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OFF-LINE ESTIMATION OF DYNAMIC SYSTEMS PARAMETERS IN THE PRESENCE OF OUTLIERS IN OBSERVATIONS

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Abstract. In the previous papers (Pupeikis, 1990; 1991; 1992) the problems of model oder determination and recursive estimation of dynamic systems parameters in the presence of outliers in observations have been considered. The aim of the given paper is the development, in such a case, of classical off-line algorithms for systems of unknown parameters estimation using batch processing of the stored data. An approach, based on a substitution of the corresponding values of the sample covariance and cross-covariance functions by their robust analogues in respective matrices and on a further application of the least square (LS) parameter estimation algorithm, is worked out. The results of numerical simulation by IBM PC/AT (Table 1, 2) are given.

Key words: LS algorithm, covariance analysis, outlier, robustness.

1. Statement of the problem. By identification and parameter estimation of real objects it is often assumed that an additive noise affecting the output of a dynamic system is Gaussian. However in many cases this assumption is not valid because of outliers in the sample data set, used for system parameter estimation. That's why robust off-line algorithms, based on the calculation of M-estimates by processing the whole data set, are worked out (Novovičova, 1987). It is known, that these algorithms are iterative, stepwise procedures requiering an inversion of the corresponding matrices at each calculation step and some respective initial conditions. On the other hand in this case a robust covariance analysis and a ordinary classical LS algorithm can be used.

Consider a single input x_k and single output y_k linear discrete-time system described by the difference equation

$$y_k = -a_1 y_{k-1} - \ldots - a_n y_{k-n} + b_1 x_{k-1} + \ldots + b_n x_{k-n} \tag{1}$$

Suppose that y_k is observed with an additive noise ξ_k^*. i.e.,

$$u_k = y_k + \xi_k^*, \tag{2}$$

then

$$\begin{aligned} u_k = &- a_1 u_{k-1} - \ldots - a_n u_{k-n} + b_1 x_{k-1} + \ldots \\ &+ b_n x_{k-n} + \xi_k^* + a_1 \xi_{k-1}^* + \ldots a_n \xi_{k-n}^* \end{aligned} \tag{3}$$

or

$$u_k = \frac{B(z^{-1})}{1 + A(z^{-1})} x_k + W(z^{-1}; h)\xi_k, \tag{4}$$

by introducing the backward shift operator z^{-1} defined by $z^{-1}x_k = x_{k-1}$, where

$$\xi_k = (1 - \gamma_k)v_k + \gamma_k \eta_k \tag{5}$$

is a sequence of independent identically distributed variables with an ε – contaminated distribution of the form

$$p(\xi_k) = (1 - \varepsilon)N(0, \sigma_1^2) + \varepsilon N(0, \sigma_2^2), \tag{6}$$

$p(\xi_k)$ is a probability density distribution of the sequence ξ_k; γ_k is a random variable, taking values 0 or 1 with the probabilities $p(\gamma_k = 1) = \varepsilon$, $p(\gamma_k = 0) = 1 - \varepsilon$; v_k, η_k are sequences of independent Gaussian variables with zero means and variances σ_1^2, σ_2^2 respectively,

$$c^T = (a^T, b^T), \quad a^T = (a_1, \ldots, a_n), \quad b^T = (b_1, \ldots, b_n), \tag{7}$$

$$B(z^{-1}) = \sum_{i=1}^{n} b_i z^{-i}, \quad A(z^{-1}) = \sum_{i=1}^{n} a_i z^{-i} \tag{8}$$

n is the order of difference equation (1), respectively;

$$\xi_k^* = W(z^{-1};h)\xi_k, \tag{9}$$

$W(z^{-1};h)$ is a noise filter transfer function; h is a parameter vector.

It is assumed that the roots of $A(z^{-1})$ are outside the unit circle of the z^{-1} plane. The true orders of the polynomials $A(z^{-1})$, $B(z^{-1})$ are known. The input signal x_k is persistent excitation of an arbitrary order according to Åström and Eykhoff (1971).

Here we deal with the estimation of unknown parameters $c^T = (a^T, b^T)$ of difference equations (3) or (4) by means of the covariance analysis and an ordinary least squares (LS) algorithm in the presence of outliers in observations.

2. Parameter estimation in the absence of outliers in observations. Suppose that $\varepsilon = 0$ in equation (6), therefore $p(\xi_k) = N(0, \sigma_1^2)$. In this case, as it is shown in Åström and Eykhoff (1971); Kazlauskas and Pupeikis (1991) to estimate the vector of unknown parameters $c^T = (a^T, b^T)$ multivariate approaches are worked out. On the other hand, it is known that in the case when

$$W(z^{-1};h) = [1 + A(z^{-1})]^{-1}, \tag{10}$$

the ordinary classical LS parameter estimation algorithm is used. Then the vector $\hat{c}^T = (\hat{a}^T, \hat{b}^T)$ of the estimates

$$\hat{a}^T = (\hat{a}_1, \ldots, \hat{a}_n), \quad \hat{b}^T = (\hat{b}_1, \ldots, \hat{b}_n)$$

of the respective parameters (7) is calculated using the classical LS of the form

$$\hat{c} = (\phi^T \phi)^{-1} \phi^T U, \tag{11}$$

where

$$\phi^T \phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}, \tag{12}$$

$$\phi_{11} = \begin{pmatrix} R_u(0) & R_u(1) & \dots & R_u(n-1) \\ & R_u(0) & \dots & R_u(n-2) \\ & & \ddots & \vdots \\ & & & R_u(0) \end{pmatrix}, \tag{13}$$

$$\phi_{22} = \begin{pmatrix} R_x(0) & R_x(1) & \dots & R_x(n-1) \\ & R_x(0) & \dots & R_x(n-2) \\ & & \ddots & \vdots \\ & & & R_x(0) \end{pmatrix} \tag{14}$$

are $n \times n$ – symmetric submatrices;

$$\phi_{12} = \phi_{21} = \begin{pmatrix} -R_{ux}(0) & -R_{xu}(1) & \dots & -R_{xu}(n-1) \\ -R_{ux}(1) & -R_{ux}(0) & \dots & -R_{xu}(n-2) \\ \vdots & \vdots & & \vdots \\ -R_{ux}(n-1) & -R_{ux}(n-2) & \dots & -R_{ux}(0) \end{pmatrix} \tag{15}$$

are $n \times n$ submatrices.

$$\phi^T U = \begin{pmatrix} -R_u(1) \\ \vdots \\ -R_u(n) \\ R_{xu}(1) \\ \vdots \\ R_{xu}(n) \end{pmatrix} \tag{16}$$

is a $2n$ vector;

$$R_x(i) = \frac{1}{s-i} \sum_{k=1}^{s-i} (x_k - \bar{x})(x_{k-i} - \bar{x}), \quad i = \overline{0,m} \tag{17}$$

are values of the covariance function of input x_k;

$$R_u(i) = \frac{1}{s-i} \sum_{k=1}^{s-i} (u_k - \bar{u})(u_{k-i} - \bar{u}), \quad i = \overline{0,m} \tag{18}$$

are values of the covariance function of output u_k;

$$R_{ux}(i) = \frac{1}{s-i}\sum_{k=1}^{s-i}(u_k - \bar{u})(x_{k-i} - \bar{x}),$$

$$R_{xu}(i) = \frac{1}{s-i}\sum_{k=1}^{s-i}(x_k - \bar{x})(u_{k-i} - \bar{u}), \quad i = \overline{0,m} \tag{19}$$

are values of cross-covariance functions which are calculated using the sequences x_k and u_k of sample size s;

$$\bar{x} = s^{-1}\sum_{k=1}^{s} x_k, \qquad \bar{u} = s^{-1}\sum_{k=1}^{s} u_k.$$

Now let us consider such a case, when the assumption (10) is invalid. Then the classical LS of the form (11), used to estimate a vector $c^T = (a^T, b^T)$ of the unknown parameter of a mathematical model of the dynamic object (1) – (9), is of little use.

Let us assume that

$$W(z^{-1}; h) = \frac{1 + P(z^{-1})}{1 + R(z^{-1})}, \tag{20}$$

where

$$h^T = (p^T, r^T), \quad p^T = (p_1, \ldots, p_{n_p}), \quad r^T = (r_1, \ldots, r_{n_r}),$$

$$P(z^{-1}) = \sum_{i=1}^{n_p} p_i z^{-i}, \quad R(z^{-1}) = \sum_{i=1}^{n_r} r_i z^{-i},$$

n_p and n_r are orders known beforehand of an autoregressive moving average model (20).

In this case the LS algorithm based on the covariance analysis displays remarkable properties (Isermann, 1974), therefore this algorithm can be applied here. Now, if we multiply both sides of difference equation (3) by $x_{k-\tau}$, then we receive an equation of the form

$$\begin{aligned} R_{xu}(\tau) = &- a_1 R_{xu}(\tau-1) - a_2 R_{xu}(\tau-2) - \ldots \\ &- a_n R_{xu}(\tau-n) + b_1 R_x(\tau-1) + \ldots + b_n R_x(\tau-n) \\ &+ R_{x\xi}(\tau) + a_1 R_{x\xi}(\tau-1) + \ldots + a_n R_{x\xi}(\tau-n), \end{aligned} \tag{21}$$

where $R_{x\xi}(\cdot)$ is the cross-covariance function of x_k, ξ_k.

In (21) we choose τ from the interval

$$-p_1 \leqslant \tau \leqslant p_2,$$

where p_1 and p_2 are determined so, that when $\tau < -p_1$ the function $R_x(\tau) \cong \text{const}$, and when $\tau > p_2 - R_{xu}(\tau) \cong \text{const}$.

Then one can rewrite equation (21) rewrite in such a form

$$R = \Psi c + \omega, \tag{22}$$

where

$$R = \Big(R_{xu}(-p_1 + n) \ldots R_{xu}(-1) R_{xu}(0) \ldots R_{xu}(p_2)\Big)^T \tag{23}$$

is a $(p_1 + p_2 - n + 1)$ vector;

$$\Psi = \begin{pmatrix} -R_x(-p_1+n-1) & \ldots & -R_{xu}(-p_1) & R_x(-p_1+n-1) & \ldots & R_x(p_1) \\ \vdots & & \vdots & \vdots & & \vdots \\ -R_{xu}(-1) & \ldots & -R_{xu}(-n) & R_x(-1) & \ldots & R_x(-n) \\ -R_{xu}(0) & \ldots & -R_{xu}(1-n) & R_x(0) & \ldots & R_x(1-n) \\ \vdots & & \vdots & \vdots & & \vdots \\ -R_{xu}(-p_2-1) & \ldots & -R_{xu}(p_2-n) & R_x(p_2-1) & \ldots & R_x(p_2-n) \end{pmatrix} \tag{24}$$

is a $(p_1 + p_2 - n + 1) \times 2n$ covariance matrix;

$$\omega = \big(R_{x\xi}(-p_1 + n) \ldots R_{x\xi}(-1) R_{x\xi}(0) \ldots R_{x\xi}(p_2)\big)^T$$

is a $(p_1 + p_2 - n + 1)$ vector.

It should be mentioned that we determine the values $R_{xu}(-j)$ in the matrix (24) using formula (19) and the nonsymmetryc property, i.e.:

$$R_{xu}(-j) = R_{ux}(j),$$

for $j = \overline{0, m}$.

Thus, one can obtain the parameter estimates vector in the form

$$\hat{c} = (\Psi^T \Psi)^{-1} \Psi^T R. \tag{25}$$

3. Parameter estimation in the presence of outliers in observations. In equation (6) it was assumed that $\varepsilon = 0$. Now let us consider the case when this assumption is invalid. It is known (Novovičova, 1987) that in such a case M-estimates of unknown parameters of linear dynamical systems (1) – (10) can be calculated using three procedures:

1) the S-algorithm

$$\begin{aligned} \hat{c}^{(j+1)} =& \hat{c}^{(j)} + \hat{\sigma} \left[\sum_{t=1}^{o} \psi'(e_t^{(j)}/\hat{\sigma}) \varphi_t^{(j)} \varphi_t^{T(j)} \right]^{-1} \\ & \times \sum_{t=1}^{o} \psi(e_t^{(j)}/\hat{\sigma}) \varphi_t^{(j)}, \end{aligned} \tag{26}$$

2) the H-algorithm

$$\begin{aligned} \hat{c}^{(j+1)} =& \hat{c}^{(j)} + \hat{\sigma} \left[\sum_{t=1}^{o} \varphi_t^{(j)} \varphi^{T(j)} \right]^{-1} \\ & \times \sum_{t=1}^{o} \psi(e_t^{(j)}/\hat{\sigma}) \varphi_t^{(j)}, \end{aligned} \tag{27}$$

3) and the W-algorithm

$$\begin{aligned} \hat{c}^{(j+1)} =& \hat{c}^{(j)} + \hat{\sigma} \left[\sum_{t=1}^{o} w_t^{(j)} \varphi_t^{(j)} \varphi_t^{T(j)} \right]^{-1} \\ & \times \sum_{t=1}^{o} \psi(e_t^{(j)}/\sigma) \varphi_t^{(j)}. \end{aligned} \tag{28}$$

Here

$$\hat{c}^{T(j)} = (\hat{a}^T, \hat{b}^T)^{(j)} = (\hat{a}_1, \ldots, \hat{a}_n, \hat{b}_1, \ldots, \hat{b}_n)^{T(j)}$$

are the estimates of parameters (7), which are calculated at the j-th iteration using the abovementioned algorithms; $\hat{\sigma}$ is a scale value of the robust estimate; $\psi(e_t^{(j)}/\hat{\sigma})$ is a ψ-vector which can be chosen according to Stockinger and Dutter (1987), Novovičova (1987); whereas $\psi(v)/v$ is non-increasing for $v > 0$ and $\lim_{v \to 0} \psi(v)/v = \rho_0'' < \infty$; $\psi'(e_t^{(j)}/\hat{\sigma})$ is the first order partial derivative of the $\psi(e_t^{(j)}/\hat{\sigma})$

$$\varphi_t^{(j)} = (-u_{t-1}, \ldots, -u_{t-n} x_{t-1}, \ldots, x_{t-n})^{T(j)}$$

is the vector of n observations of input x_k and output u_k;

$$e_t^{(j)} = u_t - \varphi_t^{T(j)} \hat{c}^{(j)}$$

is a generalized equation error at the j-th iteration;

$$w_t^{(j)} = \begin{cases} \hat{\sigma}\psi(e_t^{(j)}/\hat{\sigma})/e_t^{(j)} & \text{for } e_t^{(j)} \neq 0 \\ \rho_0'' & \text{for } e_t = 0 \end{cases}.$$

The M-estimates, obtained by means of the $S-$, $H-$ and $W-$ algorithms, are solutions of the respective nonlinear equations requiring an inversion of the corresponding matrices at each step and some initial conditions. Moreover, the problem of stopping calculations of the M-estimates will arise here too. That is why we shall try to use in this case the robust covariance analysis and an ordinary LS algorithm for parameter estimation. It is known (Gnanadesikan and Kettenring, 1972; Hampel *et al.*, 1989; Huber, 1984) that equations (17) – (19) give then strongly biased estimates of sample covariance functions and therefore the estimates $\hat{c}^T = (\hat{a}^T, \hat{b}^T)$ of the parameters $c^T = (a^T, b^T)$, obtained using the LS algorithm, will be biased too. In order to increase its efficiency it is necessary to replace the respective averaging linear operators in matrices (13), (15), (24) and vectors (16), (23) by their nonlinear robust analogues according to Pupeikis (1990). For this purpose the values of the sample covariance and cross-covariance functions $R_u(0), R_u(1), \ldots, R_u(n-1), R_u(n), R_{ux}(0), R_{ux}(1), \ldots, R_{ux}(n-1), R_{ux}(n)$

are replaced in respective matrices by their robust analogues, i.e., $r(u_k^2)$, $r(u_k u_{k-1})$, ..., $r(u_k u_{k-n+1})$, $r(u_k u_{k-n})$, $r(u_k x_k)$, $r(u_k x_{k-1})$, ..., $r(u_k x_{k-n+1}), r(u_k x_{k-n})$.

Then, in equation (12)

$$\phi_{11} = \begin{pmatrix} r(u_k^2) & r(u_k u_{k-1}) & \dots & r(u_k u_{k-n+1}) \\ & r(u_k^2) & \dots & r(u_k u_{k-n+2}) \\ & & \ddots & \vdots \\ & & & r(u_k^2) \end{pmatrix},$$

$$\phi_{12} = \phi_{21} = \begin{pmatrix} -r(u_k x_k) & -r(x_k u_{k+1}) & \dots & -r(x_k u_{k+n+1}) \\ -r(u_k x_{k-1}) & -r(u_k x_k) & \dots & -r(x_k u_{k-n+2}) \\ \vdots & \vdots & & \vdots \\ -r(u_k x_{k-n+1}) & -r(u_k x_{k-n+2}) & \dots & -r(u_k x_k) \end{pmatrix}$$

and in equation (16)

$$\phi^T U = \begin{pmatrix} -r(u_k u_{k-1}) \\ \vdots \\ -r(u_k u_{k-n}) \\ r(x_k u_{k-1}) \\ \vdots \\ r(x_k u_{k-n}) \end{pmatrix}$$

On the other hand in equation (24) the matrix Ψ will be of the form

$$\Psi = \begin{pmatrix} -r(x_k u_{k+p_1-n+1}) & \dots & -r(x_k u_{k+p_1}) & R_x(-p_1+n-1) \\ \vdots & & \vdots & \vdots \\ -r(x_k u_{k+2}) & \dots & -r(x_k u_{k+1+n}) & R_x(-2) \\ -r(x_k u_{k+1}) & \dots & -r(x_k u_{k+n}) & R_x(-1) \\ -r(x_k u_k) & \dots & -r(x_k u_{k-1+n}) & R_x(0) \\ \vdots & & \vdots & \vdots \\ -r(x_k u_{k+p_2+1}) & \dots & -r(x_k u_{k-p_2+n}) & R_x(p_2-1) \end{pmatrix}$$

$$\begin{matrix} \dots & R_x(p_1) \\ & \vdots \\ \dots & R_x(-1-n) \\ \dots & R_x(-n) \\ \dots & R_x(1-n) \\ & \vdots \\ \dots & R_x(p_2-n) \end{matrix} \Bigg)$$

and the vector R of the form

$$R = \left(r(x_k u_{k+p_1-n}) \dots r(x_k u_{k+1}) r(x_k u_k) \dots r(x_k u_{k-p_2})\right)^T.$$

In this case various robust estimates of the corresponding covariance functions can be used (Gnanadesikan and Kettenring, 1972).

There are three approaches to $\hat{c}$ calculation according to Chen *et al.*, (1989). The first approach is based on solving the normal equation by Gaussian elimination or by forming the Cholesky decomposition of $\phi^T\phi$ in (11) or $\Psi^T\Psi$ in (25), the second one rests on an orthogonal decomposition of ϕ or Ψ and the third one – on a singular value decomposition of the same matrices. Each of these approaches has some advantages and disadvantages (Chen *et al.*, 1989). It ought to be mentioned that we prefer the last two approaches to the first one when the matrices $\phi^T\phi$ and $\Psi^T\Psi$ are ill-conditioned.

4. Simulation results. As an example we consider the discrete-time object of the form

$$u_k + 0.7 u_{k-1} = x_{k-1} + \xi_k, \tag{29}$$

where $c^T = (0.7, 1)$ are real parameters, whose estimates will be obtained using formula (11), whereas matrice (12) can be rewritten in the form

$$\phi^T\phi = \begin{pmatrix} R_u(0) & -R_{ux}(0) \\ -R_{ux}(0) & R_x(0) \end{pmatrix},$$

and vector (16) in the form

$$\phi^T U = s \begin{pmatrix} -R_u(1) \\ R_{xu}(1) \end{pmatrix}.$$

Then

$$s(\phi^T \phi)^{-1} = q^{-1} \begin{pmatrix} R_x(0) & R_{ux}(0) \\ R_{ux}(0) & R_u(0) \end{pmatrix},$$

and the vector $\hat{c}^T = (\hat{a}_1, \hat{b}_1)$ is of the shape

$$\begin{pmatrix} \hat{a}_1 \\ \hat{b}_1 \end{pmatrix} = q^{-1} \begin{pmatrix} -R_x(0)R_u(1) + R_{ux}(0)R_{xu}(1) \\ -R_{ux}(0)R_u(1) + R_u(0)R_{xu}(1) \end{pmatrix}, \tag{30}$$

where

$$q = R_x(0)R_u(0) - R_{ux}^2(0).$$

In order to calculate the robust estimates of a_1 and b_1 it is necessary to substitute the robust analogues in matrice (30) instead of the respective values of covariance and cross-covariance functions. Then we obtain

$$\begin{pmatrix} \hat{a}_1 \\ \hat{b}_1 \end{pmatrix} = q_r^{-1} \begin{pmatrix} -R_x(0)r(u_k u_{k-1}) + r(u_k x_k)r(x_k u_{k-1}) \\ -r(u_k x_k)r(u_k u_{k-1}) + r(u_k^2)r(x_k u_{k-1}) \end{pmatrix}, \tag{31}$$

where

$$q_r = R_x(0)r(u_k^2) - r(u_k x_k).$$

As robust analogues of the respective values of covariance and cross-covariance functions we choose here

$$r(u_k u_{k-1}) \equiv \operatorname{med}(\tilde{u}_k \tilde{u}_{k-1}) = \begin{cases} (\tilde{u}_k \tilde{u}_{k-1})_{\frac{s+1}{2}} & \text{for odd } s, \\ \frac{1}{2}\left[(\tilde{u}_k \tilde{u}_{k-1})_{\frac{s}{2}-1} + (\tilde{u}_k \tilde{u}_{k-1})_{\frac{s}{2}+1}\right] & \text{for even } s, \end{cases}$$

$$r(u_k x_k) \equiv \operatorname{med}(\tilde{u}_k \dot{x}_k) = \begin{cases} (\tilde{u}_k \dot{x}_k)_{\frac{s+1}{2}} & \text{for odd } s, \\ \frac{1}{2}\left[(\tilde{u}_k \dot{x}_k)_{\frac{s}{2}-1} + (\tilde{u}_k \dot{x}_k)_{\frac{s}{2}+1}\right] & \text{for even } s, \end{cases}$$

$$r(x_k u_{k-1}) \equiv \operatorname{med}(\dot{x}_k \tilde{u}_{k-1}) = \begin{cases} (\tilde{u}_k \dot{x}_{k-1})_{\frac{s+1}{2}} & \text{for odd } s, \\ \frac{1}{2}\left[(\dot{x}_k \tilde{u}_k)_{\frac{s}{2}-1} \right. \\ \quad \left. +(\dot{x}_k \tilde{u}_k)_{\frac{s}{2}+1}\right] & \text{for even } s, \end{cases}$$

$$r(u_k^2) \equiv \operatorname{med}(\tilde{u}_k^2) = \begin{cases} (\tilde{u}_k^2)_{\frac{s+1}{2}} & \text{for odd } s, \\ \frac{1}{2}\left[(\tilde{u}_k^2)_{\frac{s}{2}-1} \right. \\ \quad \left. +(\tilde{u}_k^2)_{\frac{s}{2}+1}\right] & \text{for even } s, \end{cases}$$

where

$$\tilde{u}_k = u_k - \operatorname{med}(u_k),$$
$$\dot{x}_k = x_k - \bar{x}_k,$$

$$\operatorname{med}(u_k) = \begin{cases} (u_k)_{\frac{s+1}{2}} & \text{for odd } s, \\ \frac{1}{2}\left[(u_k)_{\frac{s}{2}-1} \right. \\ \quad \left. +(u_k)_{\frac{s}{2}+1}\right] & \text{for even } s. \end{cases}$$

Realizations of independent Gaussian variables ζ_k with zero mean and unitary dispersion and the sequence of the second order AR model of the form

$$x_k = x_{k-1} - 0.5x_{k-2} + \zeta_k, \qquad k = \overline{1,100}, \tag{32}$$

were used as the input sequence. A realization of the discrete AR process was generated as the additive noise according to equation (10), where $A(z^{-1}) = 0.7z^{-1}$; ξ_k is a sequence of independent identically distributed variables of shape (5) with the ε – contaminated distribution (6) and $\sigma_1^2 = 1$, $\sigma_2^2 = 100$. Ten experiments with different realizations of noise ξ_k^* were carried out at the noise level $\lambda = \sigma_{\xi^*}^2/\sigma_y^2 = 0.1$. In each i-th experiment the estimates of parameters $a_1 = 0.7$ and $b_1 = 1$ of equation (29) were obtained using formulas (30), (31) and $s = 100$.

Table 1 illustrates the estimates $\hat{b}$ and $\hat{a}$ calculated for $\lambda = 0$ and averaged by 10 experiment values $\bar{b}$, $\bar{a}$ of the abovementioned parameters and their confidence intervals obtained by the formulas

$$\Delta_1 = \pm t_\alpha \frac{\hat{\sigma}_b}{\sqrt{L}},$$

$$\Delta_2 = \pm t_\alpha \frac{\hat{\sigma}_a}{\sqrt{L}}, \tag{33}$$

for $\varepsilon = 0.25$.

Here $\hat{\sigma}_b$ and $\hat{\sigma}_a$ are the estimates of the variances σ_b and σ_a, respectively, $\alpha = 0.05$ is the significance level; $t_\alpha = 2.26$ is the $100(1-\alpha)\%$ point of Student's t distribution with $\nu = L - 1$ degrees of freedom; $L = 10$ is the number of experiments.

In this connection in Table 1 the first line for each λ corresponds to the estimates, obtained by using formula (30) and the second one – to the estimates, obtained by applying formula (31).

Table 1. Estimates $\hat{b}$, $\hat{a}$ and averaged values $\bar{b}$, $\bar{a}$ and their confidence intervals (33) for different λ

λ	$\hat{b}$ and $\bar{b} \pm \Delta_1$	$\hat{a}$ and $\bar{a} \pm \Delta_2$
	1. $x_k \equiv \zeta_k$	
0	0.98	0.69
	0.38	0.50
0.1	0.97 ± 0.01	0.70 ± 0.01
	0.36 ± 0.01	0.48 ± 0.03
	2. x_k is the sequence of the form (32)	
0	0.99	0.71
	0.55	0.94
0.1	0.98 ± 0.02	0.71 ± 0.01
	0.52 ± 0.03	0.79 ± 0.04

It follows from the simulation results, presented in Table 1, that for different inputs and $\lambda = 0$ the accuracy of the estimates calculated by formula (30) will be higher. On the other hand, the accuracy of the averaged estimates calculated by formula (31) for $\lambda = 0.1$ will be not higher than that of the same estimates, obtained by formula (30).

Further we changed the observation u_{50} in the following way

$$u_{50}^{*} = u_{50} + 100|u_{50}| \tag{34}$$

and used it with the other observations in formulas (30) and (31).

In Table 2 the averaged by 10 experiments the estimates $\bar{b}$ and $\bar{a}$ and their confidence intervals, obtained by the formulas (33) and calculated for different inputs are given. The first line for different x_k corresponds to the estimates, obtained by using formula (30) and the second one – to the estimates, obtained by applying formula (31).

Table 2. Averaged values $\bar{b}$, $\bar{a}$ and their confidence intervals (33) for $\lambda = 0.1$ and u_{50}^* of the form (34)

	$\hat{b} \pm \Delta_1$	$\hat{a} \pm \Delta_2$
1.	$x_k \equiv \zeta_k$	
	-2.769 ± 0.163	0.011 ± 0.001
	0.326 ± 0.011	0.533 ± 0.024
2.	x_k is the sequence of the form (32)	
	-1.658 ± 0.277	0.024 ± 0.024
	0.505 ± 0.025	0.775 ± 0.037

From the simulation results, presented in Table 2, it follows, that the accuracy of the averaged estimates calculated by formula (31) will be higher than that of the same estimates, obtained by formula (30). That is why we prefer the approach, based on robust parameter estimation, to the classical one, when the noise, acting on the output of the dynamical system (4) has very large outliers. On the other hand for first order object (29) the ordinary classical LS parameter estimation algorithm shown its efficiency even in a case of ε – contaminated observations, when $\lambda = 0.1$.

5. Conclusions. The results of numerical simulation carried out by computer, prove the efficiency of the robust approach, based on a substitution of the corresponding values of sample covariance and cross-covariance functions by their robust analogues in respective matrices and on a further application of the ordinary classical LS parameter estimation algorithm. The above mentioned approach can be used in place of the iterative M-procedures.

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