the previous steps.

5. Illustrative Example

In the following, a decision making problem comprising the selection of research and development (R&D) projects is provided to demonstrate the application of the new approach. Given the choice of the suitable R&D projects is an important driving force for national competitive advantage, governments and enterprises around the world have attempted to stimulate R&D investments by supporting R&D projects. The identification of the most promising R&D project from finite alternatives based on multiple attributes constitutes an instance for MAGDM problem. What is more, the R&D projects selection is a highly complex decision process owing to the underlying uncertainty and consequent use of fuzzy numbers. Therefore, we present the application of the PFWIGOWA operator in a MAGDM problem concerning the selection of R&D projects. Suppose that a venture capital firm is trying to invest in a R&D project. The market survey and preliminary screening show that the five potential R&D projects are considered for further assessment. To secure profits and reduce risk, three experts are invited to assess these five projects from the following attributes (criteria) (adapted from Zeng *et al.*, 2018):

(1) C_1: organizing ability;
(2) C_2: credit quality;

Table 1
PF decision matrix-expert 1.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.9, 0.3)	(0.5, 0.8)	(0.7, 0.6)	(0.6, 0.3)	(0.6, 0.3)
A_2	(0.6, 0.3)	(0.7, 0.6)	(0.7, 0.7)	(0.4, 0.4)	(0.3, 0.4)
A_3	(0.4, 0.7)	(0.8, 0.1)	(0.9, 0.2)	(0.5, 0.3)	(0.5, 0.3)
A_4	(0.8, 0.4)	(0.6, 0.2)	(0.7, 0.5)	(0.7, 0.4)	(0.7, 0.4)
A_5	(0.7, 0.2)	(0.8, 0.4)	(0.8, 0.2)	(0.6, 0.6)	(0.6, 0.6)

Table 2
PF decision matrix-expert 2.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.8, 0.4)	(0.6, 0.7)	(0.8, 0.6)	(0.8, 0.3)	(0.6, 0.5)
A_2	(0.4, 0.3)	(0.7, 0.4)	(0.3, 0.7)	(0.4, 0.6)	(0.5, 0.4)
A_3	(0.5, 0.7)	(0.8, 0.5)	(0.9, 0.2)	(0.6, 0.3)	(0.5, 0.6)
A_4	(0.6, 0.6)	(0.7, 0.2)	(0.7, 0.5)	(0.6, 0.4)	(0.7, 0.3)
A_5	(0.7, 0.5)	(0.9, 0.3)	(0.6, 0.4)	(0.7, 0.6)	(0.7, 0.1)

Table 3
PF decision matrix-expert 3.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.8, 0.6)	(0.5, 0.8)	(0.7, 0.6)	(0.5, 0.5)	(0.6, 0.3)
A_2	(0.7, 0.4)	(0.6, 0.1)	(0.7, 0.5)	(0.9, 0.2)	(0.5, 0.6)
A_3	(0.5, 0.6)	(0.8, 0.1)	(0.9, 0.2)	(0.5, 0.3)	(0.4, 0.3)
A_4	(0.9, 0.2)	(0.6, 0.2)	(0.5, 0.6)	(0.6, 0.1)	(0.7, 0.4)
A_5	(0.6, 0.1)	(0.9, 0.2)	(0.8, 0.2)	(0.5, 0.6)	(0.6, 0.4)

(3) C_3: level of research and development;
(4) C_4: profitability;
(5) C_5: debt servicing ability.

The weight vector of experts is given as $\omega = (0.3, 0.4, 0.3)^T$. Owing to uncertainties associated with the phenomena under analysis, the evaluated values of each alternative with respect to each attribute provided by the experts are expressed in PFNs, as shown in Tables 1–3.

First, the PFWA operator is used to convert all the individual information matrix into a collective one. The aggregation results are given in Table 4.

In this problem, the weight vector of the PFWIGOWA operator is assumed to be $W = (0.15, 0.1, 0.25, 0.3, 0.2)^T$. The order-inducing variables are listed in Table 5.

According to the information given above, one can summarize the rows of the PF collective decision matrix and thus obtain the comprehensive value of the alternatives. The aggregation results rendered by the PFWIGOWA ($\lambda = 2$) operator are given in Table 6.

The aggregation results in Table 6 indicate that the ordering of the five alternatives is $A_3 \succ A_5 \succ A_4 \succ A_1 \succ A_2$. Thus, A_3 appears to be the best choice in this case.

Table 4
PF collective results.

	C_1	C_2	C_3	C_4	C_5
A_1	(0.85, 0.40)	(0.53, 0.77)	(0.74, 0.60)	(0.66, 0.35)	(0.60, 0.25)
A_2	(0.59, 0.33)	(0.67, 0.31)	(0.63, 0.63)	(0.68, 0.37)	(0.44, 0.45)
A_3	(0.46, 0.67)	(0.80, 0.16)	(0.90, 0.20)	(0.53, 0.30)	(0.47, 0.37)
A_4	(0.80, 0.37)	(0.63, 0.20)	(0.65, 0.53)	(0.64, 0.26)	(0.70, 0.37)
A_5	(0.67, 0.21)	(0.87, 0.30)	(0.76, 0.25)	(0.61, 0.60)	(0.63, 0.31)

Table 5
Order-inducing variables.

	C_1	C_2	C_3	C_4	C_5
u	0.9	0.6	0.7	0.5	0.8

Table 6
Aggregation results and ranking rendered by the PFWIGOWA operator.

	Aggregation result	Score	Rank
A_1	(0.7193, 0.5097)	0.2576	4
A_2	(0.6302, 0.4012)	0.2362	5
A_3	(0.7746, 0.2755)	0.5241	1
A_4	(0.6938, 0.3237)	0.3767	3
A_5	(0.7633, 0.2940)	0.5008	2

Table 7
Aggregation results and ranking rendered by the PFIGOWA operator.

	Aggregation result	Score	Rank
A_1	(0.7033, 0.5097)	0.2130	5
A_2	(0.6384, 0.3976)	0.2495	4
A_3	(0.7766, 0.2594)	0.5358	1
A_4	(0.6820, 0.3107)	0.3686	3
A_5	(0.7715, 0.3099)	0.4992	2

Next, we explore the aggregation results by using the traditional PFIGOWA operator, whose order-inducing variables are just used to induce the order of argument, but not explicitly used in moderating weights. The results of the PFIGOWA ($\lambda = 2$) operator are shown in Table 7.

Thus, the ordering of the alternatives based on the PFIGOWA operator is:

$$A_3 \succ A_5 \succ A_4 \succ A_2 \succ A_1,$$

therefore, the most desirable alternative is A_3. Evidently, the best alternative based on the two operators is all the A_3. However, as can be observed, the ranking the alternatives obtained by these two operators is different and can be found in alternatives A_1 and A_2. The main reason is that the order-inducing variables of PFWIGOWA operator play double roles, one is to induce the order of the arguments, and then to moderate the associated weights. However, in the PFIGOWA operator, the order-inducing variables only play role in order of argument, which often causes a loss of the intrinsic variation information, thus obtaining a biased aggregation result. Moreover, it is possible to consider numerous special cases of the PFWIGOWA operator as described in Section 3 for a specific application according to the actual needs. Therefore, this approach is rather flexible as it enables more choices for decision maker to select the aggregation schemes in regards to actual needs or his/her interests.

6. Conclusions

In this paper, we put forward the PFWIGOWA operator, which can improve the existing operators with its ability to reflect the intrinsic variations on the order-inducing variables. Moreover, it uses the main advantages of the IOWA and the generalized means in the same formulation to handle uncertain environment that can be represented by means of PFNs. The particular cases and main properties of the proposed operator are investigated. A MAGDM method based on the PFWIGOWA operator are further developed and an application in selection R&D projects is explored, which has shown its usefulness and effectiveness.

In all, the main contributions of the paper concerning the existing induced aggregation operator are summarized as follows: (1) Much improvement has been achieved in enhancing the induced weighted operator methodology. To effectively handle and process the PF information, this study introduces a new PF induced weighted operator, which can improve the existing aggregation operators by extending the roles of its order-inducing variables. Thus, it enables us to capture the variations in the final aggregation results caused by the order-inducing variables. Some properties are also illustrated to prove the proposed method's advantages. (2) Based on the developed method, this paper proposes a novel model for PF MAGDM problems. The model shows a useful and adaptable way to deal with PF preference information and incorporate decision makers' attitudinal in real situations. (3) A comparative analysis with existing approach has been developed in the paper. A real-word case related to R&D projects selection is provided to show the calculation process and the feasibility of the introduced model. The comparative analysis with the existing method illustrates that this model can lead to a better result in the PF environment.

In our future work, combining the weighted induced approach with other aggregation methods is an interesting and important issue. For example, Zeng *et al.* (2018) proposed an integrated aggregation method for aggregating the PF information. One can extend Zeng *et al.*'s integrated aggregation method with the weighted induced approach for PF information. Moreover, further extensions to this method by using hesitant fuzzy elements and other application areas would also be considered, such as military, and for supply chain management (Wang and Wei, 2018; Rostamzadeh *et al.*, 2017).

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