INFORMATICA, 1994, Vol. 5, No. 1–2, 79 –97

SOME PROPERTIES OF THE QUASIHOMOGENEOUS AUTOREGRESSIVE RANDOM FIELDS

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Abstract. The problem of the classification, description by the difference equations and possible models of quasihomogeneous autoregressive random fields, existing in one-dimensional space R^1, is considered. The properties of the quasihomogeneous areas as well as of the parameters changing by not the jumps areas of such fields are considered also. The quasihomogeneous areas determination algorithm is proposed.

Key words: random field, autoregressive field, quasihomogeneous field.

1. Introduction. The random processes are divided into stacionary and nonstacionary in accordance with independence or dependence its characteristics on the time (Anderson, 1976; Box and Jenkins, 1974; Kendall and Stuart, 1976; Kapustinskas, 1983). In similar way the random fields are divided into homogeneous and nonhomogeneous (Rytov, 1978). The statistical characteristics of a homogeneous field are constant, i.e., they are independent on the space and time coordinates, and on the contrary the such characteristics of a nonhomogeneous fields are dependent on the space and the time coordinates. The homogeneous autoregressive (AR) fields are described by a difference equations with the constant parameters. Some its properties were considered and the indentification methods were proposed by the author in the recent papers (Kapustinskas, 1985a, 1985b, 1986a, 1986b, 1987, 1988, 1989a, 1989b, 1990, 1992). The nonhomogeneous AR fields are described by a difference equations with inconstant parameters, i.e., these parameters are functions of the space and the time coordinates. The quasihomogeneous (QH) fields belongs to the class of the nonhomogeneous fields. The statistical characteristics of such fields are jump functions of the space and time coordinates.

The QH AR fields, existing in one-dimensional space R^1 and the time, are considered below in this paper.

2. The problem. Let AR field exists in one-dimensional space R^1 and the time, i.e., the values of the field are located at the points of the space (x, t), where x, t are discrete values of the space and the time coordinates $\big(x,\ t \in (-\infty, \infty)\big)$. The field is considered in the area Ω, which we shall call as the field consideration (FC) area. The homogeneous AR is described by following difference equation (Kapustinskas, 1985a)

$$\xi_t^x = \sum_{k=1}^{n_t} \sum_{i=-n'_x}^{n''_x} a_k^i \, \xi_{t-k}^{x+i} + b\, \delta_t^x, \tag{1}$$

where ξ_t^x is the value of the field at point x and moment t $\big(\xi_t^x \in (-\infty, \infty)\big)$, n_t is the order of the field with regard to coordinate t, $\{n'_x,\ n''_x\}$ is the order of the field with regard to space coordinate x, $\{\delta_t^x\}$ is a sequence of independent normal random values with zero average and finite variance $\sigma_g^2 = 1$ (a white noise field), $\{a_k^i\}$, b are the parameters of the field.

The coefficients of the difference equation of a homogeneous AR field are constant, i.e., are independent on the coordinates x, t (a_k^i = const, b = const,) inside of the FC area Ω. It is known, that the variance σ_ξ^2 of these fields is certain function of the parameters a_k^i, b (Box and Jenkins, 1974; Kapustinskas, 1983). It follows, that the variance of such fields is constant in the all FC area ($\sigma_\xi^2 = \text{const},\ x,\ t \in \Omega$), because its parameters a_k^i, b are constant.

We shall call QH AR field such field, variance σ_ξ^2 of which is a jump function at any rate of the one of the coordinates x, t. The problem is to describe such fields by a difference equations, to propose its classification and possible models and to consider the properties of such fields.

3. Classification of the quasihomogeneous AR fields. Because the variance of the QH AR field is jump function at any rate of the one of the coordinates x, t, it is obvious, that it changes by one of five possible ways:

1) changes by jumps in the space and the time

$$\sigma_\xi^2 = \sigma_\xi^2(x,t) = \sigma_\xi^2(j) = \text{const}, \tag{2}$$

2) is constant in the time and changes by jumps in the space

$$\sigma_\xi^2 = \sigma_\xi^2(x) = \sigma_\xi^2(j) = \text{const}, \tag{3}$$

3) is constant in the space and changes by jumps in the time

$$\sigma_\xi^2 = \sigma_\xi^2(t) = \sigma_\xi^2(j) = \text{const}, \tag{4}$$

4) changes by jumps in the space and by not the jumps in the time

$$\sigma_\xi^2 = \sigma_\xi^2(x,t) = \sigma_\xi^2(j,t), \quad \left(\sigma_\xi^2 = \sigma_\xi^2(j)\big|_t = \text{const}\right), \tag{5}$$

5) changes by jumps in the time and by not the jumps in the space

$$\sigma_\xi^2 = \sigma_\xi^2(x,t) = \sigma_\xi^2(x,j), \quad \left(\sigma_\xi^2 = \sigma_\xi^2(j)\big|_x = \text{const}, \tag{6}$$

where

$$x,\, t \in \Omega_j \subset \Omega \quad (j = \overline{1, N}). \tag{7}$$

Because the variance σ_ξ^2 is constant or changes by not a jumps alongside one of the axes x or t inside all area Ω_j, we shall call these areas as the QH areas.

As the dispersion of the QH AR field can change by one of above five ways, it is naturally to divide such fields into following five classes:

1) the quasihomogeneous–quasistationary (QHQS) AR fields, when variance σ_ξ^2 changes by jumps in the space and the time in accordance with Eq. (2).

2) the quasihomogeneous–stationary (QHS) AR fields, when variance σ_ξ^2 is constant in the time and changes by jumps in the space in accordance with Eq. (3).

3) the homogeneous–quasistationary (HQS) AR fields, when variance σ_ξ^2 is constant in the space and changes by jumps in the time in accordance with Eq. (4).

4) the quasihomogeneous–nonstationary (QHNS) AR fields, when variance σ_ξ^2 changes by jumps in the space and by not a jumps in the time in accordance with Eq. (5).

5) the nonhomogeneous–quasistationary (NHQS) AR fields, when variance σ_ξ^2 changes by jumps in the time and by not the jumps in the space in accordance with Eq. (6).

Let us denote the parameters a_k^i, b as

$$c = \{c_p\} = \{a_k^i,\, b\} \qquad (p = 1,\, n_c), \tag{8}$$

where n_c – the total number of the parameters c:

$$n_c = \{n_x' + n_x'' + 1\}\, n_t. \tag{9}$$

Because the variance σ_ξ^2 of any AR field is dependent on the parameters a_k^i, b, it is obvious, that the variance σ_ξ^2 of the

QH AR field may change by jumps only in the case, when at any rate one parameter of such field changes by jumps in the space or the time. Therefore the QH AR field is described by the same Eq. (1) as the homogeneous AR field, if at any rate one of the coefficients of this equation is jump function of the space or the time.

Let Δ_j be areas, inside of which the parameters c are constant or changes by not jumps in the space or the time. We shall call these areas as the parameters changing by not the jumps (PCNJ) areas.

4. Models of the QH AR fields. We shall call the parameters a_k^i as the A group and b as the B group of the parameters. It is possible to divide these groups into changing by jumps, by not the jumps and not changing (constant) parameters also. We shall use the upper indexes for denoting parameters changing by jumps, lower – for changing by not the jumps and index c – for constant parameters (Table 1). For example, A^{xt} means that the parameters a_k^i are changing by jumps in the space and the time, and B_t^x – that the parameter b is changing by jumps in the space and by not the jumps in the time.

Then it is obvious, that the parameters of the QH AR fields and its classes must be built only from the elements of following sets of the parameters:

$$\begin{aligned}\text{QH:}\quad & c = \{A,\, B\} \\ & = \{A^c, A^x, A^t, A^{xt}, A_x, A_t, A_t^x, A_x^t, \\ & \qquad B^c, B^x, B^t, B^{xt}, B_x, B_t, B_t^x, B_x^t\}, \qquad (10)\end{aligned}$$

$$\begin{aligned}\text{QHQS:}\quad & c = \{A,\, B\} \\ & = \{A^c, A^x, A^t, A^{xt}, B^c, B^x, B^t, B^{xt}\}, \qquad (11)\end{aligned}$$

$$\text{QHS:}\quad c = \{A,\, B\} = \{A^c, A^x, B^c, B^x\}, \qquad (12)$$

$$\text{HQS:}\quad c = \{A,\, B\} = \{A^c, A^t, B^c, B^t\}, \qquad (13)$$

Table 1. The denoting of the parameters changing manner

The parameters changing manner	Parameters	
	a_k^i	b
Constant parameters	A^c	B^c
Parameters changes by jumps:		
– with regard to x, t,	A^{xt}	B^{xt}
– with regard to x and are constant with regard to t,	A^x	B^x
– with regard to t and are constant with regard to x,	A^t	B^t
– with regard to x and not by jumps with regard to t,	A_t^x	B_t^x
– with regard to t and not by jumps with regard to x.	A_x^t	B_x^t
Parameters changes not by jumps:		
– with regard to x and are constant with regard to t,	A_x	B_x
– with regard to t and are constant with regard to x.	A_t	B_t

$$\text{QHNS:} \quad c = \{A,\, B\} = \{A^c, A^x, A_t, A_t^x, B^c, B^x, B_t, B_t^x\}, \tag{14}$$

$$\text{NHQS:} \quad c = \{A,\, B\} = \{A^c, A^t, A_x, A_x^t, B^c, B^t, B_x, B_x^t\}. \tag{15}$$

The various combinations of the A and B parameters may be built from the elements of these sets for every class of the

Table 2. The models of quasihomogeneous–quasistationary (QHQS) AR fields

Possible combinations of A parameters	Possible combinations of B parameters			
	B^{xt}	B^x	B^t	B^c
A^c	A^cB^{xt}	--	--	--
A^x	A^xB^{xt}	--	A^xB^t	--
A^t	A^xB^{xt}	A^tB^x	--	--
A^{xt}	$A^{xt}B^{xt}$	$A^{xt}B^x$	$A^{xt}B^t$	$A^{xt}B^c$
A^cA^x	$A^cA^xB^{xt}$	--	$A^cA^xB^t$	--
A^cA^t	$A^cA^tB^{xt}$	$A^cA^tB^x$	--	--
A^cA^{xt}	$A^cA^{xt}B^{xt}$	$A^cA^{xt}B^x$	$A^cA^{xt}B^t$	$A^cA^{xt}B^c$
A^xA^t	$A^xA^tB^{xt}$	$A^xA^tB^x$	$A^xA^tB^t$	$A^xA^tB^c$
A^xA^{xt}	$A^xA^{xt}B^{xt}$	$A^xA^{xt}B^x$	$A^xA^{xt}B^t$	$A^xA^{xt}B^c$
A^tA^{xt}	$A^tA^{xt}B^{xt}$	$A^tA^{xt}B^x$	$A^tA^{xt}B^t$	$A^tA^{xt}B^c$
$A^cA^xA^t$	$A^cA^xA^tB^{xt}$	$A^cA^xA^tB^x$	$A^cA^xA^tB^t$	$A^cA^xA^tB^c$
$A^cA^tA^{xt}$	$A^cA^tA^{xt}B^{xt}$	$A^cA^tA^{xt}B^x$	$A^cA^tA^{xt}B^t$	$A^cA^tA^{xt}B^c$
$A^xA^tA^{xt}$	$A^xA^tA^{xt}B^{xt}$	$A^xA^tA^{xt}B^x$	$A^xA^tA^{xt}B^t$	$A^xA^tA^{xt}B^c$
$A^cA^xA^tA^{xt}$	$A^cA^xA^tA^{xt}B^{xt}$	$A^cA^xA^tA^{xt}B^x$	$A^cA^xA^tA^{xt}B^t$	$A^cA^xA^tA^{xt}B^c$

Table 3. The models of quasihomogeneous–stationary (QHS) and homogeneous–quasistationary (HQS) AR fields

Field	Possible combinations of A parameters	Possible combinations of B parameters		
		B^x	B^c	B^t
QHS	A^c	A^cB^x	--	--
	A^x	A^xB^x	A^xB^c	-
	A^cA^x	$A^cA^xB^x$	$A^cA^xB^c$	--
HQS	A^c	--	--	A^cB^t
	A^t	--	A^tB^c	A^tB^t
	A^cA^t	--	$A^cA^tB^c$	$A^cA^tB^t$

QH AR fields. For example, are possible only three combinations of the A parameters – $\{A^c\}$, $\{A^x\}$, $\{A^c A^x\}$ and only two of the B parameters – $\{B^c\}$, $\{B^x\}$ for the QHS AR fields. The combination $\{A^c\}$ means, that all parameters A of the field are constant, $\{A^x\}$ – that all parameters A changes by a jump in the space and are constant in the time, $\{A^c A^x\}$ – that a part of A parameters are constant and remain part – changes by the jumps in the space and are constant in the time. Such number of the possible parameters combinations is existing and for HQS AR fields (Table 1). The 15 A parameters combinations and 4 B parameters combinations are existing for remain fields (Tables 2, 4, 5).

Table 4. The models of quasihomogeneous–nonstationary (QHNS) AR fields

Possible combinations of A parameters	Possible combinations of B parameters			
	B_t^x	B_t	B^c	B^x
A^c	$A^c B_t^x$	--	--	--
A^x	$A^x B_t^x$	$A^x B_t$	--	--
A_t	$A_t B_t^x$	--	--	$A_t B^x$
A_t^x	$A_t^x B_t^x$	$A_t^x B_t$	$A_t^x B^c$	$A_t^x B^x$
$A^c A^x$	$A^c A^x B_t^x$	$A^c A^x B_t$	--	--
$A^c A_t$	$A^c A_t B_t^x$	--	--	$A^c A_t B^x$
$A^c A_t^x$	$A^c A_t^x B_t^x$	$A^c A_t^x B_t$	$A^c A_t^x B^c$	$A^c A_t^x B^x$
$A^x A_t$	$A^x A_t B_t^x$	$A^x A_t B_t$	$A^x A_t B^c$	$A^x A_t B^x$
$A^x A_t^x$	$A^x A_t^x B_t$	$A^x A_t^x B_t$	$A^x A_t^x B^c$	$A^x A_t^x B^x$
$A_t A_t^x$	$A_t A_t^x B_t^x$	$A_t A_t^x B_t$	$A_t A_t^x B^c$	$A_t A_t^x B^x$
$A^c A^x A_t$	$A^c A^x A_t B_t^x$	$A^c A^x A_t B_t$	$A^c A^x A_t B^c$	$A^c A^x A_t B^x$
$A^c A_t A_t^x$	$A^c A_t A_t^x B_t^x$	$A^c A_t A_t^x B_t$	$A^c A_t A_t^x B^c$	$A^c A_t A_t^x B^x$
$A^x A_t A_t^x$	$A^x A_t A_t^x B_t^x$	$A^x A_t A_t^x B_t^x$	$A^x A_t A_t^x B^c$	$A^x A_t A_t^x B^x$
$A^c A^x A_t A_t^x$	$A^c A^x A_t A_t^x B_t^x$	$A^c A^x A_t A_t^x B_t$	$A^c A^x A_t A_t^x B^c$	$A^c A^x A_t A_t^x B^x$

Therefore 6 combinations of the parameters A, B are possible for QHS and HQS classes of AR fields and 60 – for remain

Table 5. The models of nonhomogeneous–quasistationary (NHQS) AR fields

Possible combinations of A parameters	Possible combinations of B parameters			
	B_x^t	B_x	B^c	B^t
A^c	$A^c B_x^t$	--	--	--
A_x	$A_x B_x^t$	$A_x B_x$	--	$A_x B^t$
A^t	$A^t B_x^t$	$A^t B_x$	--	--
A_x^t	$A_x^t B_x^t$	$A_x^t B_x$	$A_x^t B^c$	$A_x^t B^t$
$A^c A_x$	$A^c A_x B_x^t$	--	--	$A^c A_x B^t$
$A^c A^t$	$A^c A^t B_x^t$	$A^c A^t B_x$	$A^c A^t B^c$	$A^c A^t B^t$
$A^c A_x^t$	$A^c A_x^t B_x^t$	$A^c A_x^t B_x$	$A^c A_x^t B^c$	$A^c A_x^t B^t$
$A^x A^t$	$A^x A^t B_x^t$	$A^x A^t B_x$	$A^x A^t B^c$	$A^x A^t B^t$
$A_x A_x^t$	$A_x A_x^t B_x^t$	$A_x A_x^t B_x$	$A_x A_x^t B^c$	$A_x A_x^t B^t$
$A^t A_x^t$	$A^t A_x^t B_x^t$	$A^t A_x^t B_x$	$A^t A_x^t B^c$	$A^t A_x^t B^t$
$A^c A_x A^t$	$A^c A_x A^t B_x^t$	$A^c A_x A^t B_x$	$A^c A_x A^t B^c$	$A^c A_x A^t B^t$
$A^c A^t A_x^t$	$A^c A^t A_x^t B_x^t$	$A^c A^t A_x^t B_x$	$A^c A^t A_x^t B^c$	$A^c A^t A_x^t B^t$
$A_x A^t A_x^t$	$A_x A^t A_x^t B_x^t$	$A_x A^t A_x^t B_x$	$A_x A^t A_x^t B^c$	$A_x A^t A_x^t B^t$
$A^c A_x A^t A_x^t$	$A^c A_x A^t A_x^t B_x^t$	$A^c A_x A^t A_x^t B_x$	$A^c A_x A^t A_x^t B^c$	$A^c A_x A^t A_x^t B^t$

classes. However, not every of these combinations are admissible. For example, the combination $\{A^c B^c\}$ is inadmissible, because such field is not a QHS AR field – it is homogeneous AR field. We shall call AR fields with admissible combinations of the parameters A, B as the models of the QH AR fields. In all there are 45 various models of the QHQS, QHNS and NHQS AR fields (Tables 2, 4, 5) and 5 models of the QHS and HQS AR fields (Table 3). All these models are described by the difference Eq. (1) with adequate combinations of the parameters A, B.

5. Properties of PCNJ and QH areas. The PCNJ areas Δ_j^p $(j = \overline{1, N_\Delta^p},\ p = \overline{1, n_c})$ of the parameters a_k^i, b are called an areas, inside of which these parameters are constant or changing by not the jumps with regard to x or t coordinate.

Therefore the PCNJ areas of everyone parameter do not cross each other (Faure, 1966)

$$\Delta^p_{j'} \cap \Delta^p_{j''} = \varnothing \qquad (j', j'' = \overline{1, N^p_\Delta}). \tag{16}$$

Otherwise an areas should exist, inside of which the parameters may obtain various values at the same moment. But that is a nonsense.

The other property – the PCNJ areas of each parameter fulfills the FC area, i.e.,

$$\bigcup_{j=1}^{N^p_\Delta} \Delta^p_j = \Omega. \tag{17}$$

Otherwise some areas should exist inside the area Ω, which are not identical with any PCNJ area and the values of the parameters inside of them should be indefinite. However that is impossible, because the Eq. (1) describes the field inside the whole area Ω.

The PCNJ areas of different parameters may cross each other. At least one pair of the PCNJ areas of different parameters exists always which crosses each other or even coincides with each other, i.e.,

$$\Delta^{p'}_{j'} \cap \Delta^{p''}_{j''} \neq \varnothing. \tag{18}$$

The constant parameters A^c, B^c and the parameters A_x, A_t, B_x, B_t, changing by not the jumps, has only one PCNJ area ($N^p_\Delta = 1$) and this area coincides with the FC area Ω (Fig. 3 a,b), i.e.,

$$\Delta^p_j = \Delta = \Omega \qquad (j = \overline{1, N^p_\Delta},\ N^P_\Delta = 1). \tag{19}$$

The PCNJ areas of the parameters, changing by jumps, are smaller than the FC area Ω always. At least two PCNJ areas may be disposed inside this area ($N^p_\Delta \geqslant 2$) (Fig. 3a,b).

The QH areas are the areas of the field, the variance σ_ξ^2 of which is constant or changing by not the jumps with regard to coordinates x or t inside of them. Some properties of the QH areas are the same as of the PCNJ areas, i.e., the QH areas have not cross each other

$$\Omega_{j'} \cap \Omega_{j''} = \varnothing \qquad (j', j'' = \overline{1, N}). \tag{20}$$

The QH areas must fulfill the FC area also, i.e.,

$$\bigcup_{j=1}^{N} \Omega_j = \Omega. \tag{21}$$

As the FC as the QH or PCNJ areas may be of various configuration – rectangle or nonrectangle (Fig. 1). The size of them may be various also. The rectangle areas are considered below only (we note, that the FC area may be and nonrectangle also even in the case of rectangle QH or PCNJ areas).

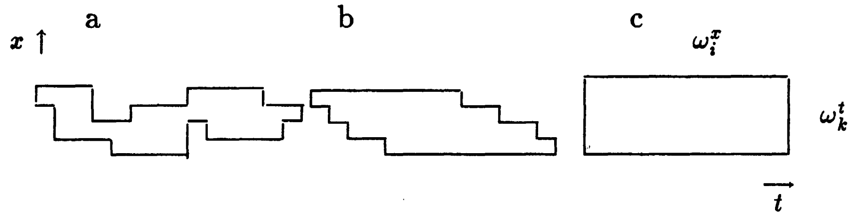

Fig. 1. The examples of the FC, PCNJ and QH areas configurations: a, b – nonrectangle, c – rectangle areas.

The location (structure) of the QH and the PCNJ areas inside the FC area is various also. The location of rectangle QH and PCNJ areas may be regular or irregular (Fig. 2). A net

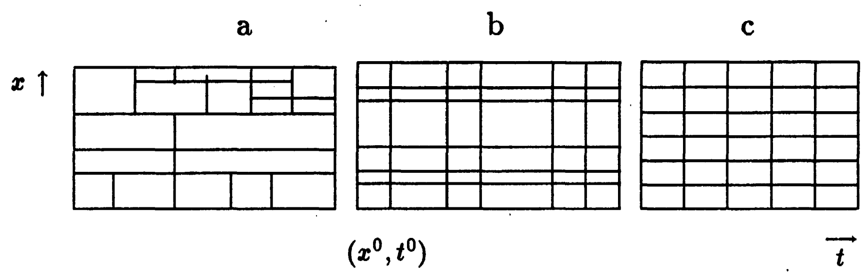

Fig. 2. The location of the PCNJ and QH areas inside the area FC. a – irregular, b – regular net type with variable step, c – regular net type with constant step.

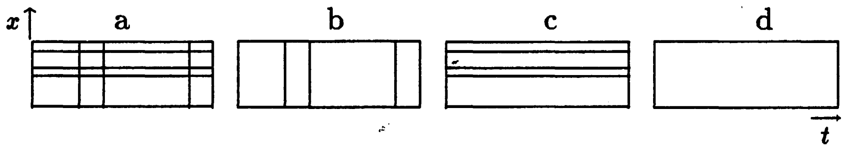

Fig. 3. The PCNJ and QH areas of the QH AR fields. The PCNJ areas of the parameters: a – A^{xt}, B^{xt}; b – A^t, A^t_x, B^t, B^t_x; c – A^x, A^x_t, B^x, B^x_t; d – A^c, A_x, A_t, B^c, B_x, B_t. The QH areas in the case, when among the parameters there are the parameters, changing by jumps: a – in the space and the time, b – in the time only, c – in the space only.

type location is example of regular structure (Fig. 2b, c). The step of the net may be constant (Fig. 2c) or variable (Fig. 2b).

It is convenient to mark the QH and PCNJ areas in the case of net type location by the help of a pair of indexes, for example, ω_{ik}, where i, k ($i = \overline{1, N_x}$, $k = \overline{1, N_t}$) are the numeration of the QH areas along the directions of the x and t axes. Then it is enough to have the coordinates of one of the tops of the FC area (x^0, t^0) and the dimensions $\omega_i^x \times \omega_k^t$ of the QH areas Ω_{ik} (Fig. 2b) for description of such QH areas. The coordinates of any point of the QH area Ω_{ik} are inside following intervals

$$\begin{aligned} &x^0 + (i-1)\,\omega_i^x \leqslant x < x^0 + i\,\omega_i^x - 1 \quad (i < N_x),\\ &x^0 + (i-1)\,\omega_i^x \leqslant x \leqslant x^0 + i\,\omega_i^x - 1 \quad (i = N_x),\\ &t^0 + (k-1)\,\omega_k^t \leqslant t < t^0 + k\,\omega_k^t - 1 \quad (i < N_t),\\ &t^0 + (k-1)\,\omega_k^t \leqslant t \leqslant t^0 + k\,\omega_k^t - 1 \quad (i = N_t). \end{aligned} \tag{22}$$

The similar intervals exists and for description of the PCNJ areas also, if the adequate dimensions of such areas are used instead of the dimensions $\omega_i^x \times \omega_k^t$.

The location of the PCNJ areas of the constant and variable parameters inside the FC area is shown in the Fig. 3. The PCNJ areas possesses the net type location in the case of the parameters, changing by jumps as in the time as in the space only, i.e., in the case of the parameters A^{xt}, B^{xt} (Fig. 3a). The band location possesses the PCNJ areas of the parameters, changing by jumps only in the space or the time only (the parameters A^t, A_x^t, B^t, B_x^t) (Fig. 3b, c). The parameters A_x, A_t, B_x, B_t, changing by not the jumps, and constant parameters A^c, B^c has a single area, coincidental with the FC area (Fig. 3d).

6. Connection between the PCNJ and the QH areas. There exist any connection between the PCNJ and the QH areas. Really, the QH areas are a part of the PCNJ areas, common for all parameters. Therefore, it is possible to determine the QH areas and its location form, if it is known the FC area, divided into the PCNJ areas. Let the rectangle FC area be divided into the PCNJ areas by the help of a rectangle net and let the dimensions $a_{ip}^x \times a_{kp}^t$ $(i = \overline{1, N_{\Delta x}^p},\ k = \overline{1, N_{\Delta t}^p},\ p = \overline{1, n_c})$ be known. We shall determine the QH areas Ω_{ik}, i.e., we shall determine the total number $N_x \times N_t$ and dimensions

$\omega_i^x \times \omega_k^t$ $(i = \overline{1, N_x},\ k = \overline{1, N_t})$. At first, we shall analyse some characteristic cases.

1. The case of single parameter, changing by jumps. Let be one parameter only, changing by jumps. Let the dimensions of its PCNJ areas Δ_{ik} be $a_i^x \times a_k^t$ $(i = \overline{1, N_{\Delta x}},\ k = \overline{1, N_{\Delta t}})$. It is obvious, that the QH areas coincides with the PCNJ areas of the parameter, changing by jumps, i.e.,

$$\begin{aligned} &\Omega_{ik} = \Delta_{ik} \qquad (i = \overline{1, N_x},\ k = \overline{1, N_t}), \\ &\omega_i^x = a_i^x, \ \ \omega_k^t = a_k^t \quad (i = \overline{1, N_x},\ k = \overline{1, N_t}), \\ &N_x = N_{\Delta x}, \quad N_t = N_{\Delta t}. \end{aligned} \tag{23}$$

This case is met in the QHQS model A^cB^{xt}, QHS model A^cB^x, HQS model A^cB^t, QHNS models $A^cB_t^x$, $A_tB_t^x$, $A^cA_tB_t^x$, A_tB^x, $A^cA_tB^x$ and in the NHQS models $A^cB_x^t$, $A_xB_x^t$, $A^cA_xB_x^t$, A_xB^t, $A^cA_xB^t$ always, because the parameter b is the single parameter in these models, changing by jumps. Also this case is met in the QHQS models $A^{xt}B^c$, $A^cA^{xt}B^c$, QHS model $A^cA^xB^c$, HQS model $A^cA^tA^c$, QHNS models A^xB_t, $A_t^xB_t$, $A^cA^xB_t$, $A^cA_t^xB_t$, $A^xA_tB_t$, $A_tA_t^xB_t$, $A^cA^xA_tB_t$, $A^cA_tA_t^xB_t$, $A_t^xB^c$, $A^cA_t^xB^c$, $A^xA_tB^c$, $A_tA_t^xB^c$, $A^cA^xA_tB^c$, $A^cA_tA_t^xB^c$ and in the NHQS models A^tB_x, $A_x^tB_x$, $A^cA^tB_x$, $A^cA_x^tB_x$, $A_xA^tB_x$, $A_xA_x^tB_x$, $A^cA_xA^tB_x$, $A_x^tB^c$, $A^cA_x^tB^c$, $A_xA^tB^c$, $A_xA_x^tB^c$, $A^cA_xA^tB^c$, if the groups A^{xt}, A^x, A^t, A_t^x, A_x^t are formed of single parameter, which changes by jumps. The location of the QH areas of the QHQS models has the net shape (Fig. 3a), of the QHS and QHNS models – the shape of the bands, parallel to t axes (Fig. 3c), and of the HQS and NHQS models – the shape of the bands, parallel to x axes (Fig. 3b).

2. The case of the two parameters, changing by jumps. Let be two parameters only, changing by jumps: one of them changes only in the space, other – in the time only. The location of the PCNJ areas of such parameters has the shape of

the bands, parallel to t or to x axes (Fig. 3b,c). Let the dimensions of the PCNJ areas of these parameters be $a_i^x \times W^x$ and $a_k^t \times W^t$ ($i = \overline{1, N_{\Delta x}}$, $k = \overline{1, N_{\Delta t}}$, where $W^x \times W^t$ are the dimensions of the FC area. Then it is easy to see, that the dimensions of the QH areas in this case are following

$$\begin{aligned} &\omega_i^x = a_i^x, \quad \omega_k^t = a_k^t \qquad (i = \overline{1, N_x}, \ k = \overline{1, N_t}), \\ &N_x = N_{\Delta x}, \quad N_t = N_{\Delta t}. \end{aligned} \tag{24}$$

The location of the QH areas always is of the net type (Fig. 3a). An example of this case is the model $A^x B^t$, if A^x means the single parameter only, changing in the space.

3. The case, when the parameters changes by jumps and has the same PCNJ areas. Let the dimensions of the parameters be the same, i.e., $a_i^{px} = a_i^x$, $a_k^{pt} = a_k^t$ and $N_{\Delta x}^p = N_{\Delta x}$, $N_{\Delta t}^p = N_{\Delta t}$ for all p. Then the dimensions of the QH areas are determined by the Eq. (24) and its location has the same shape as the PCNJ areas: the net shape, if the PCNJ areas are located as net (Fig. 3a), the form of the bands, parallel to the x axes or to t axes, if the PCNJ areas are located in the same manner (Fig. 3b, c). This case is met in all models.

Now we shall consider, how to determine the QH areas in the other cases. Let the dimensions $a_i^{px} \times a_k^{pt}$ ($i = \overline{1, N_{\Delta x}^p}$, $k = \overline{1, N_{\Delta t}^p}$, $p = \overline{1, n_c}$) of the PCNJ areas be known. Then we can calculate the sets $\{x_{pi}^{\Delta}\}$, $\{t_{pk}^{\Delta}\}$ of the x and t coordinates of the PCNJ areas Δ_{ik}^p by the following equations

$$x_{pi}^{\Delta} = x_{p,i-1}^{\Delta} + a_i^{px} \qquad (x_{p0}^{\Delta} = x^0), \tag{25}$$

$$t_{pk}^{\Delta} = t_{p,k-1}^{\Delta} + a_k^{pt} \qquad (t_{p0}^{\Delta} = t^0), \tag{26}$$

where x^0, t^0 are the coordinates of the FC area. The total number of the PCNJ areas is $\left(\sum\limits_{p=1}^{n_c} N_{\Delta x}^p\right) \times \left(\sum\limits_{p=1}^{n_c} N_{\Delta t}^p\right)$. As the QH areas $\Omega_{lm}(L = \overline{1, N^x},\ m = \overline{1, N^t})$ are the parts of the PCNJ areas, common for all parameters, it is easy to see, that x coordinate x_1^Ω of the areas ω_{1m} is the smallest value of the set $\{x_{pi}^\Delta\}$ and t coordinate t_1^Ω of the areas Ω_{l1} is the smallest value of the set $\{t_{pk}^\Delta\}$, i.e.,

$$x_1^\Omega = \min_{p,i} x_{pi}^\Delta, \qquad t_1^\Omega = \min_{p,k} x_{pk}^\Delta. \tag{27}$$

The x coordinates $x_l^\Omega$for $l = 2, 3, \ldots$ are determined as the next by size values x_{pi}^Δ and t coordinates t_m^Ω for $m = 2, 3, \ldots$ – as the next by size values t_{pk}^Δ. The calculation of the x_l^Ω is continued while the current smallest value x_{pi}^Δ is less or equal to W^x and the calculation of the t_m^Ω – while the current smallest value t_{pk}^Δ is less or equal to W^t. Then the total number N^x of the QH areas is equal to number of calculated x_l^Ω and the number N^t – to the number of calculated t_m^Ω. The dimensions ω_l^x, ω_m^t are calculated by such equations

$$\begin{aligned} \omega_l^x &= x_l^\Omega - x_{l-1}^\Omega \qquad (x_0^\Omega = x^0), \\ \omega_m^t &= t_m^\Omega - t_{m-1}^\Omega \qquad (t_0^\Omega = t^0). \end{aligned} \tag{28}$$

Therefore the dimensions ω_l^x, ω_m^t and the numbers N^x, N^t of the QH areas may be calculated by the following algorithm.

Step 1. The one-dimensional array X of length $\sum\limits_{p=1}^{n_c} N_{\Delta x}^p$ is cleaned.

Step 2. The one-dimensional array T of length $\sum\limits_{p=1}^{n_c} N_{\Delta t}^p$ is cleaned.

Step 3. The values x_{pi}^Δ $(p = \overline{1, n_c},\ i = 1, N_{\Delta x}^p)$ are calculated by the Eq. (25) and located into the array X.

Step 4. The values t_{pk}^Δ $(p = \overline{1, n_c},\ i = 1, N_{\Delta k}^p)$ are calculated by the Eq. (26) and located into the array T.

Step 5. The values of the array X and T are lined by the size.

Step 6. The numbers N^x, N^t are calculated. They are equal to the total numbers of the nonzero values ω_l^x, ω_m^t.

Step 7. The dimensions ω_l^x, ω_m^t of the QH areas are calculated by the Eq. (28).

7. Conclusions. The quasihomogeneous (QH) AR field is described by the same difference equation as the homogeneous AR filed, if at least one of the equation coefficients changes by jumps in the space or time. Such fields can be divided into five classes: quasihomogeneous–quasistationary (QHQS), quasihomogeneous–stationary (QHS), homogeneous–quasistationary (HQS), nonhomogeneous–quasistationary (NHQS) AR fields. Some sets of the models may be built in every classes of such fields (Tables 2–5). The field consideration (FC) area may be divided into quasihomogeneous (QH) areas, inside of which the dispersion of the field is constant or changes by not the jumps. The QH areas have not cross each other. The FC area may be divided into parameters changing by not the jumps (PCNJ) areas also, inside of which the parameters are constant or changes by not the jumps. The properties of the PCNJ areas are similar to properties of the QH areas. A connection exists between the QH and the PCNJ areas. The QH areas may be built by help of proposed algorithm, if are known PCNJ areas.

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Received December 1993

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