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DATA COMPRESSION METHODS. APPLICATION TO ECG

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Abstract. A review of electrocardiographic (ECG) data compression methods is presented. It shows what data compression techniques are available and what the implementation considerations are for each technique.

Key words: electrocardiographic (ECG) data, data compression.

1. Introduction. There are many papers and books concerning data compression methods and their applications. In this paper we restrict our attention to the methods suitable for signal compression, i.e., the case when data are signal samples. We present a review of compression methods for one of such a data type-electrocardiograms (ECG). The review can be useful for applications to another real signal, too. It describes the situation of data compression in general because most of the available data compression methods are tested on ECG.

The problem of digitized electrocardiogram compression arose together with the computer analysis of ECG. A necessity to store and transmit a huge amount of data stimulated investigators to apply available methods of data compression to compress ECG data. In each application there is a need to know what compression techniques are available and what the implementation considerations are for each technique. We present here a review of ECG data compression methods which is an obvious example of data compression applications for signal data. It illustrates the present situation in this area and may be useful not only for the application of data compression methods to ECG but also to other signal as well. One of the first publications on this subject is the paper by Cox et al (1968) on the AZTEC algorithm, which uses the zero order interpolator. At present, as far as we know, the number of publications on ECG data reduction exceeds a hundred. The first publications were focussed on the methods for compression of resting ECGs. Later on, in 80-ties new investigations were accomplished on the reduction of Hotter ECGs. Regardless of the type of ECG (resting, exercise or Holter) the motivation for data compression is the same – the necessity to store and transmit via telephone lines data of high density.

The main principles applied to ECG data compression are the following:

1. Bandwidth limitation and reduction of the sampling rate.
2. Redundancy reduction, i.e. reproducible data compression.
3. Information reduction, i.e. irreversible data compression.

The main aspects for evaluation of data compression methods are: compression ratio (CR), various errors, computing expenditure, stability to transmission errors, portability to other computers. The following errors are mainly used to describe the differences between the original ECG $X(n)$ and reconstructed ECG $Y(n)$ obtained from the compressed data:

1. Absolute error:

$$R(n) = |X(n)| - |Y(n)|.$$

2. Maximal absolute error:

$$\text{Max } R(n) = \max_n \left[|X(n)| - |Y(n)|\right].$$

3. Relative error:

$$RE(n) = \frac{X(n) - Y(n)}{\text{Max }[X(n)]}.$$

4. Root mean square error:

$$RMS = \sqrt{\frac{\sum_{n=1}^{m} [X(n) - Y(n)]^2}{m}}.$$

5. Percentile root mean square difference error:

$$PRD = \frac{\sum_{n=1}^{m} [X(n) - Y(n)]^2}{\sum_{n=1}^{m} [X(n)]^2} \cdot 100.$$

It is not sufficient to use only one of these errors to evaluate the performance of a compression procedure. Usually we must estimate several of them.

The compression ratio (CR) is usually defined as N/M, where N is the number of input samples, and M is the number of reconstructed ECG samples.

In Fig. 1.1 we have classified the available ECG data compression techniques which are reviewed further by such a scheme.

2. Bandwidth limitation and reduction of sampling rate. The problem of what frequency limits are to be taken has been the subject of discussion for many years (Kerwin, 1953; Langner, 1960; Reynolds, 1967; Flowers, 1971; Golden, 1973; Anderson, 1975; Cappelini, 1976; Berson, 1977 a); Berson, 1977 b); Sapoznikov, 1977; Schick, 1978; Riggs, 1979; Goldberger, 1980; Bhargava, 1981; Brag-Remchel, 1982; Nichols, 1985). However, this problem has been investigated in different aspects. The main question is what the upper frequency limit must be in order that ECG not be distorted. For example, Kerwin (1953) and Longner (1960) found the upper limit frequency 1300 Hz and 1000 Hz. In other investigations Flowers (1971), Goldberger (1980) studied the influence of the upper frequency on the reproducibility of slurs and notches in ECG and upper limits of approximately 500 Hz were set by them.

The sampling rate for adequate digitizing ECGs was frequently discussed as well (Barr, 1977; Pahlm, 1979). Sampling rates of 250 S/s (Samples/second) and 500 S/s are mainly used in digitization. Theoretically these sampling rates should be sufficient, however, in ECG processing the signal usually is not reconstructed by the theoretically required function $\sin\left(\frac{t}{T}\right)$ between sample points, but linear interpolation is used. For this reason short ECG waves are

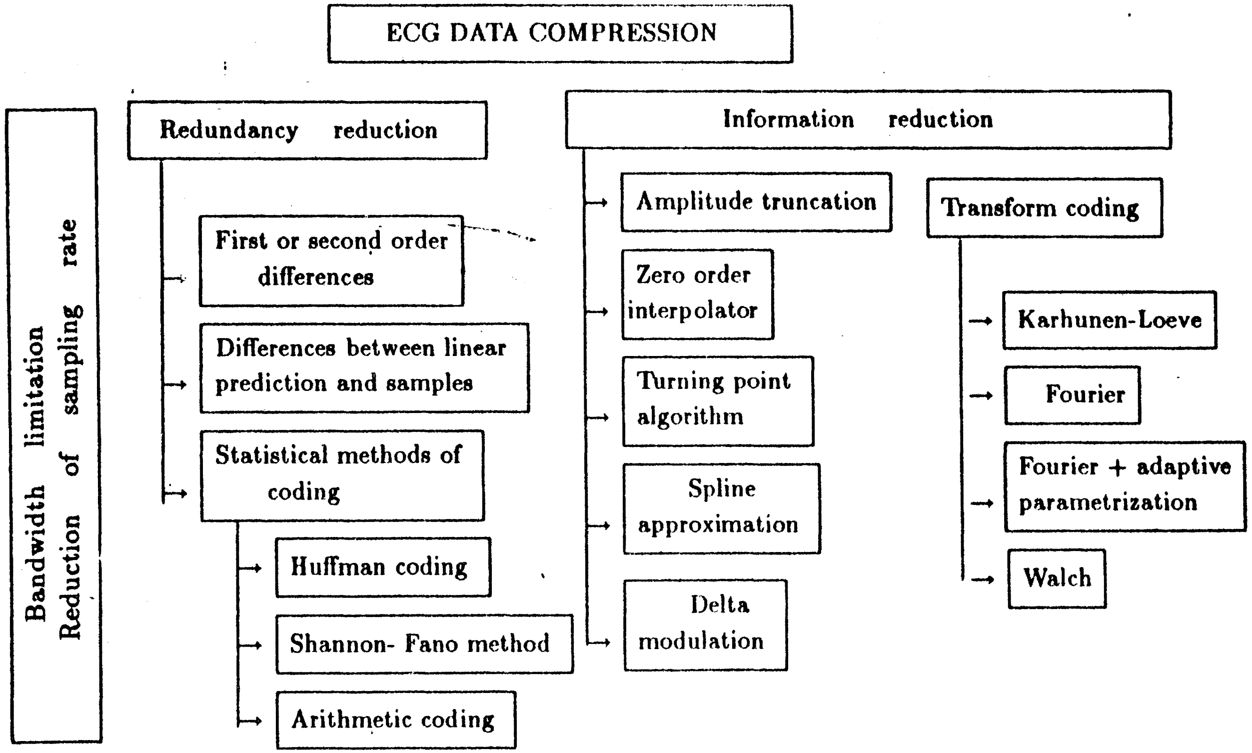

Fig. 1.1. The main methods used for ECG compression.

omitted or estimated with great errors (Zywietz, 1983). Reasonable results (amplitude errors < 10 %) are obtained if the sampling interval is smaller than approximately 1/8 of the duration of the wave. The essential conclusion from this observation is that the sampling rate and measurability of minimum ECG waves are strongly interdependent. International Electrotechnical Comission and American Heart Association (Pipberger, 1975) recommend bandwidth of 0.05 Hz - 250 Hz and the sampling rate of 500 S/s for processing the resting ECGs. Reduction of one of them would result in information reduction.

3. Redundancy reduction. From the point of view of data encoding there are two reasons of ECG data redundancy - correlation between samples (neighbouring samples are not statistically independent of each other) and dissimilar frequency of quantized signal amplitudes.

3.1. Computation of differences. A widespread technique to reduce redundancy due to intersample correlation is to compute first or second order differences between the samples. Let $X(n) = x_1, \ldots, x_N$ be a sequence of original samples. The first order differences

$$\Delta_1 x_i = x_{i+1} - x_i \tag{3.1.1}$$

remove constant baseline amplitudes. The second order differences are

$$\Delta_2 x_i = \Delta_1 x_{i+1} - \Delta_1 x_i = x_{i+2} - 2x_{i+1} + x_i. \tag{3.1.2}$$

As can be seen from experimental results there is no great difference in code word lengths of first and second order differences [Zywictz, 1989]. When using differences of ECG samples we suggest a modification. A converter A/C gives ECG samples as a sequence of integers. One needs 2 bytes of memory for storage of an integer variable. Meanwhile in the experiments we noticed that after taking differences, the values $\Delta_1 x_i$ are usually not large (very often they are < 256), most of them fit in one byte. Thus, we suggest separating numbers stored in first and second bytes. So instead of

$\{\Delta_1 x_i\}$ we get two sequences $\{y_1(i)\}$ and $\{y_2(i)\}$:

$$\begin{cases} y_1(i) = \Delta_1 x_i \, {}^0\backslash_0 \, 256, \\ y_2(i) = [\Delta_1 x_i / 256], \end{cases} \tag{3.1.3}$$

where [ ] is a mark of entire function and ${}^0\backslash_0$ means the remainder from a division by 256. Thus,

$$y_1(i) = \begin{cases} \Delta_1 x_i, & \Delta_1 x_i \leqslant 256, \\ r_i, & \Delta_1 x_i > 256, \end{cases} \tag{3.1.4}$$

(r_i is the remainder from a division by 256). The variables $y_1(i)$ and $y_2(i)$ must be defined as characters (type "char") so their storage needs only one byte. The sequence $\{y_2(i)\}$ consist of small values, for ECGs they are mostly zeros, and when one applies, for example, statistical methods of data compression (considered further) separately to the sequence $\{y_1(i)\}$ and $\{y_2(i)\}$, a good compression ratio can be obtained. When restoring a signal one has to calculate

$$\Delta_1 x_i = y_2(i) \cdot 256 + y_1(i). \tag{3.1.5}$$

3.2. Prediction. Reduction in the variance of the distribution of successive differences can be obtained if instead of the original successive difference only the difference between the Δ_2-value and its prediction is stored. Prediction values $\Delta_2 \hat{x}_n$ can be obtained, e.g., from a linear combination of previous samples by the equation

$$\Delta_2 \hat{x}_n = \sum_{k=1}^{p} a_k \Delta_2 x_{n-k}. \tag{3.2.1}$$

The weight factors a_k are chosen so that the expected mean square error between the original values and the estimated samples becomes minimal (p is the number of samples employed for the prediction). An algorithm for calculating these factors can be found in the book of Box (1970), p. 72.

3.3. Statistical coding. The redundancy due to dissimilar frequency distribution of amplitudes can be removed by the methods of statistical coding. They operate by encoding symbols one at a time. The symbols are encoded into output codes, the length of which varies dependent on the probability of the symbol. Low probability symbols are encoded using many bits, high probability symbols are encoded using fewer bits. All statistical methods of data compression use the estimates of symbol probabilities. We shall mention three most important statistical methods-Huffman, Shannon-Fano and arithmetic coding.

Huffman method (Huffman, 1952; Apiki, 1991) is probably the best-known technique of data compression. It works on the premise that some symbols are used more often than others in data representation. Huffman coding formalizes the idea of relating the symbols length to the probability of a symbol's occurrence. It requires to have a table of probabilities (estimates) before you begin compressing of data. The compressor and decompressor can construct an encoding tree with this probability information. This tree is a binary tree with one leaf for each symbol. To construct the tree the compressor starts with the two symbols of lowest probability. It then combines these two as two leaf branches under a node; this node, in turn, is assigned the sum of the two probabilities. The compressor then consideres this node along with the rest of the symbols in the probability list and it again selects the two least probable items. It continues to build and combine node until it builds up a single tree, with the probability at the root equal to 1. The resulting tree has the leaves of varying distance from the root. The leaves that represent the symbols with the highest probability are closest to the root, while those with the lowest probability are most away. To encode a symbol, the compressor finds the path from the root of tree to the symbol's leaf.

EXAMPLE 1. Let us have the probability Table 3.3.1.

Table 3.3.1. Probabilities of symbols

Symbol	A	B	C	D	E	F	G	H
Probability	0.22	0.2	0.16	0.16	0.1	0.1	0.04	0.02

Computations to construct a tree are carried out in such a way:

0.22	0.22	0.22	0.26	0.32	0.42	0.58	1
0.2	0.2	0.2	0.22	0.26	0.32	0.42	
0.16	1.16	1.16	0.2	0.22	0.26		
0.16	0.16	0.16	0.16	0.2			
0.10	0.10	0.16	0.16				
0.10	0.10	0.10					
0.04	0.06						
0.02							

The corresponding binary tree is presented in Fig. 3.3.1.

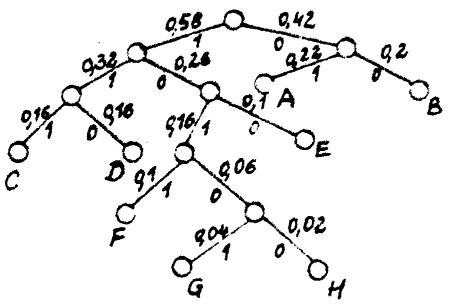

Fig. 3.3.1 Huffman binary tree.

Thus, the corresponding codes of symbols are as described in Table 3.3.2.

Table 3.3.2. Huffman codes

symbol	A	B	C	D	E	F	G	H
code	0.1	00	111	110	100	1011	10101	10100

When encoding the text consisting of these symbols we shall maximally save bits if use this table.

Shannon-Fano method (Dmitryjev, 1989, p. 186) is close to Huffman method. The probabilities of symbols and code length are related but the procedure of prescribing codes is different. At first all the probabilities are lined up from the greatest to the lowest. Afterwards the probabilities are devided into two groups. Thus the sum of probabilities of both groups will be equal (or approximately equal). For all symbols of the first group 1 is prescribed, for all symbols of the second group–0. Then, each group is devided into two groups again. For each symbol of the first group 1 is prescribed again, for each symbol of the second group 0 is prescribed. We go on deviding until only one symbol remains in each group.

EXAMPLE 2. Let us consider the same example but construct the codes according to Shannon-Fano method:

A	0.22	1	1			
B	0.20	1	0			
C	0.16	0	1	1		
D	0.16	0	1	0		
E	0.10	0	0	1		
F	0,.10	0	0	0	1	
G	0.04	0	0	0	0	1
H	0.02	0	0	0	0	0

The resulting codes are presented in Table 3.3.3

Table 3.3.3. Shannon-Fano codes

symbol	A	B	C	D	E	F	G	H
code	11	10	011	010	001	0001	00001	00000

One may get different Shannon-Fano codes when using the same probability table because of freely chosen groups.

Arithmetic coding (Witten, 1987; Nelson, 1991) bypasses the idea of replacing an input symbol by a specific code. Instead, it takes a stream of input symbols and replaces it by a single floating point output number. The longer (and more complex) the message the more bits are needed in the output number. The output from an arithmetic coding process is a single number less than 1 and greater or equal to 0. To construct the output number the symbols being encoded have to have set probabilities assigned to them.

EXAMPLE 3. Let us have to encode the message BILL GATES (the example from Nelson, 1991). Then the estimates of probabilities are like Table 3.3.4.

Table 3.3.4. Estimates of probabilities

Symbol	Estimate of probability	Range
space	0.1	0 - 0.1
A	0.1	0.1 - 0.2
B	0.1	0.2 - 0.3
E	0.1	0.3 - 0.4
G	0.1	0.4 - 0.5
I	0.1	0.5 - 0.6
L	0.2	0.6 - 0.8
S	0.1	0.8 - 0.9
T	0.1	0.9 - 1

Once the character probabilities are known the individual symbols need to be assigned a range along a "probability line". It doesn't matter which characters are assigned which segment of the range as long as it is done in the same manner by both the encoder and the decoder. The nine-character symbol set used here would look like in Table 3.3.4.

The most significant portion of the arithmetic coded message belongs to the first symbol to be encoded. When encoding the message BILL GATES, the first symbol is B. For the first character to be decoded properly, the final coded message has to be a number greater than or equal to 0.2 and less than 0.3. To encode this number, we keep the track of the range within which this number could fall. So after the first character is encoded, the low end of this range is 0.2 and the high end is 0.3. During the rest of the encoding process, each new symbol to be encoded will further restrict the possible range of the output number. The next character to be encoded, I, owns the range 0.5 through 0.6. We say that I owns the range corresponding to 0.5 – 0.6 in a new subrange of 0.2 – 0.3. This means that the new encoded number will have to fall somewhere in the 50 to 60th percentile of the currently established range. Applying this logic will further restrict our number to range between 0.25 and 0.26; the next interval will be 0.256 – 0.258, etc. until we encode the last symbol:

B	0.2	0.3
I	0.25	0.26
L	0.256	0.258
L	0.2572	0.2576
	0.25720	0.25724
G	0.257216	0.257220
A	0.2572184	0.2572168
T	0.25721676	0.2572160
E	0.257216772	0.257216776
S	0.2572167752	

The final low value 0.2572167752 will be the code of our message.

Decoding is the inverse procedure. The range is expanded in proportion to the probability of each symbol as it is extracted.

We briefly reviewed the main ideas of the statistical methods for data compression. In practice these methods are often used together in order to get a greater compression ratio.

4. Irreversible data compression. The methods which partly reduce information give greater compression. Such methods are called irreversible because after the compression ECG data cannot be exactly reproduced. Among the irreversible data compression methods one can exclude a group of methods which use mathematical transformation–Fourier, Karhunen–Loeve, etc.

4.1. Amplitude truncation. The amplitude truncation – an increase of quantization by an amplitude step – considerably increases the compression ratio and decreases the code word length (the number of bits per sample). Basically the amplitude precision is determined by the quantization level of the A/C converter. The American Heart Association and Association for Advancement of Medical Instrumentation (American National Standard..., 1982) allow the amplitude quantization in $10\mu V$ steps. In (The CSE Working Party ..., 1985) there are recommendations to use a wave with the minimum amplitude of $20\mu V$. The problem of errors here is of great importance. From the medical point of view errors of 2 – 5% could be acceptable (Furth, 1988) and this accuracy can be maintained by using 6 bits for amplitude quantization (Pipberger, 1975; American..., 1982; Zywietz, 1989). We present Fig. 4.4.1 from the paper (Zywietz, 1989) where one can see an interesting dependence of truncation on the compression ratio, word length and errors. The data were the averages from ten digitally recorded ECGs and the second succesive differences were encoded in 3,6 or 15 bits, respectively. The truncation being 3 with maximum amplitude error of $4 \mu V$ the average word length was 2 bits/S. In the paper of Andrew et al. (1967) an adaptive data compression algorithm has been described where for each sample only the most significant 6 bits (out of 12 A/C converter word) are used. The position of 6 bits is coded by additional 2 bits. The movable amplitude window provides a constant relative accuracy of digitized amplitudes.

4.2. Zero order interpolator. One of the first algorithms on ECG compression AZTEC (Cox, 1968) is based on the zero order interpolator. An ECG signal is approximated by a stepped function, i.e., an amplitude is regarded constant as long as the signal remains within a lower and an upper threshold and only the number of samples and the mean value of the amplitude window is stored. The compression ratio is reported to be approximately 10. This algorithm has been extended (Furth, 1988) in order to obtain a better approximation of the ECG signal within the QRS-complex.

4.3. Spline approximation. The main principle is to approximate an ECG curve by straight line segments (Shakin, 1981; Ishijama, 1983). An acceptable amplitude error can be preset and the error does not exceed this preset threshold. In 1988 Bertinelli suggested a sample skipping algorithm which is a modification of the spline approximation–not only amplitude but also time error limitation is done as well. The area subtended by original and reconstructed signal is maintainable within preset limits.

4.4. Turning point algorithms. The algorithms are based on the analysis of ECG curvature. Specific turning points are selected and amplitude and time windows are used to approximate

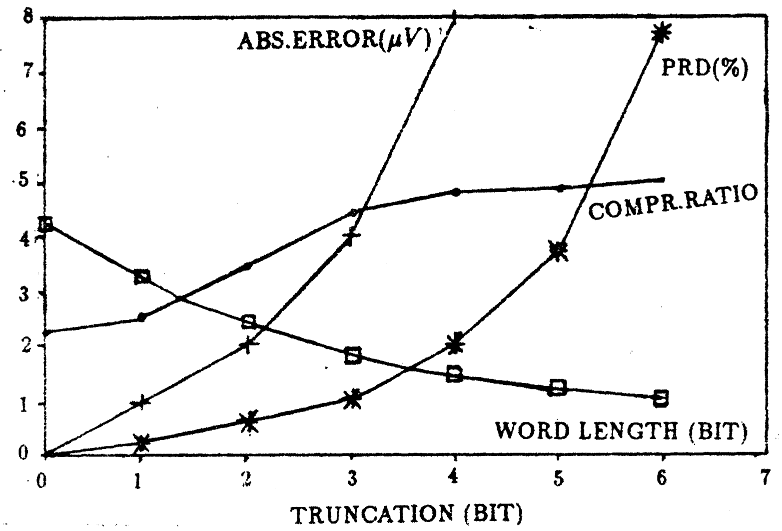

Fig. 4.1.1. Dependence of truncation on the compression ratio, word length and errors.

the signal between the selected points (Muller, 1978; Lamberti, 1988). CR is reported to be from 2 to 10. This basic concept of referring to turning points within the signal has been used and extended by several authors for development of other schemes. A combination of the AZTEC and the Turning point algorithm (Abenstein, 1982) gave a $CR = 8$. Moody (1987) combined the turning point algorithm with the piecewise linear approximation of the ECG signal ($CR = 10$).

4.5. Delta modulation. The basic concept is prediction of the next sample by adding or subtracting a step value Δ and encoding the sum or the difference into one bit:

$$y_n = y_{n-1} \pm \Delta.$$

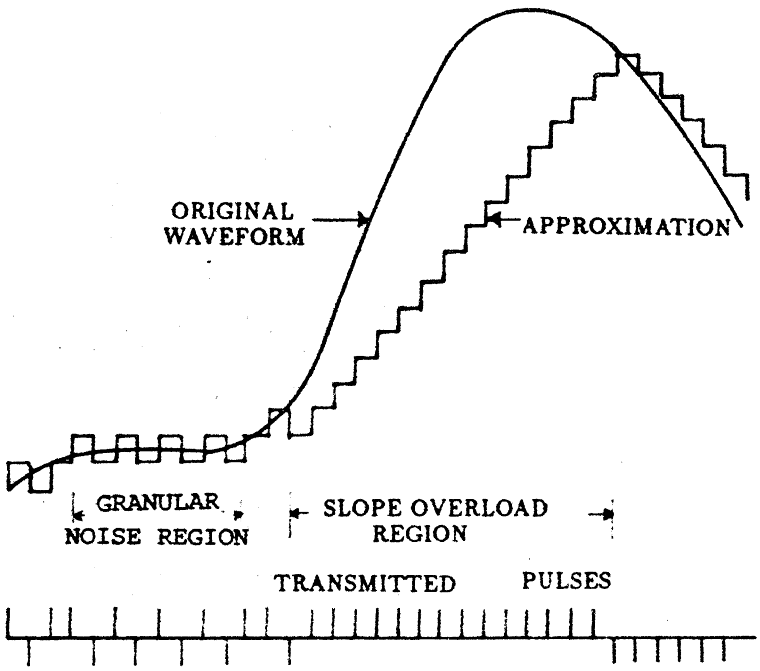

Fig. 4.5.1. The performance of Δ modulation.

Each sample as an integer needs 16 bits thus, $CR = 16$ is possible. However, there are some problems with errors. Fig. 4.5.1 depicts the performance and problems of this method (Zywietz, 1989). Granular noise and slope overload are the limiting factors of application. By increasing the step size Δ one may reduce or avoid slope overload, but that increases the granular noise, a reduction of Δ reduces the granular noise but increases a distortion by slope overload. Several adaptive schemes have been proposed. In the paper of Jayant (1984) step size is increased by 50% each time the Δ value has the same polarity as in the previous sample (potential slope overload region) and v.v. Coppellini (1976) has used a varying step Δ, i.e., small Δ_1 for signal parts in which high precision is required and a larger Δ_2 for other signal parts. At the RMS error = 1.8% CR was reported to be 6.3.

4.6. Transform coding. Transform coding is applied not to the whole ECG records but only to representative cycles of ECG. It is a procedure where a block of N input samples is linearly transformed into a set of transform coefficients are then encoded for transmission and a reconstructio.ı of the original signal $x(n)$ is obtained using an inverse transform operation on these coefficients.

In matrix notation, let $\vec{x}$ be a vector representing the ECG input sequence:

$$\vec{x}^T = \{x(i),\ i = 0, 1, \ldots, N-1\}. \tag{4.6.1}$$

The transform is given by

$$\vec{w} = A\vec{x}, \tag{4.6.2}$$

where $A(N \times N)$ is the transform matrix. The coefficients are encoded and stored or transmitted. A reconstructed ECG is obtained by the inverse transformation

$$\vec{x} = B\vec{w}, \tag{4.6.3}$$

where $F = A^{-1}$. The columns of the matrix B b_k $(k = 0, \ldots, N-1)$ are called the basis vectors and they determine the type of transformation. This procedure allows data compression because the number of transform coefficients to be transmitted can be chosen $M < N$ on the basis of MSE error allowed in the reconstruction. Apart from that, the coefficients can be quantized with fewer bits than that used to represent the samples of the original signal. Thus, the scheme in Fig. 4.6.1 describes the transform coding.

Three main transformations have been used in ECG processing: principal components (Karhunen-Loeve), discrete Fourier (sometimes other version of this transformation – discrete cosine transformation – is used), and Walch transformation. The parametric Fourier transformation of Poliakov et al (1986) should be mentioned where additional parameters are used to get a better approximation.

Discrete *Fourier* transformation is known best. The basis vectors are

$$b_k = \left(1,\ e^{-j\frac{2\Pi k}{N}},\ e^{-j\frac{4\Pi k}{N}},\ \ldots,\ e^{-j\frac{2(N-1)\Pi k}{N}}\right)',$$

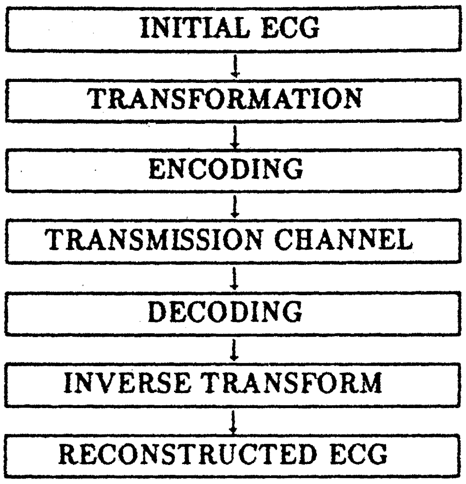

Fig. 4.6.1 Data compression by transform coding

$k = 0, 1, \ldots, N-1$. The transformation coefficients $w(k)$, $k = 0, 1, \ldots, N-1$ are calculated by the formula

$$w(k) = \frac{1}{N} \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi km}{N}}, \quad k = 0, 1, \ldots, N-1. \tag{4.6.4}$$

The corresponding inverse transformation is

$$x(m) = \sum_{k=0}^{N-1} w(k) e^{j\frac{2\pi km}{N}}, \quad m = 0, 1, \ldots, N-1. \tag{4.6.5}$$

Results of Fourier transformation application are presented in the paper of Reddy (1986). When PRD errors for complex QRS and for segment S-Q are estimated separately, $PRD_{QRS} < 5\%$, $PRD_{S-Q} < 10\%$, CR is reported to be ~ 7.4.

Karhunen–Loeve transformation minimizes the mean error of representation (4.6.3), therefore, it is regarded to be optimal. Many other transformations have been compared to it in the performance. Hotelling (1933) was the first to derive and publish this transformation under the name "principal components". This was a discrete transformation. Karhunen (1947) and Loeve (1948) have developed the corresponding continuous transformation. At present the discrete transformation is often called Karhunen–Loeve, too. In this transformation the basis vectors are eigenvectors of a signal covariance matrix. Thus, at first a covariance matrix is estimated by ECG data, afterwards the eigenvectors of this matrix are calculated. In order to calculate the transform coefficients one need to invert the matrix B formed of eigenvectors. Thus, this method is rather complicated, it requires many calculations, operations with large matrixes and, being optimal, it is useful for comparison with other methods (Poliakov, 1986). The point which makes the Karhunen–Loeve transformation experiments particularly suitable at this moment is the present availability of tools for the easy manipulation with large matrixes (Zywietz, 1989). The data compression is obtained because only $M (M < N)$ eigenvectors, that correspond to the M highest eigenvalues of the covariance matrix, are used to recover the signal. Many groups of researchers have applied Karhunen–Loeve transformation to various ECG data compression (Karlson, 1967; Womble, 1977; Hsu, 1981; Shakin, 1981; Poliakov, 1986; Marcus, 1987; Moody, 1989). Because of different error limitations and different data it is difficult to compare the results. Using Karhunen–Loeve transform for data compression CR was obtained from 5 to 19.

Walsh transformation (sometimes it is called Hadamard–Walsh) uses a fixed transformation matrix corresponding to the set of basis orthonormal functions. For example, for $N = 8$ the transformation matrix is

$$W_n = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \end{bmatrix}$$

This matrix corresponds to orthonomal rectangular basis functions described in Fig. 4.6.2.

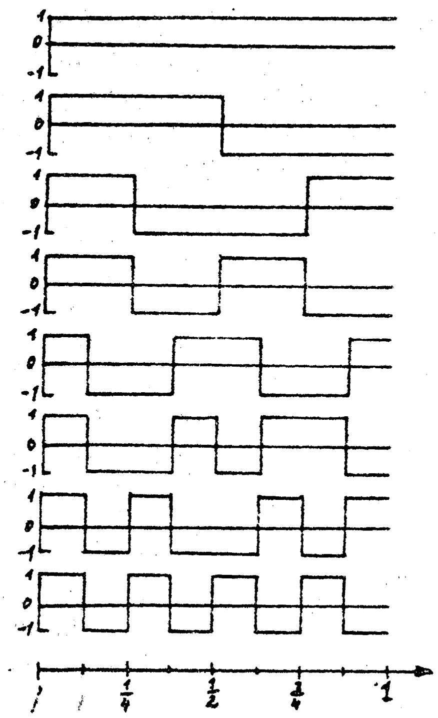

Fig. 4.6.2. The first eight Walsh functions.

The transformation is determined by equations

$$\vec{w} = \frac{1}{N} W_N \vec{x}, \qquad \vec{x} = W' \cdot \vec{w}. \tag{4.6.6}$$

Applications of the Walsh transformation to ECG described in the papers of Ahmed (1975) and Kuklinski (1983). CR is not reported. Though the implementation is feasible reconstructed signals are degraded.

The parametrical approximation and its application to ECG compression was developed by a group of authors (Poliakov, 1986).

The curve $g[\varphi_0(t)]$ (from a family of curves), which approximates ECG, is searched for ECG as for parametrical curve $x[\varphi(t)]$. Any of the transformations discussed earlier can be used for approximation but the authors used Fourier operator. The parametrical function $\varphi(t)$ is chosen satisfying the following condition:

$$\left|\frac{d}{dt}(g\varphi(t))\right| = \lambda\left|\frac{d}{dt}A_n\big[g(\varphi(t));\ t\big]\right|, \tag{4.6.8}$$

where λ is const. and λ_n is Fourier operator, i.e., $\varphi(t)$ is chosen so that the approximation not change the velocity of moving along the curve. Yet another choice of the parametrical function is possible, too. An algorithm of approximation is determined by three things:

1) a choice of an approximation operator;
2) a condition for the choice of a parametrical function;
3) an algorithm for the solution of (4.6.8).

The authors indicated that the application of the parametrical description increases the efficiency of transform coding. If a cardiocycle is determined by a fixed number of parameters the compression ratio possible is 20.

While the methods described in sections 2, 3, 4.1 – 4.5 can be applied to complete ECG records, the transform coding is applied to single representative cycles of ECGs. Since no objective criterion for the truncation of the series after M terms is given in many instances, it makes no sense to compare the values of CR possibly obtained to the values of M chosen by reconstructed signals of different quality. Besides, it would be better to estimate the overall bit gain Nn/Mn which takes into account both the number n of bits used to quantize the input samples and the number m of bits used to represent the output coefficients.

5. Conclusions. The decision what method is most suitable first of all depends on the further use of compressed and decompressed ECG data.

If the data are used in repeated quantitative analysis and serial comparison, then high resolution data are necessary.

If the data are used only for visual verification of the processing result , then the reproducibility requirements can be lowered.

It must be found a compromise between the requirements for accuracy and the cost of compression.

The redundancy reduction methods enable us to reproduce ECG so that errors of each sample do not exceed the preassigned threshold. Only these methods provide reproducible ECG records.

The methods of section 4 are good for Holter ECG. The requirements for resting ECG are greater so the redundancy reduction methods should be applied. For compression of resting ECG the methods described in 4.1 – 4.5 should be used only if the records are used for the visual verification of processing. The tolerable amplitude errors are in order of $\pm 40 - 50\mu V$ (0.5 mm) at $1mV/cm$ amplitude coding (Zywietz, 1989).

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