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THE INVESTIGATION OF TWO COMPETITIVE SYSTEMS

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Abstract. The paper deals with a simple model of the competition of two queuing systems, providing the same service. Each system may vary its service price and its service rate. The customers choose the system with less total service price, that depends on the waiting time and on the service price. The possibility for the existence of equilibrium is investigated. Simple cases are investigated analytically. It is shown that the Nash equilibrium exists in special cases only. A modification of the Stakelberg equilibrium is proposed as a model of competition with a prognosis. This prognosis helps form more stable prices and more stable strategies of competitors. The case of social economics is investigated, too. The dynamics of the competition of more realistic stochastic queuing systems is investigated by Monte Carlo simulation. The simulative analysis is realized by means of a rule–based simulation system.

Key words: competition, equilibrium, simulation, rule–based system.

1. Introduction. The models of competitive systems are wide used in various areas of economics, engineering, society, etc. This paper is an attempt of composing the methods of game theory, of queuing theory, as well as those of the rule–based qualitative simulation in order to examine the competitive models. A permanent competition of two enterprises is investigated. The objective of the investigation is to understand what dynamics of the competition is, what decisions are used by competitors, and what mechanism forms the prices. Two enterprises are modeled as two competitive queuing systems, providing the same service, see Mockus (1994). We use such notation:

- y_i is the price of a service for a customer in the i-th system;
- a is the average of the total number of customers, which come per unit of time;
- a_i is the average of the number of customers, which come to the i-th system per unit of time;
- x_i is the service rate: the average of the number of customers, which may be served in the i-th system per unit of time, when the maximum capacity of the system is used;
- $x = (x_1, x_2),\ y = (y_1, y_2),\ z_i = (x_i, y_i),\ z = (z_1, z_2)$;
- φx_i is the cost price of the i-th system per unit of time, $\varphi > 0$;
- fa_iy_i is the income of the i-th enterprise per unit of time, $f > 0$, the coefficient f represents the reduction of income (e.g., due to taxes);
- $u_i(a_i, z_i)$ is the profit of the i-th enterprise per unit of time, $u_i = fa_iy_i - \varphi x_i$;
- G_i is the cost of waiting in the i-th system.

When a customer comes, he chooses the i-th system, if

$$y_i + G_i < y_j + G_j. \tag{1.1}$$

If $y_i + G_i = y_j + G_j$, then a customer chooses randomly. Then the customer stays in a queue in the chosen system. Obviously a_i depends on z according to (1.1): $a_i = a_i(z_1, z_2)$. Further we will use such notation: $u_i = u_i(a_i, z_i) = u_i(z_1, z_2)$.

The i-th system may vary its service price y_i and its service rate x_i (its strategy z_i). Various approaches of optimizing those strategies are investigated in the paper.

Usually, the equilibrium approach is used in the game theory. Let $z_i = t_i(z_j),\ j \neq i$ is the choice of the i-th competitor, when another competitor keeps its strategy equal to z_j, and let the operator $T(z_1, z_2) = (t_1, t_2)$. The reasonable strategy is

$$t_i = \arg\max_{z_i}\{u_i(a_i, z_i)\}. \tag{1.2}$$

So, t_i will be the optimal strategy of the i-th system, if another competitor keeps its strategy z_j, and if the queuing processes are stationary. In a stationary case all customers, which come to the queuing system, leave this system with the same fixed rate of coming.

If there are strategies z^* such that

$$T(z_1^*, z_2^*) = (z_1^*, z_2^*),$$

then the Nash equilibrium (Moulin, 1985) is satisfied, and z^* is the fixed point (Todd, 1976) of the operator T. Naturally, the strategies of competitors will tend to z^*, if this equilibrium exists and is stable (Moulin, 1985), and if competitors use the optimality criterion (1.2). So, the strategies z_i^* will form the prices in the market.

In the Section 2 a deterministic flow of customers is investigated analytically. We show, that the Nash equilibrium exists in special cases only. This is, due to the fact that the competitor do not forecast the answer of the partner. This forecast is natural, as the competition is permanent and the history of the behaviour of competitors is known. Further a modification of the Stakelberg equilibrium (Moulin, 1985) is proposed as a model of competition with a prognosis.

Some stochastic cases of the social optimization approach are investigated in the Section 3.

In Section 4 a qualitative simulation approach realized by the rule–based system is presented. The dynamics of rather a realistic competition with unstationary queuing is simulated by this system, and simulation results are given. In the Section 5 the conclusions are presented.

2. Deterministic case. In this section it is assumed that all customers come and are served according to the fixed schedule, i.e., customers come at the fixed moments with the fixed period $1/a$, and they are served at the fixed period $1/x_i$. So the waiting time γ_i of the i-th system satisfies

$$\gamma_i = \begin{cases} 0. & x_i \geqslant a_i; \\ \to \infty. & x_i < a_i. \end{cases} \tag{2.1}$$

In this simple case the investigation is unsophisticated, but the results are rather general. We assume here, that $G_i = \gamma_i$.

2.1. Urgent service. In this subsection it is assumed that a customer goes away only in the case the systems are busy. Thus,

$$a_1 + a_2 = \min\{a, x_1 + x_2\}. \tag{2.2}$$

In this case there exist stationary trivial queues of the length equal to zero. This case is very special, but the results of the investigation find the conditions, when the equilibrium may exist.

The number of customers, served in the i-th system per unit of time, is such:

$$a_i(z) = \begin{cases} \min\{a, x_i\}, & y_i < y_j; \\ \min\{x_i, \max\{a/2, (a - x_j)\}\}, & y_i = y_j; \\ \min\{x_i, \max\{0, (a - x_j)\}\}, & y_i > y_j. \end{cases} \tag{2.3}$$

Optimal strategies and the optimal profit of the i-th competitor, calculated in accordance with (1.2, 2.3) for fixed x_j and y_j, are such:

$$
\sup_{z_i} u_i =
$$

$$
= \begin{cases} \lim\limits_{y_i \to \infty} u_i = \infty, & \\ \qquad (y_i \to \infty, x_i > 0), & x_j < a; \\ \lim\limits_{y_i \to y_j} u_i = a(f y_j - \varphi), & \\ \qquad (y_i \nearrow y_j,\ x_i = a), & \varphi/f < y_j,\ x_j \geqslant a; \\ 0,\ (x_i = 0), & y_j < \varphi/f,\ x_j \geqslant a; \\ 0,\ (y_i = y_j,\ x_i \leqslant a/2), & y_j = \varphi/f,\ x_j \geqslant a. \end{cases} \tag{2.4}
$$

Theorem 1 follows from (2.4):

Theorem 1. *There is no Nash equilibrium situation in the deterministic case for urgent service.*

Mathematical formulation: The operator T has no fixed point in the domain $y_i \geqslant 0,\ x_i \geqslant 0,\ n = 1, 2$, when (2.1, 2.2) are satisfied.

Proof. We see from 2.4. (case 1 and case 2) that the i-th competitor may gain by changing its strategy. In case 1 the i-th competitor is a monopolist for $a - x_j$ customers, and it may gain by raising its price y_i. In case 2 the i-th competitor may gain by reducing the price y_i until it becomes less than y_j. In cases 3 and 4 the i-th competitor gains by reducing its capacity x_i, and then the j-th competitor may gain like the i-th one in case 1 of (2.4). So, for any point $z = (z_1, z_2)$ there exists a point z', where $u_i' > u_i,\ i \in \{1, 2\}$ and $z_j' = z_j,\ j \neq i$.

2.2. Customers constrained by price. Here we consider the case, when a customer goes away, if the total minimal price $\min(y_i + G_i)$ exceeds some critical level $y_{\max}$. We will show that the Nash equilibrium exists only when $y_{\max} = \varphi/f$. In this case the optimal profit is calculated analogously to (2.4). In the case $y_{\max} < \varphi/f$ the optimal profit equals 0, and the optimal strategy is to be quite lazy: $x_i = 0$. This lazy strategy is a trivial equilibrium situation.

In the case $y_{\max} = \varphi/f$ the optimal profit is:

$$\sup_{x_i} u_i = \begin{cases} 0, \ \big(y_i = y_{\max}, \\ \qquad x_i \leqslant \max\{0, a - x_j\}\big), & y_j < y_{\max}; \\ 0, \ \big(y_i = y_{\max}, \\ \qquad x_i \leqslant \max\{a/2, a - x_j\}\big), & y_i = y_{\max}. \end{cases} \tag{2.5}$$

The equilibrium is at $y_i = y_j = y_{\max} = \varphi/f$, $x_1 + x_2 \leqslant a$, $u_1 = u_2 = 0$.

In the case $y_{\max} > \varphi/f$ the optimal profit is

$$\sup_{x_i} u_i =$$

$$= \begin{cases} \max\{(fy_{\max} - \varphi)(a - x_j), \\ \qquad a(fy_j - \varphi)\}, & x_j < a; \\ a(fy_j - \varphi), \\ \qquad (y_i \nearrow y_j, \ x_i = a), & \varphi/f < y_j, \ x_j \geqslant a; \\ 0, \ (x_i = 0), & y_j < \varphi/f, \ x_j \geqslant a; \\ 0, \ (y_i = y_j, \ x_i \leqslant a/2), & y_j = \varphi/f, x_j \geqslant a. \end{cases} \tag{2.6}$$

In this case the i-th competitor may gain reducing the price y_i until it becomes less than y_j or icreasing its price up to $y_{\max}$, if $x_j < a$. The set of optimal strategies of the i-th competitor doesn't contain the fixed strategy of the j-th competitor. So, there is no equilibrium in this case, and the folowing theorem is valid.

Theorem 2. *In the case when customers are constrained by price, the nontrivial equilibrium situation exists if and only if* $y_1 = y_2 = y_{\max} = \varphi/f$, $x_1 + x_2 \leqslant a$ *and* $u_1 = u_2 = 0$.

The critical level $y_{\max} = \varphi/f$ is defined naturally, if a third enterprise is introduced:

$$u_3 = a_1(\varphi/f - y_1) + a_2(\varphi/f - y_2),$$
$$0 \leqslant a_1 \leqslant x_1, \qquad 0 \leqslant a_2 \leqslant x_2.$$

I.e., the customer may serve himself with the cost φ/f, which is equal to the minimal cost φ plus taxes. The customer chooses the third enterprise, when $y_i > \varphi/f,\ i = 1,2$. In this case $a_i = 0,\ i = 1,2$. If $y_i < \varphi/f$, then $a_i = x_i$. If $y_i = \varphi/f$, then, according to the customer's choice, a_i is arbitrary, and the equilibrium is formed according to Theorem 2.

2.3. The equilibrium with a prognosis. There are some reasons to criticize the strategy (1.2), which defines the Nash equilibrium. First, the competitor optimizes his strategy, despite the optimization by another competitor. Second, the equilibrium exists only when the maximal price is equal to the minimal cost plus taxes. But usually customers may pay prices which are significantly higher than the cost. We consider the strategy of the informed leader in the Stakelberg equilibrium (Moulin, 1985) as a model for a long period prognosis. But differently from Stakelberg we suppose that both competitors forecast the strategy of the partner. So, the optimal strategies are:

$$\begin{aligned} z_1^* &= \arg(u_1^*) = \arg\ \max_{s_1}\{u_1(s_1, \arg\ \max_{s_2} u_2(s_1,s_2))\},\\ z_2^* &= \arg(u_2^*) = \arg\ \max_{s_2}\{u_2(\arg\ \max_{s_1} u_1(s_1,s_2), s_2)\}. \end{aligned} \tag{2.7}$$

We suppose that there exists the equilibrium at z^*, if both competitors gain not less than they had forecasted:

$$\begin{aligned} u_1^* &\leqslant u_1(z^*),\\ u_2^* &\leqslant u_2(z^*). \end{aligned} \tag{2.8}$$

Then such a theorem follows from (2.6):

Theorem 3. *In the deterministic case* (2.1., 2.2.) *the equilibrium* (2.7, 2.8) *exists at such strategies:*

$$\begin{aligned} y_1 &= y_2 = 0.5(y_{max} - \varphi/f) + \varphi/f,\\ x_1 &= x_2 = a/2. \end{aligned} \tag{2.9}$$

Proof. The conditions of the equilibrium folow from (2.7), e.g., case 1, and from (2.6) case 1:

$$\begin{aligned}&\max_{x_1,y_1}(fy_1-\varphi)x_1,\\&(fy_{max}-\varphi)(a-x_1)\geqslant a(fy_1-\varphi).\end{aligned}\tag{2.10}$$

The solution of (2.10) is (2.9). I.e., the equilibrium with a forecast generates nonzero profit.

3. Social model. Now let us consider a social model. In this case a minimized objective function, the social service cost, is defined as

$$\min_x \sum_{i=1}^{2}(a_i\overline{g}_i+x_i),\quad x_i>a_i,\tag{3.1}$$

where $\overline{g}_i$ is the average time a customer stays in the i-th enterprise, while waiting for service and being served. (We define that 1 unit of time = 1 unit of money, and $f=\varphi=1$).

We suppose that the rates of customers a_i satisfy the condition

$$a_1+a_2=a,$$

where a is the average total number of comming customers per unit of time. Here customers don't have a possibility to give up a service, when the service price is too high for them, i.e., all the comming customers without fail are served in a certain enterprise. In our case the service prices y_i, $i=1,2$, for one customer in the i-th enterprise are fixed and equal in each enterprise

$$y_1=y_2.$$

Note that if the service price y_i is fixed so that the profit of a social enterprise equals zero

$$a_iy_i-x_i=0,$$

then the social service cost becomes equal to customers losses.

We may suppose that the number of customers, comming to one enterprise, is distributed corresponding to the Poisson law, and the service time has an exponential distribution. So, the formula to calculate the average waiting time $\overline{g}$ is (see Kleinrock, 1975)

$$\overline{g}_i = \frac{1}{x_i - a_i}, \qquad x_i > a_i.$$

Case **A. Distributed customer flow.** Let us investigate the case when the distribution of customer flows is fixed. It is natural to suppose that in social enterprises the rates of customers a_i, $i = 1, 2$ are equal

$$a_1 = a_2 = \alpha, \quad \alpha > 0.$$

Thus, the minimized objective function is

$$\min_x \sum_{i=1}^{2} \left(\frac{\alpha}{x_i - \alpha} + x_i \right), \quad x_i > \alpha,\ \alpha > 0,\ \forall i. \tag{3.2}$$

In solving this problem we find that

$$x_1 = x_2 = \alpha + \sqrt{\alpha}$$

are the optimal values for the social service cost function (3.2). Note this solution as x_i^{A}, $\forall i$.

Hence, we may write the minimum of the objective function (3.2)

$$L^{\mathrm{A}} = \min_x \sum_{i=1}^{2} \left(\frac{\alpha}{x_i - \alpha} + x_i \right) = 2(\alpha + 2\sqrt{\alpha}).$$

Case **B. One social enterprise.** If the rates of customers in each enterprise of the social system are determined, it is reasonable to unite these enterprises. Thus, instead of some (in our case two) enterprises working side by side, there appears one enterprise with a service rate equal to the total service rate of former enterprises. Then, the customers, instead of queuing in different enterprises, queue up only in one enterprise and are served in turn.

Let us consider the case of one social enterprise, where the rate of customers is equal to a. Here, the minimized social service cost is

$$\min_x (a\overline{g} + x). \tag{3.3}$$

As in the previous case, we suppose the service price y to be fixed and the average time, spent by a customer in the enterprise, $\overline{g}$ to be expressed as

$$\overline{g} = \frac{1}{x-a}, \qquad x > a. \tag{3.4}$$

Replacing the average time, that a customer stays in the enterprise, $\overline{g}$ by (3.4) in (3.3), the objective function becomes equal to

$$\frac{a}{x-a} + x.$$

Analogously to case A we seek the optimal service rate, that gives the minimal social service cost. The optimal service rate is

$$x^{\mathrm{B}} = a + \sqrt{a},$$

and the minimal social service cost is

$$L^{\mathrm{B}} = \min_x \left(\frac{a}{x-a} + x \right) = a + 2\sqrt{a}.$$

Thus, when the price for a service is fixed, the amalgamation of enterprises reduces the total service rate and the social service cost

$$x^{\mathrm{A}} = x_1^{\mathrm{A}} + x_2^{\mathrm{A}} = a + \sqrt{2a} > a + \sqrt{a} = x^{\mathrm{B}},$$

$$L^{\mathrm{A}} = a + 2\sqrt{2a} > a + 2\sqrt{a} = L^{\mathrm{B}}.$$

Therefore, in the social system that is more preferable to have only one enterprise instead of two ones. In particular, that is a good explanation of the social system practice of building only one polyclinic or only one supermarket in one residential district of a town.

4. Rule–based Monte Carlo simulation of the competition.

4.1. The rule–based qualitative simulation method. In order to investigate the competitive model (consisting of 2 service enterprises and one common customers' flow) the rule–based discrete time simulation method (Maskeliūnas, 1993) was applied. The qualities of this method are the following:

- convenience for realization of research prototypes;
- convenience for modification of simulation system components, subsystems, parameters;
- faster and simpler debuging of the simulation system because of its hierarchical organization;
- good visualization capabilities, applicability in getting acquainted of novices with the investigated problem area.

The simulation here proceeds in 2 phases:

1) a qualitative evaluation of the current state and selection of the corresponding set of systems parameter change equations;

2) calculation of system state changes in the next simulation step (according to the selected set of equations of phase 1).

So, a simulation system can be organized as a diagnostic, evaluation system with feed–back links and state–change calculations of the simulated process.

Up till now the proposed rule–based simulation method was used for simulating a water heater (Maskeliūnas, 1993). Using it for the competitive system simulation two additional important features were introduced: 1) variable duration of the simulation step; 2) combination with the global optimization.

1) The simulation process here is a combination of three asynchronous sub-processes (i.e., the income flow of customers, serving at the 1st and the 2nd service enterprise). The checked time moments are such, when a qualitative change occurs in any of the simulated subsystems. The duration of a simulation step varies, dependent on the concrete calculated sequence of qualitative changes. Such an improvement allows to realize any usual qualitative simulation (Kuipers, 1986), (Trave–Massuyes, 1990), not only constant step simulations. Thus, qualitative simulations can be realized by the rule–based system (i.e., by the available expert system shell), without applying specialized qualitative simulation tools.

2) The search for the optimal strategy (cost&price positioning) was simulated. The rule–based simulation method was combined with the Monte Carlo global optimization method, realized using the same rule–based system. The simulation process includes 2 subprocesses: search for the optimal case, and simulation with the selected best cost&price levels. After the current simulation phase both competing enterprises suppose that the competitor will keep the current cost&price of its service and seek (using a generator of random numbers and a trial simulation run for every generated alternative) his best possible cost&price combination. I.e., the optimizer (1.2) is used, and the Nash equilibrium is sought. When both competitors have their prepared cost&price pair, the simulation

using them is executed. The checked number of random generated alternatives and the number of optimization steps are given by the user.

The control parameters of the competitive enterprise simulator are: the price and cost range; the means and variances of customers' flow and service durations at every service enterprise; the factors of service duration/service cost (for both enterprises); the duration of a separate trial simulation and the number of the last checked time moments of a qualitative change (the results gained at those moments are averaged for decreasing effects of a concrete stopping moment); the number of checked Monte Carlo alternatives; and the number of optimization steps.

The content of the main simulation videogram given to the user during the simulation time is presented in Fig. 1.

At any moment of the simulation process the user can temporarily interrupt it and: print the current simulation state data, change simulation parameters, continue or terminate the simulation. The final results of simulation are optimal prices and costs of both enterprises (y_i, x_i, $i = 1, 2$) and their means calculated during the whole simulation process. The results of one concrete simulation and the conclusions of gained experience are presented in subsections 4.2 and 4.3.

4.2. The experiment with two competitive servers. The simulation system was used to construct a model of two competitive servers, say, hairdressing saloons. We assume that the service prices range from 10Lt to 50Lt. Clients come every 10 minutes with the deviation of 5 minutes; one trial simulation (one work day) continues 12 hours. So, one day simulation processes are not yet steady, and rather a realistic dynamics of competition is investigated. Obviously, the results essentially depend on limitations of the simulation duration, limitations of prices and so on. The queues are not extended to the next day, therefore the asymptotic optimal strategy may become

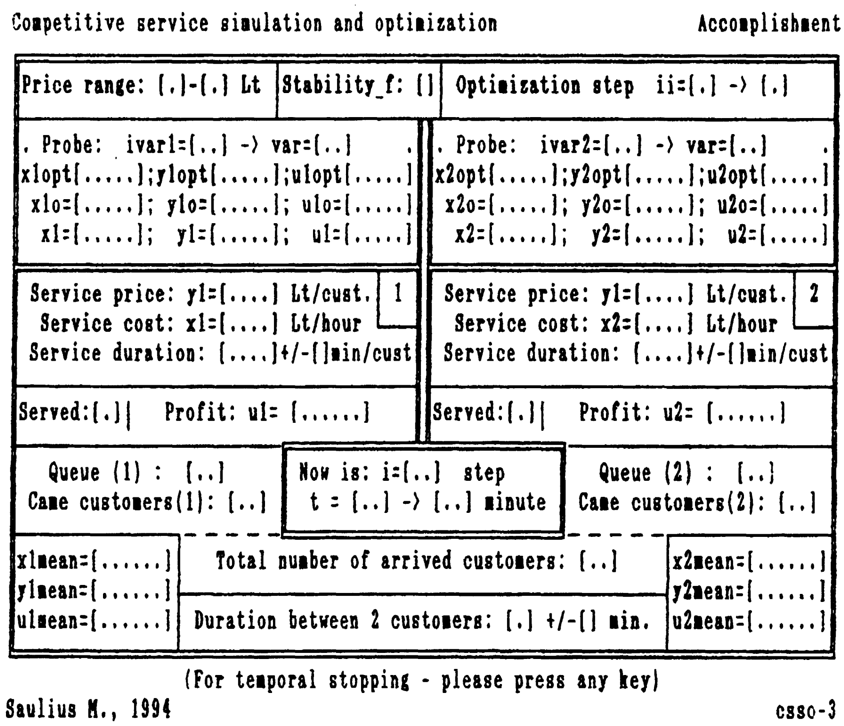

Fig. 1. The main videogramme of the rule–based simulation system.

a bad one. The service rate is measured in h^{-1}. The deviation of the service rate is 4 minutes. The rate of cost price $\varphi = 15$Lt. The taxes are negligible: $f = 1$.

The competitive hairdressing saloon uses the object of optimization (1.2) and the Monte Carlo optimizer in the way described in the previous section. Each of them looks over a lot of random versions of its own service price for one day simulation with the purpose to maximize its profit, expecting that the competitor's service price will be the same as today. Next day both competitors change their service prices according to their yesterday's calculations. And everything is repeated. We set the number of random versions (looked over by every hairdressing saloon) equal to 200. An example of one simulation run is presented in Table 1. The influence of initial values of x_i, y_i, $i = 1, 2$ is insignificant: in all cases the analogous process is reached finally.

4.3. Rule–based Monte Carlo simulation experience and results. In the investigation of the competitive model several different simulation versions were tried, gradually increasing the complexity and improving the simulation system.

The simulation versions were these:

- the step–wise simulation of competition;
- the competition simulation combined with the Monte Carlo optimization, giving a broad menu of control possibilities to the user;
- a significantly accelerated Monte Carlo simulation (the acceleration was gained by limiting control possibilities in the intermediate simulation states, and by informing the user about the simulation current stage only when requested, not at every simulation step);
- the application of additional heuristics.

Analysing the competitive system the simulation parameters and the applied assumptions got gradually complicated from the simplest to more realistic cases of competition. At first, the deterministic case was probed. Later on, the random deviations of the customers' flow and of the service durations were introduced. At the next investigation stage the simulation parameters were adjusted to some hairdressing saloons competition case. Finally, the additional features were introduced: (1) increasing the stability of the simulation results (i.e., decreasing the fluctuations by averaging the re-

Table 1. Dynamics of the service price change in the competitive model

day	φx_1	y_1	u_1	φx_2	y_2	u_2
1	28.07	30.07	-34.37	25.84	27.84	33.97
2	25.19	26.19	26.18	28.20	29.20	-30.06
3	27.93	28.43	-27.93	24.70	25.20	22.36
4	22.87	23.87	19.36	26.41	26.91	-23.82
5	24.85	25.85	-16.98	22.10	22.60	15.66
6	19.40	19.90	9.04	24.47	24.97	-11.53
7	21.68	23.68	-3.77	16.94	18.44	5.32
8	16.27	18.27	4.32	21.82	22.32	-8.06
9	18.28	20.28	0.18	16.18	18.18	4.56
10	13.86	17.36	2.40	17.73	18.73	1.06
11	15.78	18.28	4.49	17.63	19.63	-0.30
12	15.83	18.33	-0.75	16.35	16.85	3.10
13	18.18	20.18	-0.68	16.19	18.19	4.21
14	16.65	18.15	2.52	16.67	18.67	-0.62
15	16.86	18.36	0.56	14.89	17.89	3.35
16	15.31	17.31	2.67	16.89	17.89	-0.84
17	16.81	17.31	1.79	15.89	17.89	-0.19
18	16.43	17.43	-0.27	15.15	17.15	3.14
19	16.06	18.06	-0.63	15.15	17.15	1.49
20	14.64	16.64	2.07	14.93	17.93	1.23
21	14.56	17.56	2.59	16.48	18.98	1.16
22	15.48	18.48	3.00	14.32	17.32	2.08
23	15.48	18.48	0.48	16.67	18.17	2.89
24	15.68	17.68	2.19	16.63	18.13	-0.89
25	15.42	17.92	1.09	14.84	16.34	1.67
26	15.20	16.20	1.26	16.93	18.93	-1.00
27	16.76	18.76	-1.15	15.98	18.48	4.92
28	16.17	17.67	3.06	15.98	18.48	0.44
29	17.17	17.67	-1.15	16.51	17.51	1.83
30	16.33	17.33	2.98	15.98	17.48	-1.36
31	16.33	17.33	-0.13	14.83	16.83	2.12
32	17.05	19.55	5.02	14.33	16.83	2.12
33	18.71	20.71	-5.94	17.92	18.42	4.61

sults gained at the indicated number of last checked qualitative change moments of the trial simulations); (2) setting the maximal price levels the customer can pay for his/her service; (3) introducing the heuristics for seeking better (x_i, y_i) pairs (as the complement to the Monte Carlo generation of service rates and prices).

Some regularities of the competition dynamics, compatible with the theory presented in Section 2, may be derived from the simulation results. When the service rate x_i is small (less than the rate of clients $x_i < a_i$), every competitor raises the service price in accordance with (2.6. case 1) (in our case the service price is limited from above and from below). When the service prices become very large, competitors begin to reduce them in accordance with (2.6 cases 1, 2). The Nash equilibrium was not reached, and that is consistent with Theorems 1, 2. But at the end of the competition process the prices vary in a rather narrow zone. This variation is compatible with the reduction or the raise in conformity with (2.6). The width of the variation zone depends on the limitations of duration of one day simulation and on those of prices. The averaged final prices y_i, $i = 1, 2$ are slightly greater than the critical level φ.

The rule-based Monte Carlo simulation system enables us to vary a set of control parameters, i.e., to vary the conditions of the competition and the strategies of competitors. Some new regularities were observed in this way:

1) The influence of the initial values of x_i, y_i on the simulation results is insignificant: after the transitional period the same final process is reached;

2) The result of the competition has no steady solution (where: $T(z_1^*, z_2^*) = (z_1^*, z_2^*)$), but here a fuzzy zone exists in which x_i and y_i values fluctuate at random. Random fluctuations depend on the customers' flow and service duration random deviations, on the number of tested random alterna-

tives, and on the inner parameters of the simulated system;

3) The fluctuation character is similar to the "saw" shape, i.e., a gradual reduction with a sudden raise. On overstepping some limit level, the next reduction increases the probability of a sudden raise. The deviations from the averaged fluctuation form can be reduced by increasing the number of the tried Monte Carlo alternatives, and by averaging the results, gained at the indicated number of the last checked time moments of a qualitative change in eachtrial simulation (e.g., taking last 6 values). The last mode is especially effective because it decreases the dependence of the simulation results on the incidence of the very last event of simulation time;

4) It is reasonable to combine the random generation of (x_i, y_i) pairs with the use of heuristics. (An example of heuristic: if we know the strategy of partner's service cost and price pair generation – it is reasonable to reject the assumption that he will keep steady values of his choice). The heuristics can be derived as the effective conclusions of the simulation process regularities, that, in their turn, were cleared up by the analysis of previous simulation. That allows us to increase the efficiency, the quality, and the stability of searching for (x_i, y_i) pairs. Various heuristics can be tested assigning them to different simulated competitors. The use of the rule–based system for combining the Monte Carlo simulation with strategic heuristics is especially promising for future investigations.

5. Conclusions. Both analytical and simulation investigations show that there are models of competition, where wide used Nash equilibrium does not exist. This occurs as the competitor optimizes his strategy, without considering the optimization of another competitor. We have proposed and investigated a modification of the Stakelberg equilibrium as a model for the optimization with a prognosis. This equilibrium forms more realistic prices and it represents more reasonable strategies of competitors.

The design, modification and experience with the simulation system have shown that the rule–based simulation approach is useful for designing prototype simulation systems, convenient for obtaining principal operations of the simulated system, for a comparative investigation of different alternative cases, educational and training purposes.

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