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Global Sensitivity Analysis of Infection Spread, Radar Search and Multiple Criteria Decision Models

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Abstract. In the present paper, the method of structure analysis for multivariate functions was applied to examine the global sensitivity of three complex models: the HIV/AIDS infection spread, radar search, and the multiple criteria decisions.

The investigation of global sensitivity exposed the most influential parameters or their groups. This knowledge makes it possible to concentrate efforts to obtain more exact values of these main parameters.

As a rule, only a small part of model parameters has a significant influence.

Key words: multivariate functions, models, sensitivity, structure, multicriterial, infection spread, radar search.

1. Introduction

The sensitivity of model outputs to simultaneous variations of several variables is very important. If the investigator finds that one or more of the variable values have a significant impact on the model outcome, an attempt may be made to reduce imprecision about these values.

Sensitivity is easily evaluated when only a single variable is at issue. When two or more parameters are under scrutiny, the issue is more complicated, because several parameters may not have a separable influence. The classic approaches to sensitivity analysis, which are predicated on local information, cannot be relied on to give global information. The global and nonlinear sensitivity analysis is mainly used by Wagner (1995) for mathematical programming models. The global sensitivity analysis was also investigated in (Cukier *et al.*, 1978) and (Abramov *et al.*, 1986).

The article suggests a different approach to sensitivity analysis of models (Šaltenis, 1989; Sobol', 1990). The structure characteristics of multidimensional functions are used to evaluate the global sensitivity in three practical complex models. These multiparameter models were elaborated and investigated by the author and his colleagues and were used to solve urgent practical problems as follows:

– investigation and forecasting of the HIV/AIDS infection spread and determination of optimal level of preventive means in Lithuania;

– investigation of radar search strategies of manoeuvring targets to minimize the average time until all targets are detected;

– multicriterial analysis of energy development alternatives and that of updating the automated control of the power system of Lithuania.

2. Decomposition into Components of Different Dimensionality and Structure Characteristics

Let the model be based on a function $f(x_1, \ldots, x_n)$, defined on the cube $K^n(0 \leqslant x_1 \leqslant 1, \ldots, 0 \leqslant x_n \leqslant 1)$. Decomposition into components of different dimensionality (Šaltenis, 1989; Sobol', 1990) and structural characteristics (Šaltenis, 1989) are the base for analyzing the structure.

We use a group of indices $i_1, \ldots, i_s$, where $1 \leqslant i_1 < \ldots < i_s \leqslant n,\ s = 1, \ldots, n$. Let us denote the sum with $2^n - 1$ terms as:

$$\sum^{\wedge} T_{i_1 \ldots i_s} = \sum_{i=1}^{n} T_i + \sum_{1 \leqslant i < j \leqslant n} \sum T_{ij} + \cdots + T_{12\ldots n}.$$

The decomposition of the objective function f

$$f = f_0 + \sum^{\wedge} f_{i_1 \ldots i_s}(x_{i_1}, \ldots x_{i_s})$$

is unique and orthogonal for each integrable on K^n function f, if f_0 is constant and

$$\int_0^1 f_{i_1 \ldots i_s}(x_{i_1}, \ldots x_{i_s})\, dx_{i_k} = 0, \quad 1 \leqslant k \leqslant s.$$

The system of structural characteristics for mutidimensional models

$$D_{i_1 \ldots i_s} = \int_0^1 \cdots \int_0^1 f^2_{i_1 \ldots i_s}\, dx_{i_1} \ldots dx_{i_s}$$

was proposed, investigated and applied in (Šaltenis, 1989). The characteristics are usually used after the normalization:

$$\sum^{\wedge} D_{i_1 \ldots i_s} = 1.$$

The structure characteristics D_i indicate the degree of influence of single variables and $D_{i_1 \ldots i_s}$ (interactions) indicate the degree of influence of the respective variable groups.

If we know the values of the function f for some points X^j $(j = 1, \dots, N)$, then the Monte-Carlo method may be used for evaluations of characteristics:

$$f_0 \approx \frac{1}{N}\sum_{j=1}^{N} f(X^j),$$

$$D + f(f_0)^2 \approx \frac{1}{N}\sum_{j=1}^{N} \left(f(X^j)\right)^2,$$

where $X^j = (x_1^j, \dots, x_n^j)$ are random points of dimensionality n, uniformly distributed in K^n.

Some part of coordinates (s coordinates $Y^j = (x_{i_1}, \dots, x_{i_s})$) must be identical in pairs of random points used for the evaluation of $D_{i_1 \dots i_s}$ (Sobol', 1990):

$$D_{i_1 \dots i_s} + (f_0)^2 \approx \frac{1}{N}\sum_{j=1}^{N} f(Y^j, Z^j) f(Y^j, U^j),$$

where Y^j are uniformly distributed random points of dimensionality s, Z^j and U^j are uniformly distributed random points of dimensionality $n - s$.

The global sensitivity analysis may conveniently use the structure characteristics as some measures of influence of parameters of a multidimensional model. Usually we analyse the function $f(x_1, \dots, x_n)$ not on the unit hypercube K^n. The domain of imprecision may be formed as a domain of possible variations $\delta_1, \dots, \delta_n$ (on both sides) from some typical parameter values $x_1', \dots, x_n'$:

$$x_1' - \delta_1 \leqslant x_1 \leqslant x_1' + \delta_1, \dots, x_n' - \delta_n \leqslant x_n \leqslant x_n' + \delta_n. \tag{1}$$

3. Investigation of the HIV/AIDS Infection Spread Model

3.1. *Model of Infection Spread*

The model of HIV/AIDS infection spread was proposed and investigated in (Dzemyda *et al.*, 1992; 1994, 1995, 1996). The choice of partners, transmission of the infection, processes of birth, death, and migration are represented in this model. The transition of individuals and spread of the infection within and among the risk groups (e.g., homosexual men, highly promiscuous heterosexual men and women, drug users) is evaluated. The model describes any risk group by three differential equations, which simulate the dynamics of active susceptible x_i, active infected y_i, and passive (not spreading the infection) infected individuals z_i:

$$\frac{dx_i}{dt} = u_i^x - (\mu^i + \mu_2^i)x_i - \frac{x_i}{x_i + y_i}\sum_{j=1}^{n} b_{ij}p_{ij}y_j, \tag{2}$$

$$\frac{dy_i}{dt} = u_i^y + \frac{x_i}{x_i + y_i}\sum_{j=1}^{n} b_{ij}p_{ij}y_j - (\mu^i + \mu_1 + \mu_2^i + k_i + k_a\nu^i)y_i, \tag{3}$$

$$\frac{dz_i}{dt} = u_i^z + (k_i + \mu_2^i + k_a\nu^i)y_i - (\mu_i + \mu_1)z_i, \quad i = 1, 2, \dots, n, \tag{4}$$

where n is the number of risk groups,

- u_i^x, u_i^y, u_i^z are the numbers of susceptible, active infected, and passive infected individuals, respectively, recruited into the group per time unit,
- μ^i, μ_1 are the mortality rates not due to AIDS and due to AIDS, respectively,
- μ_2^i is the rate at which an individual naturally becomes inactive (but does not die),
- k_i is the rate at which an active infected individual becomes inactive because of the positive results of blood test for HIV,
- k_a is the rate of transition from active infected to AIDS,
- ν^i are the proportions of individuals, terminating the infection spread after diagnosing HIV,
- b_{ij} is the averaged number of contacts of an individual from group j with the individuals from group i per time unit,
- p_{ij} is the probability of infecting a susceptible individual from group i by an infected one from group j per contact.

The balance of contacts must be satisfied: $(x_j + y_j) \cdot b_{ij} = (x_i + y_i) \cdot b_{ji}$. At the initial stage of the infection spread, p_{ij}, k_a, and μ_1 depend on time in the low HIV/AIDS prevalence countries because the percentage of individuals with later stages of HIV increases.

3.2. *Computational and Control Parameters*

It is not convenient to use all parameters of the model (2)–(4) directly. Some additional computational parameters are not difficult to estimate and they have a demographic and a medical meaning:

- U_i are the proportions of individuals recruited into the i-th group per year due to transition from immature to mature state,
- U_i^x are the numbers of susceptible individuals recruited into the i-th group per year due to migration from other countries,
- U_i^y are the numbers of infected but not disclosed individuals recruited into the i-th group per year due to migration from other countries,
- U_i^z are the numbers of infected and disclosed individuals recruited into the i-th group per year due to migration from other countries,
- $K^i$are the proportions of individuals tested for the AIDS per year,
- P_{ij} is the transmission probability per unprotected contact,
- α is a proportion of protected sexual contacts.

They are related with the parameters of the model in such a way:

$$u_i^x = U_i(x_i^0 + y_i^0 + z_i^0) + U_i^x,$$

$$u_i^y = U_i^y(1 - \nu^i)U_i^z,$$
$$u_i^z = \nu^i U_i^z,$$
$$k_i = \nu^i K^i,$$
$$p_{ij} = P_{ij}(1 + \alpha(r - 1)),$$

where r is the probability of protection (condom) failure (usually $r = 0.1$).

We must also set the initial values of model variables x_i, y_i and z_i at the beginning of the modelling period. Let us denote them by x_i^0, y_i^0 and z_i^0.

The infection spread in Lithuania was investigated for three risk groups: homo/bisexual men, promiscuous heterosexual men and women ($n = 3$). Oiler's method was used to solve the system of differential equations; the integration was performed by a one-month step. Then the model output is the total number of all infected at the end of the two-year period. It depends on 48 variables: $f(w_1, \dots, w_{48})$.

Table 1

Variables w_i, parameters of the model and their values

Variable	Parameter of model	Value of parameter	Variable	Parameter of model	Value of parameter
w_1	α	0.15	w_{25}	μ_2^1	0.0102
w_2	k_α	0.02	w_{26}	μ_2^2	0.0102
w_3	μ_1	0.01	w_{27}	μ_2^3	0.0191
w_4	U_1	0.024	w_{28}	x_1^0	93 000
w_5	U_2	0.024	w_{29}	x_2^0	216 000
w_6	U_3	0.022	w_{30}	x_3^0	168 000
w_7	U_1^x	-48	w_{31}	y_1^0	22
w_8	U_2^x	-141	w_{32}	y_2^0	60
w_9	U_3^x	-79	w_{33}	y_3^0	0
w_{10}	U_1^y	1	w_{34}	z_1^0	7
w_{11}	U_2^y	3	w_{35}	z_2^0	12
w_{12}	U_3^y	2	w_{36}	z_3^0	1
w_{13}	U_1^z	0.2	w_{37}	b_{11}	60
w_{14}	U_2^z	4	w_{38}	b_{12}	2
w_{15}	U_3^Z	2	w_{39}	b_{22}	1
w_{16}	K^1	0.065	w_{40}	b_{13}	5
w_{17}	K^2	0.065	w_{41}	b_{23}	95
w_{18}	K^3	0.065	w_{42}	b_{33}	0
w_{19}	ν^1	30	w_{43}	P_{11}	0.002
w_{20}	nu^2	30	w_{44}	P_{12}	0.002
w_{21}	ν^3	30	w_{45}	P_{22}	0.002
w_{22}	μ^1	0.0118	w_{46}	P_{13}	0.0008
w_{23}	μ^2	0.0118	w_{47}	P_{23}	0.0004
w_{24}	μ^3	0.0029	w_{48}	P_{33}	0

The correspondence of variables w_i and parameters of the model and their typical values is presented in Table 1.

Only seven parameters: α, K^i, and ν^i $(i = 1, 2, 3)$ of all may be controlled or influenced by prevention means. They may be called as control parameters.

3.3. *The Global Sensitivity Analysis of the Model*

The global sensitivity analysis of the model was made in the domain (1), where the variations of parameters $\delta_1, \ldots, \delta_n$ were equal to 5% of typical parameter values $x'_1, \ldots, x'_n$. The evaluation of each structure characteristic is based on 10–50 thousand calculations of the model. The results of the investigation in per cents are presented in Table 2 (single characteristics) and in Table 3 (interactions).

The characteristics not included in Tables are negligible.

It is obvious that the parameters mainly influencing the model results are: w_{31} and w_{32} (the initial numbers of active infected homo/bisexual and promiscuous heterosexual men). So the gratest effort must be put to obtain as exact as possible values of these parameters.

The interactions of the parameters were relatively small.

The influence of seven control parameters was investigated separately in a wide variation domain:

– α (per cent of protected sexual contacts) was changed from 10 to 90;

– K^i (per cents of individuals tested for the AIDS per year) were changed from 0 to 30;

– ν^i (the annual per cents of individuals, terminating the infection spread after diagnosing HIV) were changed from 10 to 60.

α was detected as the most influencing control parameter ($D_\alpha = 99.7\%$) so the protection of sexual contacts is obviously the unique infection prevention means.

Table 2

The largest single structure characteristics

D_{32}	D_{31}	D_{37}	D_{43}	D_{29}	D_{30}	D_{34}	D_{35}
68.1	17.1	2.8	2.1	1.3	0.8	0.7	0.1

Table 3

The largest double structure characteristics (interactions)

D_{37-43}	D_{31-32}	D_{31-34}	D_{34-35}
1.5	1.4	1.3	0.5

4. Investigation of a Radar Search Model

4.1. *Simulation of Radar Search Strategies*

The model (Šaltenis and Tiešis, 1992) is meant for investigating radar search strategies of manoeuvring targets in a three-dimensional space. The purpose of investigations is to minimize the average time until the moment at which all targets are detected with a given false alarm probability and the probability of detection.

Some assumptions were made on the search region and signals.

The space region in which the targets are searched is divided into a number of cells, because the technical abilities of radar equipment to separate two neighbouring targets are limited. We have $I = m_a m_b$ directions (where m_a is the number of cells along the bearing angle, m_b is the number of cells along the elevation angle) and m is the number of cells in each direction. We suppose that the number of targets is much smaller than the number of cells in the search region and that the targets are uniformly distributed and mutually independent in the region.

The radar set has a possibility to send a package of radio impulses of fixed power in each direction. The power of an impulse is constant and the time required for changing the search direction may be neglected. Therefore, the resource of a radar system may be characterised by the summary number of impulses.

The impulses sent in some direction may be reflected from the targets in each of m cells of this direction and the number of the cell may be detected. Stochastic independent noise (stationary Gaussian noise) is added to the reflected useful signal. The noise is uniform for all cells and has the Rayleigh envelope V with the density (Akimov *et al.*, 1989):

$$P_n(V, \sigma_1) = V/\sigma_1^2 \exp(-V^2/2\sigma_1^2), \tag{5}$$

where σ_1^2 is the variance of noise.

We also assume the reflected impulses to be received in a coherent way, therefore the summary amplitude A of the useful signal is equal to

$$A = tA_1,$$

where t is the number of sent impulses; A_1 is the amplitude of a single reflected impulse, which depends on the distance to the target. In the case of fixed A_1 this dependence is determined by the signal-to-noise ratio of power:

$$q = \frac{A_1^2}{2\sigma_1^2}.$$

The ratio q depends on the distance d to the target according to the equation:

$$q = (C_D/d)^4,$$

where C_D (coefficient of a transmitter) is a constant, depending on the characteristics of the atmosphere and targets, on the power of a transmitter. The envelope X of the reflected signal mixed with noise has the density corresponding to the generalized Rayleigh distribution law (Akimov, 1989; Levin, 1966):

$$p_s(X, \sigma, A) = (X/\sigma^2) \exp\left[- (X^2 + A^2)/2\sigma^2 \right] I_0(XA/\sigma^2), \tag{6}$$

where $\sigma^2 = t\sigma_1^2$, I_0 is a modified Bessel function (Abell *et al.*, 1992).

The reflected signal is detected by checking two statistical hypotheses: H_0 – the envelope of the reflected signal is distributed according to law (5) or H_1 – it is distributed according to law (6). The decision rule depends on the probability of the first order error Q_0 (false alarm probability) and that of the second order error β. The probability of detection is equal to $P_0 = 1 - \beta$.

We have a possibility to control some parameters of search strategies during the search time period. The parameters are:

– the search direction $i \in \{1, \ldots, I\}$;

– the number of impulses in the package, transmitted in the direction i;

– the probabilities P_0 and Q_0 of the direction i.

Two main approaches may be distinguished in the optimization of search strategies. The first is the static case where one can assume the targets to be motionless during the period of investigation. In this simplified case we were able to construct an analytical expression of the dependence of the average resource on search parameters and to optimize the parameters.

A more real dynamic case where the motion of targets is taken into account has been investigated by statistical simulation and optimal search parameters have been obtained.

4.2. *Dynamic Case. Search Region*

We assume that the targets change their space coordinates during the search period. We consider these changes to be greater than the size of space cells, however, on the other hand, the targets move only through some limited number of cells during the search period. Therefore, we may introduce two main assumptions on the movement of targets, that simplify the algorithms and programs of simulation. During the search period the targets do not change:

1) their altitudes;

2) direction of the movement.

The investigation of dynamic situations is carried out mainly by means of statistical simulation. The simulation of the target movement is the main part of the simulation system.

A horizontal projection of the search region $ABCD$ is presented in Fig. 1. A vertical one is similar (the figure is only turned clockwise in 90^0 and the elevation angle $\Delta\varepsilon$ is marked instead of the bearing angle $\Delta\alpha$).

The region is limited by four planes, stretching through the point O, and by two spherical surfaces with the centre at the same point O. The characteristics of the region are:

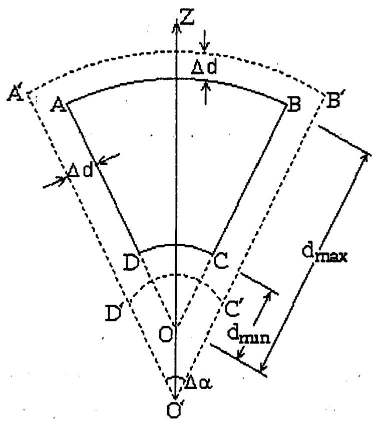

Fig. 1. A horizontal projection of the search region.

1) maximal distance $d_{\max}$;

2) minimal distance $d_{\min}$;

3) width along the bearing angle $\Delta\alpha$;

4) width along the elevation angle $\Delta\varepsilon$.

The horizontal plane divides the angle $\Delta\varepsilon$ in half.

The movements of targets are simulated only in the search region. But during the search period some new targets may appear from the surroundings of the region. Therefore, there arises a necessity of simulation of additional targets in the surroundings. The width δd of the surroundings must be chosen according to the maximal speed of targets and the search period. If we choose Δd too large, we shall have to simulate a great number of targets. Too small Δd may cause errors in the simulation.

4.3. *Generation of the Target Coordinates and the Direction of Movement*

We generate coordinates of the targets randomly and uniformly distributed in the region and its surroundings. The next algorithm of generation is proposed.

1. Uniformly distributed coordinates of the targets are generated both in the region and in the surroundings.

2. The coordinates are checked whether they are in the region.

3. The process of generation must stop as soon as the required number n of targets is generated in the region.

The generation in the region and surroundings uses new characteristics of the region, different from the four ones presented above. The differences are (see Fig. 1):

1) a new centre O' is used instead of the centre O;

2) a new maximal distance

$$d'_{\max} = d_{\max} + \Delta d\left(1 + \frac{1}{\sin\frac{\Delta\alpha}{2}}\right)$$

is used instead of $d_{\max}$.

The characteristics $d_{\min}$, $\Delta\alpha$, and $\Delta\varepsilon$ remain the same.

The generation of uniformly distributed random values α' and ε' is rather simple. The third coordinate, distance d', must be generated with the density increasing as a square of the distance. We use the following transformation of uniformly distributed in the interval (0,1) random number ξ:

$$d' = \sqrt[9]{d_{\min}^9 + \xi(d_{\max}^9 - d_{\min}^9)}.$$

These polar coordinates are calculated with respect to the centre O'. After simple trigonometric calculations we transform the coordinates to the centre O. The verification of polar coordinates whether they are in the region is equals to verification of such conditions:

$$-\frac{\Delta\alpha}{2} \leqslant \alpha \leqslant \frac{\Delta\alpha}{2}, \quad -\frac{\Delta\varepsilon}{2} \leqslant \varepsilon \leqslant \frac{\Delta\varepsilon}{2}, \quad d_{\min} \leqslant d \leqslant d_{\max}.$$

The movements in a relatively short period of search may take place only in the horizontal plane. The distribution of probabilities of movement directions must be nonuniform because the movements to the centre (the attack on the radar set) or out of the centre (the escaping manoeuvre) are more probable. The coefficient of nonuniformity C_n is the ratio between the largest and the smallest probabilities for the angles of movement directions. We use the next density of probabilities of the movement directions (angle φ):

$$p(\varphi) = \begin{cases} p_0 - \psi p_1, & \text{if } 0 \leqslant \psi < \frac{\pi}{2}, \\ p_0 - (\pi - \psi), & \text{if } \frac{\pi}{2} \leqslant \psi < \pi, \\ p_0 - (\psi - \pi)p_1, & \text{if } \pi \leqslant \psi < \frac{3\pi}{2}, \\ p_0 - (2\pi - \psi), & \text{if } \frac{3\pi}{2} \leqslant \psi < 2\pi, \end{cases}$$

$$\psi = \varphi - \alpha,$$

where α is the bearing angle of the target and

$$p_0 = \frac{C_n}{\pi(C_n + 1)},$$
$$p_1 = \frac{2(C_n - 1)}{\pi^2(C_n + 1)}.$$

Then we may generate random values of the angle φ

$$\varphi = P^{-1}(\xi),$$

where P^{-1} is the inverse function of $P(\varphi) = \int_0^{\varphi} p(x)\,dx$, ξ is a uniformly distributed random number in the interval (0,1), using the next formula:

$$\varphi = \begin{cases} \dfrac{p_0 - \sqrt{p_0^2 - 2p_1\xi}}{p_1}, & \text{if } \ 0 < \xi \leqslant 0.25, \\ \pi - \dfrac{p_0 - \sqrt{p_0^2 - 2p_1(0.5 - \xi)}}{p_1}, & \text{if } \ 0.25 < \xi \leqslant 0.5, \\ \pi + \dfrac{p_0 - \sqrt{p_0^2 - 2p_1(\xi - 0.5)}}{p_1}, & \text{if } \ 0.5 < \xi \leqslant 0.75, \\ 2\pi - \dfrac{p_0 - \sqrt{p_0^2 - 2p_1(1 - \xi)}}{p_1}, & \text{if } \ 0.75 < \xi \leqslant 1. \end{cases}$$

The velocity of targets is constant during the search period. Its value is random, limited by the maximal velocity $V_{\max}$.

During the simulation process we calculate the discrete coordinates of the moving targets at some moments of time. The process consists of the stages:

1) the orthogonal coordinates of targets are calculated on the base of:
   a) the coordinates at the time t=0;
   b) the velocity of the targets;
   c) the direction of the movement;
   d) time t from the beginning of simulation;
2) the coordinates are transformed to the polar coordinates;
3) the discrete coordinates, i.e., the numbers of cells are calculated.

The coordinates must be recalculated at each step during the simulation, because the targets may change their discrete coordinates, go out of the region or go into the region. A straightforward process of simulation requires much computer time for real radar search situations. We used the forecast of time moments when the targets change their discrete coordinates or go out of the region or go into the region. That saved the time of modelling.

4.4. *Global Sensitivity Analysis of the Model*

The model for investigating radar search strategies has numerous parameters because of the large number of search directions I (I = 900 in our case). So we restricted ourselves to the global sensitivity analysis only of four parameters of the model $f(x_1, \ldots, x_4)$. The strategy of changing search directions was fixed and independent of search results. The parameters analysed and their minimal and maximal values are presented in Table 4.

The results of investigation (the highest structure characteristics in per cents) are presented in Table 5. The evaluation of each characteristic is based on 10–50 thousand calculations of the model to provide reliable results.

The results show that the number of targets and the coefficient of a transmitter (power of the transmitter, atmosphere and target characteristics) and their interaction are the most influential parameters.

Table 4
The parameters of the model and their minimal and maximal values

Variable	Parameter of model	Minimal value of parameter	Maximal value of parameter
w_1	Number of targets n	1	3
w_2	Maximal velocity $V_{\max}$	1000	3000
w_3	Coefficient of nonuniformity C_n	1	3
w_4	Coefficient of transmitter C_D	250	300

Table 5
The highest structure characteristics

D_1	D_4	D_{1-4}
30.0	61.6	6.6

5. The Investigation of Multiple Criteria Decision Models

5.1. *Multiple Criteria Decision Support*

An optimal decision problem in the multiple criteria situation is complicated because it is impossible to optimize conflicting objectives simultaneously. The problem is to select the best alternative from a finite set of k alternatives. Any alternative is characterized by c criteria. The criteria may be expressed by some values x_{ij}, $i = 1, \ldots, k$, $j = 1, \ldots, c$ (e.g., capacity, power, weight), though verbal criteria (e.g., safety, feasibility, social effect) are possible, too. Usually multiple criteria decision support systems (DSS) are used to solve a multicriterial problem with the help of judges (experts, voters).

Three methods of an increasing complexity are proposed to be used in DSS (Dzemyda and Šaltenis, 1994; 1995; Dzemyda *et al.*, 1996): paired comparisons of alternatives (Saaty, 1977; Uppuluri, 1989), Pareto (Karpak *et al.*, 1989) and Fuzzy (Zhang Li Li *et al.*, 1992). The first method is the simplest one: the judge must only compare the alternatives, two at a time, and determine which of the two alternatives is better (or equivalent). The second method offers the Pareto subset of alternatives to the judge. The judge varies the weights of criteria and looks for the best alternative from this subset. The Fuzzy method is similar to that of Pareto, only the judge has an opportunity to doubt as to his opinion. Naturally, such an approach is most complicated. The system uses the following information from a specialized data base: names of alternatives, names and values of criteria, names of judges, intermediate dialogue actions of each judge, results of each judge, integral results.

5.2. *The Paired Comparisons Method*

The paired comparisons method is the simplest one from the user's point of view. The simplest case of the method is where a judge needs to determine only whether one alternative is more or less important (or equally important) than the other one.

Assume that the judge makes a simple decision and says the alternative i to be $a_{ij} > 0$ $(i = 1,\ldots,k-1;\ j = i,\ldots,k)$ times as important as alternative j. Thus the judge provides the $k \times k$ reciprocal matrix a_{ij} $(1 \leqslant i,j \leqslant k)$. There are two procedures to rank the alternatives on the basis of these data. The first of them suggests to rank the alternatives according to the values

$$w(i) = y_i / \sum_{j=1}^{k} y_j$$

of the i-th alternative, where $(y_1, y_2, \ldots, y_k)$ are the components of the eigenvector, associated with the highest eigenvalue of the reciprocal matrix a_{ij}.

The second is the logarithmic least squares procedure, based on the model:

$$\ln a_{ij} = \ln w(i) - \ln w(j) + \varepsilon_{ij}.$$

Let

$$g_1 = (a_{11}a_{12}\ldots a_{1k})^{1/k},$$
$$g_2 = (a_{21}a_{22}\ldots a_{2k})^{1/k},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$g_k = (a_{k1}a_{k2}\ldots a_{kk})^{1/k}.$$

Then the logarithmic least squares estimates of $w(i)$ are:

$$w(i) = g_i / \sum_{j=1}^{k} g_j.$$

We normalize the weights $w(i)$ in such a way that the largest weight is equal to one.

Each judge makes $\frac{k(k-1)}{2}$ simple decisions. Not all of them have the same influence on the final result of ranking. The aim of the global sensitivity analysis is to detect the main simple decisions.

5.3. *Energy Applications*

Topical energy problems in Lithuania are related to the nuclear safety and night power consumption of the Ignalina Nuclear Power Plant (NPP). Due to lower production costs and a higher reliability of fuel supply the Ignalina NPP has become the main producer of cheap energy. The Kruonis Pumped Storage Hydro Power Plant (PSHPP) operates jointly with the Ignalina NPP.

The choice of energy alternatives is complicated, therefore, the application of various decision support methods may increase the efficiency of solutions. The applications of DSS in (Dzemyda and Šaltenis, 1994; 1995; Dzemyda *et al.* 1996) were in the scope of decisions on the best energy development strategy for Lithuania. The alternatives covered various scenarios of nuclear plant development, fuel import, electricity export, environmental impact, and so on. Table 6 of alternatives illustrates the problem of reconstruction of the Kruonis PSHPP, Ignalina NPP and installation of the night electric heaters (Heaters).

Table 6

Description of alternatives

No. of alternative	Number of turbines in Kruonis PSHPP	Number of reactors in Ignalina NPP	Power of Heaters (MW)
1	2	2	–
2	3	2	–
3	2	2	400
4	–	2	800
5	4	2	–
6	–	1	–

Ten criteria (export of energy, fuel saving, degree of risk, and so on) were used by judges. The results of paired comparisons by one of experts and the weights of alternatives are presented in Table 7. The plus sign says that the alternative in a row is preferred over the alternative in the respective column; the minus sign has the opposite meaning; the plus-minus sign stands for equal alternatives.

Table 7

Results of paired comparisons and weights of alternatives

Number of alternative	1	2	3	4	5	6	Weight of alternative
1	*	+	±	±	+	+	1.00
2	–	*	+	±	+	+	0.88
3	±	–	*	+	±	+	0.75
4	±	±	–	*	+	–	0.50
5	–	–	±	–	*	+	0.38
6	–	–	–	+	–	*	0.25

5.4. *Global Sensitivity Analysis of the Model in Energy Application*

Each simple decision of judge a_{ij} may be treated as a parameter of the decision support model. Then we have 15 parameters with three possible values. The values in the sensitivity analysis were changed in such a way. The values corresponding to the simple decisions of Table 7 were changed to the nearest possible values with the probability 0.5. The outcome of the model was the weight of alternative No. 1 after the ranking of changed results of paired comparisons. The results of the investigation in per cents are presented in Table 8 (single characteristics) and in Table 9 (interactions).

Table 8

Results of sensitivity analysis (single characteristics)

	2	3	4	5	6
1	$D_1 = 12.6$	$D_2 = 23.0$	$D_3 = 17.1$	$D_4 = 7.1$	$D_5 = 6.5$
2		$D_6 = 0.1$	$D_7 = 1.8$	$D_8 = 0.8$	$D_9 = 1.6$
3			$D_{10} = 0.0$	$D_{11} = 0.8$	$D_{12} = 0.5$
4				$D_{13} = 0.0$	$D_{14} = 0.0$
5					$D_{15} = 0.0$

Table 9

Results of sensitivity analysis (interactions)

Interaction	Value of interaction	Interaction	Value of interaction
D_{1-2}	2.2	D_{2-4}	1.4
D_{1-3}	1.9	D_{2-5}	1.8
D_{1-4}	1.4	D_{3-4}	0.6
D_{1-5}	1.8	D_{3-5}	0.8
D_{2-3}	2.2	D_{4-5}	0.8

The largest structure characteristics are in the first row of Table 8. These paired comparisons correspond to the comparisons between the best alternative and others. So the simple decisions of judges when one of the alternatives is the best are crucial important.

5.5. *Multicriterial Analysis of the Alternatives of Updating the Automated Control of the Power System*

Multicriterial analysis of the alternatives of updating the automated control of the power system was made by means of DSS (Nemura and Šaltenis, 1996).

The existing Automated Dispatch Control System (ADCS) of the Power System of Lithuania is obsolete and inefficient. This implies the necessity of studies on modern dispatch control and the directions of updating. The following alternatives of updating the ADCS were considered:

1. ADCS is completely updated including the implementation of modern computers. The Lithuanian State Power System is considered as a control area.

2. Complete updating of ADCS, and the control area is realised in the Unified Baltic Power System.

3. ADCS has no updating.

4. ADCS is updated by implementing new programmable remote terminals as well as modern devices for receiving and transmitting data of telemeasurements and telesignals.

Nine criteria were used for estimating the alternatives. The four leading criteria were as follows:

- the cost of updating;
- the pay-off period of updating (economic efficiency criterion);
- the emission quantity of SO_2;
- the emission quantity of NO_x.

The results of paired comparisons by expert E1 and the weights of alternatives are presented in Table 10.

Table 10

Results of paired comparisons and weights of alternatives

Number of alternative	1	2	3	4	Weight of alternative
1	*	+	+	+	1.00
2	−	*	+	+	0.76
3	−	−	*	±	0.33
4	−	−	±	*	0.33

5.6. *The Global Sensitivity Analysis of the Model*

The global sensitivity analysis of the model is similar to the analysis of the previous model. We have four alternatives and six parameters. The results of the investigation in per cents are presented in Table 11 (single characteristics) and in Table 12 (interactions).

Table 11

Results of sensitivity analysis (single characteristics)

	2	3	4
1	$D_1 = 25.7$	$D_2 = 33.5$	$D_3 = 33.5$
2		$D_4 = 0.5$	$D_5 = 0.7$
3			$D_6 = 0.9$

As to the previous model we may see that the simple decisions of judges when one of the alternatives is the best are very important.

Table 12

Results of sensitivity analysis (interactions)

Interaction	Value of interaction	Interaction	Value of interaction
D_{1-2}	0.3	D_{2-5}	0.2
D_{1-3}	0.5	D_{2-6}	0.5
D_{1-4}	0.3	D_{3-4}	0.3
D_{1-5}	0.2	D_{3-6}	0.6
D_{1-6}	0.2	D_{4-5}	0.2
D_{2-3}	0.7	D_{4-6}	0.3
D_{2-4}	0.2	D_{5-6}	0.3

6. Conclusions

The investigation of global sensitivity using the method of structure analysis for various practical models exposed the most influential parameters or their groups. This knowledge makes it possible to concentrate efforts to obtain more exact values of these main parameters.

As a rule, only a small part of model parameters has a significant influence. The values of interactions are relatively small.

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References

Abell, M., and J. Braselton (1992). *The Mathematica Handbook.* Academic Press, London.

Abramov, V.I., A.P. Kartashev and A.S. Roshal' (1986). On one method of nonlinear sensitivity analysis of mathematical models. *Zurnal vytchisliteljnoj i matematitcheskoj fiziki*, **26**(3), 469–474 (in Russian).

Akimov, P.S. *et al.* (1989). *The Detection of Radio Signals.* Radio i Svyaz, Moscow, 288pp. (in Russian).

Cukier, R.I., H.B. Levine and K.E. Shuler (1978). Nonlinear sensitivity analysis of multiparameter model systems. *J. Comput. Phys.*, **26**(1), 1–42.

Dzemyda, G., and V. Šaltenis (1994). Multiple criteria decision support system: methods, user's interface and applications. *Informatica*, **5**(1–2), 31–42.

Dzemyda, G., and V. Šaltenis (1995). Optimization problems with non-precisely specified models. In J. Doležal and J. Fidler (Eds.), *Proceedings of the 3rd IFIP WG-7.6 Working Conference on Optimization-Based Computer-Aided Modelling and Design.* Institute of Information Theory and Automation, Prague, pp. 175–180.

Dzemyda, G., V. Šaltenis, V. Tiešis, S. Chaplinskas and A. Trechiokas (1992). HIV/AIDS infection spread: simulation and control. *Informatica*, **3**(4), 455–468.

Dzemyda, G., V. Šaltenis and V. Tiešis (1994). Means for the HIV/AIDS infection control. In *Proceedings of the First Asian Control Conference*, Vol. 1. Tokyo Metropolitan Institute of Technology, Tokyo, pp. 13–16.
Dzemyda, G., V. Šaltenis and V. Tiešis (1995). Optimization in simulation-based applications: approaches and software. In J. Dolezal and J. Fidler (Eds.), *Proceedings of the 3rd IFIP WG–7.6 Working Conference on Optimization-Based Computer-Aided Modelling and Design*. Institute of Information Theory and Automation, Prague, pp. 181–187.
Dzemyda, G., V. Šaltenis and V. Tiešis (1996). Decision making problems: AIDS prevention and energy development. In J. Doležal and J. Fidler (Eds.), *Proceedings of the 17th IFIP TC7 Conference on System Modelling and Optimization*. Chapman and Hall, pp. 317–324.
Karpak, B., and S. Zionts (Eds.) (1989). *Multiple criteria decision making and risk analysis using microcomputers, NATO ASI Series, Series F: Computer and Systems Sciences*, Vol. 56. Springer.
Levin, B.R. (1966). *The Theoretical Fundamentals of Statistical Radio Engineering*. Book first. Soviet Radio, Moscow (in Russian).
Nemura, A., and V. Šaltenis (1996). Multiobjective analysis of alternatives of updating the automated control of the Power System of Lithuania. *Information Technologies and Control*, **2**(3), 19–24.
Saaty, T.L. (1977). A scaling method for priorities in hierarchical structures. *J. Math. Psychology*, **15**, 234–281.
Sobol', I.M. (1990). On sensitivity estimation for nonlinear mathematical models. *Matematitcheskoje Modielirovanie* (Mathematical Modelling), **2**(1), 112–118 (in Russian).
Šaltenis, V. (1989). *Structure Analysis of Optimization Problems*. Vilnius, Mokslas Publishers, 123pp. (in Russian).
Šaltenis, V., and V. Tiešis (1992). Simulation and optimization of radar search strategies. *Informatica*, **3**(2), 256–274.
Uppuluri, V.R.R. (1989). Prioritization techniques based on stochastic paired comparisons. In B. Karpak and S. Zionts (Eds.), *Multiple Criteria Decision Making and Risk Analysis Using Microcomputers*. NATO ASI Series, Series F: *Computer and Systems Sciences*, Vol. 56. Springer, pp. 293–303.
Wagner, H. M. (1995). Global sensitivity analysis. *Operations Research*, **43**(6), 948–969.
Zhang Li Li and Chang Da Young (1992). Extent analysis and synthetic decision. In *Support Systems for Decision and Negotiation Processes*, Vol. 2. Warsaw, pp. 633–639.

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Infekcijos plitimo, radiolokacinės paieškos ir daugiakriterinių sprendimų modelių globali jautrumo analizė

Vydūnas ŠALTENIS

Daugelio kintamųjų funkcijų struktūros analizė taikyta tiriant trijų praktikoje aktualių daugiaparametrinių modelių globalinį jautrumą. Šie autoriaus ir jo kolegų sukurti ir tyrinėti modeliai apima: HIV/AIDS infekcijos plitimo prognozę, radiolokacinės paieškos valdymą, siekiant greičiau surasti visus judančius taikinius bei daugiakriterinių sprendimų priėmimą vystant energetiką.

Analizės rezultatai parodė, kad tik nedidelė tyrinėtų modelių parametrų dalis žymiau įtakoja modeliavimo rezultatą.