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DISCRETE SEQUENTIAL DETECTION OF ABRUPT OR SLOW MULTIPLE CHANGES IN SEVERAL UNKNOWN PROPERTIES OF RANDOM PROCESSES

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Abstract. An essentially new method for discrete sequential detection of abrupt or slow multiple changes in several unknown properties of random processes is considered. The method is based on a sequential nonlinear mapping into two-dimensional vectors of many-dimensional vectors of parameters which describe the properties of random process. The mapping error function is chosen and the expressions for sequential nonlinear mapping are presented along with some experimental results. Theoretical minimum amount of at the very beginning simultaneously mapped vectors is obtained.

Key words: sequential detection of change in properties, complicated random processes, sequential nonlinear mapping.

1. Introduction. The purpose of this paper is to present an essentially new method for sequential detection of many abrupt or slow changes in several unknown properties of random processes. Random processes often have rather a complicated structure. Sometimes nonstationary processes consist of stationary segments with typical properties of a certain class. This takes place when we are watching the states of various dynamic objects or technological processes and detecting their changes. Dynamic objects or technological processes may be described by various parametric models. When the state of an object changes then the parameters of the model change as well. If the object is described by a random process generated by this object, then the object state is described by the data characterizing random process. The object can have several unknown states and we need to identify the states and detect their changes sequentially and independently of the history. It is convenient to watch the properties of complicated random process and their changes marking it by some mark on the PC screen. According to the mark position we can make a decision of the properties of random process and its change if the mark position changes.

For solution of these problems it is necessary to have a method of sequential detection of many abrupt or slow changes in several unknown properties of random processes. There are many methods of detection of changes in the properties of random processes in the scientific publications (Kligienė and Telksnys, 1984; Basseville and Benveniste, 1986; Nikiforov, 1983), but there are no methods to solve the above mentioned problems, because our problem has such characteristic feature as:

- sequential detection,
- many abrupt or slow changes N,
- several unknown properties P.

In the paper we present an essentiallly new method for sequential detection of many abrupt or slow changes in several unknown properties of random processes based on sequential nonlinear mapping onto the plane of vectors of the L parameters which describe the random process. The mapping error function is chosen and the expressions for sequential nonlinear mapping are presented along with some experimental results. Theoretical minimum amount of at the very beginning simultaneously mapped vectors is obtained.

2. Statement of the problem. Let us have in general a nonstationary random process Z_t. However separate segments of the process have their own vector consisting of L constant parameters which describe the segment properties, i.e., we have a locally–stationary process. The stationary segments may be described by a proper mathematical model, e.g., an autoregressive (AR) sequence:

$$z_t - m = -\sum_{i=1}^{p} a_i(z_{t-i} - m) + bv_i, \qquad (1)$$

where m is segment average, p is AR order, a_k $(k = 1, \dots, p)$ are AR parameters, b is amplication factor of input excitation, v_t is discrete white noise distributed by $\mathcal{N}(0,1)$. The assumption of stationarity involves a condition that all the roots of the characteristic equation are less than 1 in absolute value. Then, we have $L = p + 2$ parameters: m, b, a_k $(k = 1, \dots, p)$, and the L-dimensional vectors describe stationary segments of a random process.

For identification of property of random process it is necessary to map the L-dimensional vectors sequentially and nonlinearly into two-dimensional vectors in order to represent the present property of random process by some mark on the PC screen and, having in mind the existance of particular properties, to identify the current property, a deviation from it or a transition to other property when the mark changes its position.

The main requirement of mapping the L-dimensional vectors into two-dimensional vectors is to preserve the inner structure of distances between the vectors. This is achieved using a nonlinear mapping procedure. In (Sammon, 1969) the nonlinear simultaneous mapping algorithm is presented. The stationary segments of random process are described by a proper mathematical model, and L estimates of the model parameters make up the vectors in the L-hyperspace. These L-dimensional vectors are simultaneously nonlinearly mapped into two-dimensional vectors preserving the inner structure of distances among them.

However, it can be done only after having got the whole random process. In such a case when we are working in a real time and segments of a random process are received sequentially, we need to create a sequential nonlinear mapping algorithm. Besides, one should be find cases when the sequential mapping is not available and it is necessary to find conditions which make the mapping to be possible.

3. Solution of the problem. In order to realize sequential nonlinear mapping first of all we have to nonlinearly map M vectors ($M \geqslant 2$) simultaneously. We shall use for that the expressions in (Sammon, 1969). Afterwards, we need to map sequentially and nonlinearly the receiving parameter vectors and, in such a way, to identify the present property of random process, detect its changes and deviations from it for a practically unlimited time. In order to formalize the method we denote by N this practically unlimited number of the arriving vectors.

Thus, let us have $M+N$ vectors in the L-hyperspace. We denote them $X_i,\ i=1,\dots,M;\ X_j,\ j=M+1,\dots,M+N$. The M vectors are already simultaneously mapped into two-dimensional vectors $Y_i,\ i=1,\dots,M$. Now we need to sequentially map the L-dimensional vectors X_j into two-dimensional vectors $Y_j,\ j=M{+}1,\dots,M{+}N$. Here the simultaneos nonlinear mapping expressions will change into sequential nonlinear mapping expressions, respectively. First, before performing iterations it is expedient to put the two-dimensional vectors being mapped in the same initial conditions, i.e., $y_{jk}=C_k,\ j=M+1,\dots,M+N;\ k=1,2$. Note that in the case of simultaneous mapping of the first M vectors, the initial conditions are chosen in a random way (Sammon, 1969). Let the distance between the vectors X_i and X_j in the L-hiperspace be defined by d^x_{ij} and on the plane – by d^y_{ij}, respectively. This algorithm uses the Euclidean distance measure, because, if we have no a priori knowledge concerning the data, we would have no reason to prefer any metric over the Euclidean metric.

For computing the error of distances E we can construct at least three expressions:

$$E_1 = \frac{1}{\sum_{i=1}^{M}(d^x_{ij})^2} \sum_{i=1}^{M}(d^x_{ij} - d^y_{ij})^2, \quad j = M+1, \ldots, M+N; \tag{2}$$

function E_1 reveals the largest errors independently of magnitudes of d^x_{ij};

$$E_2 = \sum_{i=1}^{M}\left(\frac{d^x_{ij} - d^y_{ij}}{d^x_{ij}}\right)^2, \quad j = M+1, \ldots, M+N; \tag{3}$$

function E_2 reveals the largest partial errors independently of magnitudes of $|d^x_{ij} - d^y_{ij}|$;

$$E_3 = \frac{1}{\sum_{i=1}^{M} d^x_{ij}} \sum_{i=1}^{M} \frac{(d^x_{ij} - d^y_{ij})^2}{d^x_{ij}}, \quad j = M+1, \ldots, M+N; \tag{4}$$

function E_3 is the useful compromise and reveals the largest product of error and partial error. So we choose the third expressions for computing the error of distances E.

For correct mapping we have to change the positions of vectors Y_j, $j = M+1, \ldots, M+N$ on the plane in such a way that the error E would be minimal. This is achieved by using the steepest descent procedure. After the r-th iteration the error of distances will be

$$E_j(r) = \frac{1}{\sum_{i=1}^{M} d^x_{ij}} \sum_{i=1}^{M} \frac{\left[d^x_{ij} - d^y_{ij}(r)\right]^2}{d^x_{ij}}, \quad j = M+1, \ldots, M+N. \tag{5}$$

Here

$$d_{ij}^{y}(r) = \sqrt{\sum_{k=1}^{2} \left[y_{ik} - y_{jk}(r) \right]^2}, \quad i = 1, \ldots, M, \quad j = M+1, \ldots, M+N. \tag{6}$$

During the $r+1$-iteration the coordinates of the mapped vectors Y_j will be

$$y_{jk}(r+1) = y_{jk}(r) - \mathbf{F} \cdot \Delta_{jk}(r), \quad j = M+1, \ldots, M+N, \quad k = 1, 2; \tag{7}$$

where

$$\Delta_{jk}(r) = \frac{\partial E_j(r)}{\partial y_{jk}(r)} \Big/ \left| \frac{\partial^2 E_j(r)}{\partial y_{jk}^2(r)} \right|. \tag{8}$$

$\mathbf{F}$ is the coefficient for correction of the coordinates and it is defined empirically to be $\mathbf{F} = 0.35$;

$$\frac{\partial E_j}{\partial y_{jk}} = H \sum_{i=1}^{M} \frac{D \cdot C}{d_{ij}^{x} \cdot d_{ij}^{y}}, \tag{9}$$

$$\frac{\partial^2 E_j}{\partial y_{jk}^2} = H \sum_{i=1}^{M} \frac{1}{d_{ij}^{x} \cdot d_{ij}^{y}} \left[\mathbf{D} - \frac{C^2}{d_{ij}^{y}} \left(1 + \frac{\mathbf{D}}{d_{ij}^{y}} \right) \right], \tag{10}$$

$$H = -\frac{2}{\sum_{i=1}^{M} d_{ij}^{x}}; \quad \mathbf{D} = d_{ij}^{x} - d_{ij}^{y}; \quad C = y_{jk} - y_{ik}.$$

When $E_j < \varepsilon$, where ε can be taken arbitrarily small, the iteration process is over and result is shown on the PC screen. In fact it is enough $\varepsilon = 0.01$. In order to have equal computing time for each mapping we can execute constant number of iterations. In practice it is enough $\mathbf{I} = 30 \div 50$.

4. Experimental results. Let random process (RP) has any property p_i of the set of possible properties: $p_i \in \mathbf{P}$. Let $\mathbf{P} = 4$ and the RP(4) is described by the 3-rd–order AR equation (1) with the parameters (see Table 1). Each collection (vector L) of AR parameters describes some property of RP(4).

Table 1. The AR parameters of the properties $1 \div 4$ of RP(4)

PROPERTY	a_1	a_2	a_3	b
1	0.312	0.62	0.35	1.0
2	0.11	0.21	0.2	1.0
3	−0.315	0.1	0.25	1.0
4	−0.12	0.52	0.41	1.0

We detect the properties of RP(4) at $M + N = 16$ time moments. First we take such a case when the number of initial simultaneous mapping of property vectors is equal to the number of stationary properties of the RP(4): $M = \mathbf{P} = 4$ and during the time moments $M = 1 \div 4$ the RP(4) passes through all its possible properties **P**. Then the view on the screen is "fixed" from the very beginning because of the automatic scale of coordinates. After that we detect the properties of RP(4) at the time moments $N = 5 \div 16$ sequentially. According to the conditions of the experiments a priori the properties of RP(4) are known at the time moments (see Table 2).

At each time moment the AR parameters are estimated from segments of 256 lenght using the Yule–Walker equations (Box and Jenkins, 1970). In Fig. 1 the results of mapping are presented, where at the first $M = 4$ time moments property vectors, mapped simultaneously, are denoted by mark $\times$ with the index which means the time moment number, and the

Table 2. The properties of RP(4) at the time moments $M + N = 4 + 12 = 16$

MAPPING	SIMULTAN (i)				SEQUENTIAL (j)											
MARK	$\times$				$+$											
TIME MOMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
PROPERTY	1	2	3	4	4	1	3	2	4	1	2	2	3	4	1	3

property vectors mapped sequentially, are denoted by mark $+$ with the respective index.

Now, let us take the case when the number of initial simultaneously mapped property vectors will be minimal $M = 2$ and does not involve all the possible properties of RP(4). After that we shall detect the properties of RP(4) at the time moments $N = 3 \div 16$. sequentially. A priori the properties of RP(4) at the time moments are the same as in the previous experiment and are presented in Table 2. In Fig. 2 the results of mapping are presented where at the first $M = 2$ time moments property vectors mapped simultaneously are denoted by $\times$ and vectors mapped sequentially denoted by $+$ respectively.

In Fig. 3. the situation similar to that of Fig. 2 is presented. The difference is at the time moments number 5 and number 11, where a slow changes of the properties of RP(4) took place, and random process at the time moments 5 and 11 changed its properties from 4 to 1 and from 1 to 2 respectively. In Fig. 3 this situation is clear.

5. Theoretically possible complicated case. In all generated situations, when amount of initial simultaneously mapped vectors of parameters of RP properties was taken to be $M = 2$, at every time moments the marks of RP properties

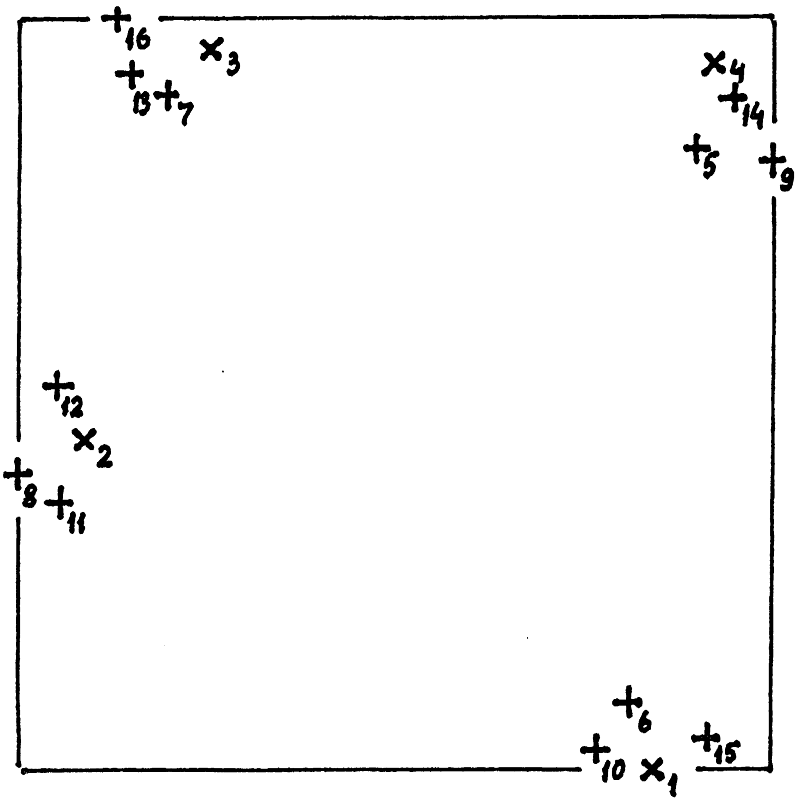

Fig. 1. The view on the PC screen of the mapped vectors of RP(4) properties for the first situation.

got into their right places on the PC screen and the properties were identified correctly (Montvilas, 1993). Even those properties which were not involved into M initial vectors of parameters had got their own places on PC screen and at every time moments the places of marks on the screen corresponded to right RP properties entirely.

However, theoretically there are possible such cases, when points (ends of parameter vectors) being in different places in the L-space can be mapped into one point on the plane, because those points have the same distances with $M = 2$

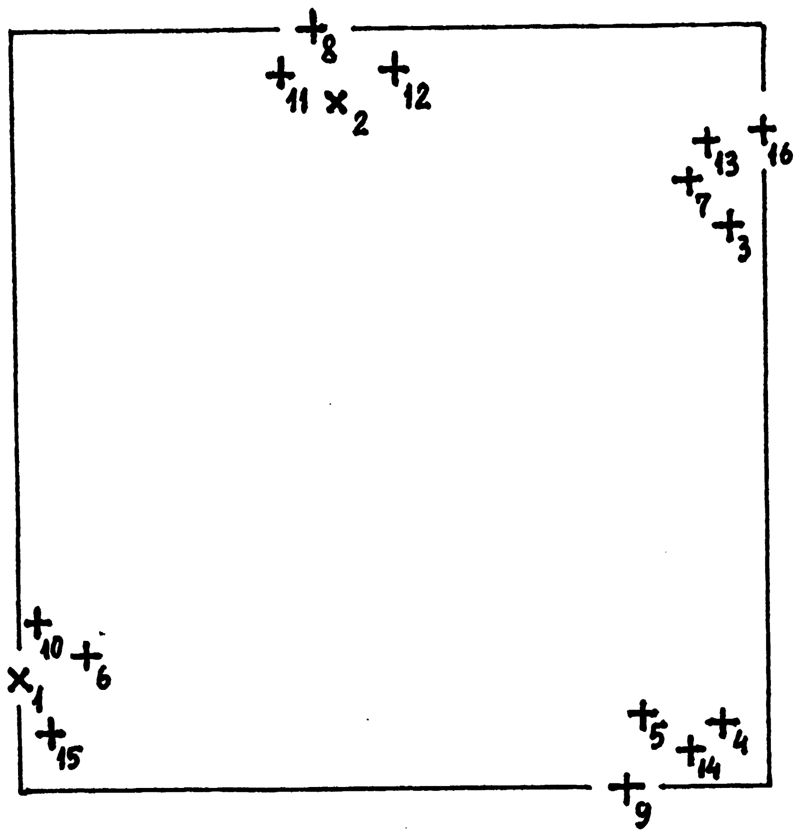

Fig. 2. The view on the PC screen of the mapped vectors of DS(4) states for the second situation.

simultaneously mapped points. By way of illustration let us map points from three-dimensional space ($L = 3$) onto the plane. Let the initial simultaneously mapped points are A and B (see Fig. 4).

Let they are being on the axis of cylinder. Then points C, D and E being on circle, which is on surface of cylinder, have the equal distances with point A and point B: $d^x_{AC} = d^x_{AD} = d^x_{AE}$ and $d^x_{BC} = d^x_{BD} = d^x_{BE}$. So points C, D and E having the same initial conditions will be mapped onto the plane into the same point. If these points mean the different

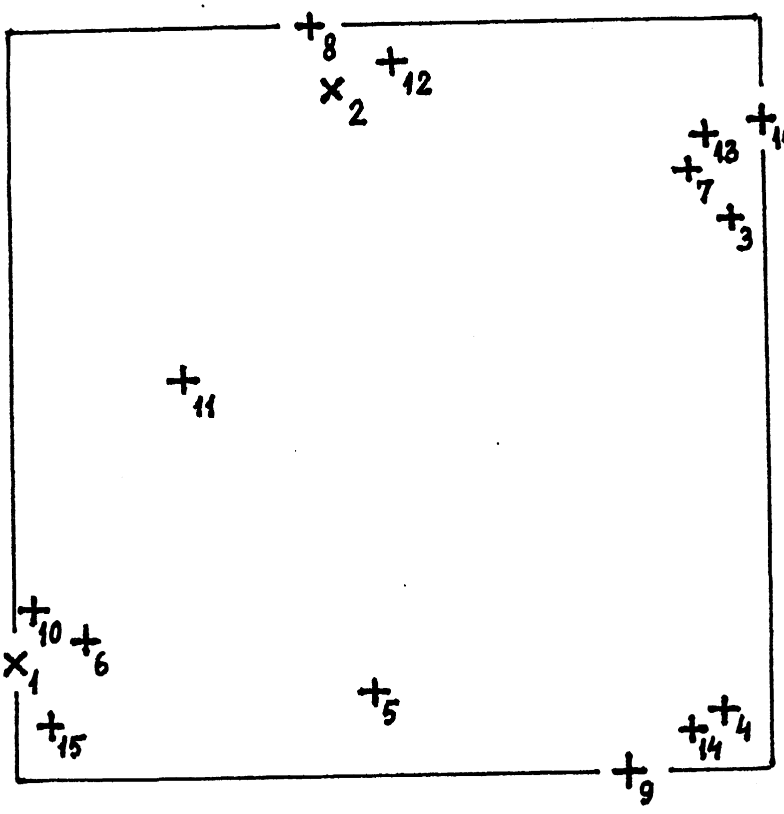

Fig. 3. The view on the PC screen for the third situation.

properties of RP then we shall have a mistake.

Now let us have three points $M = 3$ as the initial points for simultaneous mapping: A, B and C. Then one can draw any straight line orthogonal to the plane ABC. Any two points of the line G and K being on different sides of the plane ABC have equal distances with points A, B and C: $d^x_{AG} = d^x_{AK}$, $d^x_{BG} = d^x_{BK}$ and $d^x_{CG} = d^x_{CK}$. So in this case points G and K can be mapped into one point, too.

When taking $M = 4$ points (A, B, C and D) so that the fourth D point would not be on the plane ABC and the

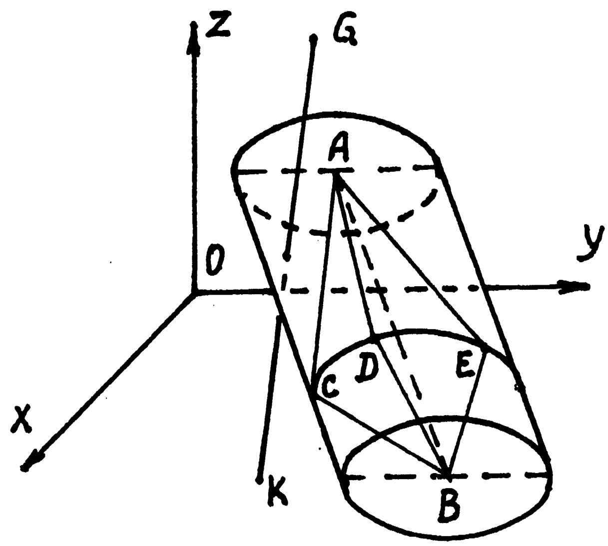

Fig. 4. The case of three-dimensional space ($L = 3$).

all $M = 4$ points would form the three-dimensional space we have a situation when even theoretically one can not find any two points, which have equal distances with all simultaneously mapped $M = 4$ points.

Thus, having analysis of various possible situations at diverse L and P values, we can draw a conclusion that for initial simultaneous mapping one need to take $M = \min(L+1, P)$ when P is known or $M = L + 1$ when P is unknown or random process can have indeterminate properties, besides, these $M = L + 1$ points have to form the L-dimensional space.

In practice, how it was mentioned above, it is enough to have $M = 2$ vectors of parameters for the initial simultaneous mapping, because cases considered here can take place only under coincidence of unexpectednes. However, in order to avoid only theoretically possible complications, we need to do the following: after having simultaneous mapping of $M = 2$ and sequential mapping of $L - 1$ vectors we have got $L + 1$ vectors already. Then we have to map simultaneously the available $L+1$ vectors again and after that to map sequentially the receiving later vectors with respect to the initial $L + 1$ vectors.

6. Conslusions. The described method enables us to sequentially detect the properties of random processes, their abrupt or slow changes and to watch it on PC screen.

Before sequential detection of the properties, it suffices to map simultaneously only $M = 2$ vectors of parameters. However, in order to avoid probably only theoretically possible complications one need to take $M = L + 1$, where L is the dimensionality of vectors of parameters which describe the properties of random processes.

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